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# On fuzzy resolvable functions

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**Abstract**

In this paper the notion of fuzzy resolvable functions between fuzzy topological spaces, is introduced and studied. The mathematical directions under which fuzzy functions defined between fuzzy spaces for being fuzzy resolvable functions, are obtained. By applying fuzzy resolvable functions, the mathematical means under which fuzzy topological spaces becoming fuzzy Baire spaces, fuzzy D-Baire spaces and fuzzy second category spaces, are explored.

**Keywords**

Fuzzy nowhere dense set, fuzzy resolvable set, fuzzy simply open set, fuzzy hyperconnected spaces, fuzzy Baire spaces, fuzzy P- spaces, fuzzy continuous function, Somewhere fuzzy continuous function.

**AMS Subject Classification**

54A40.

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## Contents



## 1. Introduction

The introduction of fuzzy sets by L.A. Zadeh [29] in 1965, as an approach to a mathematical representation of vagueness in everyday language, was realized by many researchers and has successfully been applied in every branch of Mathematics. Fuzzy topological spaces introduced by C. L. Chang [4] in 1968, had a significant role in the subsequent tremendous growth of the numerous fuzzy topological notions .

In the recent years, a considerable amount of research has been done on various types of fuzzy sets and many types of continuity between fuzzy topological spaces . The notion of resolvable sets in topological spaces were studied in [7]. Fuzzy resolvable sets in topological spaces was introduced and analyzed in [20]. In this article, a new type of fuzzy functions, namely, fuzzy resolvable functions are introduced and several characterizations of fuzzy resolvable functions are established. The mathematical directions through which the fuzzy functions defined between fuzzy spaces, for becoming fuzzy resolvable functions, are obtained. By applying fuzzy resolvable functions, the mathematical means through which fuzzy topological spaces becoming fuzzy Baire spaces, fuzzy D-Baire spaces and fuzzy second category spaces, are also explored in this paper.

## 2. Preliminaries

For the purpose of having the exposition self-contained, some basic concepts and results used in the sequel, are presented. In this paper, $(X,T)$ or simply $X$, means a fuzzy topological space due to Chang (1968). A fuzzy set $\lambda$ defined in $X$, is a mapping from the set $X$ into the unit interval $I=[0,1]$. The fuzzy set $0_X$ will be defined as $0_X(x)=0$, for all $x\in X$ and the fuzzy set $1_X$ will be defined as $1_X(x)=1$, for all $x\in X$.

**Definition 2.1.** *[4] Let $\lambda$ be any fuzzy set in the fuzzy topological space $(X,T)$. The fuzzy interior , the fuzzy closure and the fuzzy complement of $\lambda$ are defined respectively as follows :*

*(i)* $Int(\lambda)=\vee\{\mu/\mu\leq\lambda,\mu\in T\}$;

(ii) $Cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$.

(iii) $\lambda'(x) = 1-\lambda(x)$, *for all* $x \in X$.

*The notions union* $\chi = \vee_i(\lambda_i)$ *and intersection* $\delta = \wedge_i(\lambda_i)$, *are defined respectively, for the family* $\{\lambda_i/i \in I\}$ *of fuzzy sets in* $(X,T)$ *as follows :*

(iv) $\chi(x) = \sup_i\{\lambda_i(x)/x \in X\}$;

(v) $\delta(x) = \inf_i\{\lambda_i(x)/x \in X\}$.

**Lemma 2.2.** *[1] For a fuzzy set* $\lambda$ *of a fuzzy space X,*

(i) $1 - int(\lambda) = cl(1-\lambda)$ *and*

(ii) $1 - cl(\lambda) = int(1-\lambda)$.

**Definition 2.3.** *Let* $\lambda$ *be a fuzzy set in the fuzzy space* $(X,T)$. *Then* $\lambda$ *is called a*

(1) *Fuzzy regular-open if* $\lambda = intcl(\lambda)$; *Fuzzy regular-closed if* $\lambda = clint(\lambda)$ *[1].*

(2) *Fuzzy semi-open if* $\lambda \leq clint(\lambda)$; *Fuzzy semi-closed if int cl* $(\lambda) \leq \lambda$ *[1].*

(3) *Fuzzy* $\alpha$*-open if* $\lambda \leq intclint(\lambda)$*; Fuzzy* $\alpha$*-closed if cl int cl* $(\lambda) \leq \lambda$ *[3].*

(4) *Fuzzy* $G_\delta$*-set in* $(X,T)$ *if* $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, *where* $\lambda_i \in T$ *for* $i \in I$ *[2].*

**Definition 2.4.** *Let* $\lambda$ *be the fuzzy set in the fuzzy space* $(X,T)$. *Then* $\lambda$ *is called a*

(i) *Fuzzy dense set if there exists no fuzzy closed set* $\mu$ *in* $(X,T)$ *such that* $\lambda < \mu < 1$. *That is, cl* $(\lambda) = 1$, *in the fuzzy space* $(X,T)$*[11].*

(ii) *Fuzzy nowhere dense set if there exists no non-zero fuzzy open set* $\mu$ *in* $(X,T)$ *such that* $\mu < cl(\lambda)$. *That is, int* $[cl(\lambda)] = 0$, *in* $(X,T)$*[11].*

(iii) *Fuzzy first category set if* $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, *where* $(\lambda_i)'s$ *are fuzzy nowhere dense sets in* $(X,T)$. *Any other fuzzy set in* $(X,T)$ *is said to be of fuzzy second category [11].*

(iv) *Fuzzy Baire set if* $\lambda = \mu \wedge \delta$, *where* $\mu$ *is the fuzzy open set and* $\delta$ *is the fuzzy residual set in* $(X,T)$ *[9].*

(v) *Fuzzy resolvable set in* $(X,T)$ *if for each fuzzy closed set* $\mu$ *in* $(X,T), \{cl(\mu \wedge \lambda) \wedge cl(\mu \wedge [1-\lambda])\}$ *is a fuzzy nowhere dense set in* $(X,T)$ *[20].*

(vi) *Fuzzy somewhere dense set if int cl* $(\lambda) \neq 0$ *in* $(X,T)$ *[12].*

(vii) *Fuzzy simply open set if [ cl* $(\lambda) \wedge cl(1-\lambda)]$ *is the fuzzy nowhere dense set in the space* $(X,T)$ *[17].*

(viii) *Fuzzy simply* $\star$ *open set if* $\lambda = \mu \vee \delta$, *where* $\mu$ *is a fuzzy open set and* $\delta$ *is a fuzzy nowhere dense set in* $(X,T)$ *[18].*

(ix) *Fuzzy pseudo-open set if* $\lambda = \mu \vee \delta$ *where* $\mu$ *is the non-zero fuzzy open set and* $\delta$ *is the fuzzy first-category set in* $(X,T)$ *[19].*

**Definition 2.5.** *[14] Let* $\lambda$ *be the fuzzy first category set in the fuzzy topological space* $(X,T)$. *Then* $1-\lambda$ *is called the fuzzy residual set in* $(X,T)$.

**Definition 2.6.** *[21] If* $\lambda$ *is the fuzzy somewhere dense set in the fuzzy topological space* $(X,T)$, *then the fuzzy set* $1-\lambda$ *is called the fuzzy cs dense set in* $(X,T)$.

**Definition 2.7.** *Let* $(X,T)$ *be the fuzzy topological space and* $(X,T)$ *is called*

(i) *Fuzzy Baire space if int* $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, *where* $(\lambda_i)'s$ *are fuzzy nowhere dense sets in* $(X,T)$ *[14].*

(ii) *Fuzzy submaximal space if for each fuzzy set* $\lambda$ *in* $(X,T)$ *such that cl* $(\lambda) = 1, \lambda \in T$ *[2].*

(iii) *Fuzzy hyper-connected space if every non-null fuzzy open subset of* $(X,T)$ *is fuzzy dense in* $(X,T)$ *[8].*

(iv) *Fuzzy P-space if each fuzzy* $G_\delta$*-set in* $(X,T)$ *is fuzzy open in* $(X,T)$ *[10].*

(v) *Fuzzy globally disconnected space if each fuzzy semi-open set is fuzzy open in* $(X,T)$ *[27].*

(vi) *Fuzzy resolvable space if there exists fuzzy dense set* $\lambda$ *in* $(X,T)$ *such that cl* $(1-\lambda) = 1$. *Otherwise* $(X,T)$ *is called the fuzzy irresolvable space [13].*

(vii) *Fuzzy perfectly disconnected space if for any two non–zero fuzzy sets* $\lambda$ *and* $\mu$ *defined on X with* $\lambda \leq 1-\mu, cl(\lambda) \leq 1-cl(\mu)$, *in* $(X,T)$ *[26].*

(viii) *Fuzzy strongly irresolvable space if cl int* $(\lambda) = 1$ *for each fuzzy dense set* $\lambda$ *in* $(X,T)$ *[28].*

(ix) *Fuzzy D -Baire space if every fuzzy first -category set in* $(X,T)$, *is the fuzzy nowhere dense set in* $(X,T)$ *[16].*

**Definition 2.8.** *Let* $(X,T)$ *and* $(Y,S)$ *be any two fuzzy topological spaces. The function* $f:(X,T) \to (Y,S)$ *is called a*

(i) *Fuzzy continuous function if for the fuzzy open set* $\lambda$ *in* $(Y,S) f^{-1}(\lambda)$ *is fuzzy open in* $(X,T)$ *[4].*

(ii) *Fuzzy semi-continuous function if for the fuzzy open set* $\lambda$ *in* $(Y,S) f^{-1}(\lambda)$ *is fuzzy semi-open in* $(X,T)$ *[1].*

(iii) *Fuzzy contra-continuous function if* $f^{-1}(\lambda)$ *is fuzzy closed in* $(X,T)$, *for the fuzzy open set* $\lambda$ *in* $(Y,S)$ *[5].*

(iv) *Fuzzy Baire continuous function if for every non-zero fuzzy open set* $\lambda$ *in* $(Y,S), f^{-1}(\lambda)$ *is the fuzzy Baire set in* $(X,T)$ *[22].*

(v) *Somewhat fuzzy continuous function if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$, there exists a non-zero fuzzy open set $\mu$ in $(X,T)$ such that $\mu \leq f^{-1}(\lambda)$. That is, Int $[f^{-1}(\lambda)] \neq 0$, in $(X,T)$ [11].*

(vi) *Somewhere fuzzy continuous function, if whenever Int $(\lambda) \neq 0$, for the fuzzy set $\lambda$ defined on $Y$, then $f^{-1}(\lambda)$ is fuzzy somewhere dense in $(X,T)$ [21].*

(vii) *Fuzzy simply continuous function if $f^{-1}(\lambda)$ is the fuzzy simply open set in $(X,T)$, for the fuzzy open set $\lambda$ in $(Y,S)$ [17].*

(viii) *Fuzzy simply $\star$ continuous function if $f^{-1}(\lambda)$ is fuzzy the simply $\star$ open set in $(X,T)$, for a fuzzy open set $\lambda$ in $(Y,S)$ [18].*

(ix) *Fuzzy pseudo continuous function if $f^{-1}(\lambda)$ is the fuzzy pseudo-open set in $(X,T)$, for the fuzzy open set $\lambda$ in $(Y,S)$ [19].*

*(x). Fuzzy skeletal function if Int $\{f^{-1}[cl(\lambda)]\} \leq Cl f^{-1}(\lambda)$, for the fuzzy open set $\lambda \in S$ such that $f^{-1}(\lambda) \neq 0$ in $(X,T)$ [9].*

**Theorem 2.9.** *[1] In a fuzzy space X,*

(a) *The closure of a fuzzy open set is a fuzzy regular closed set.*

(b) *The interior of a fuzzy closed set is a fuzzy regular open set.*

**Theorem 2.10.** *[20] If $\lambda$ is the fuzzy resolvable set in the fuzzy topological space $(X,T)$, then $1-\lambda$ is also the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.11.** *[20] If $(\lambda_i)'s$ are fuzzy resolvable sets in the fuzzy topological space $(X,T)$, then $\wedge_{i=1}^{\infty}[\lambda_i \vee (1-\lambda_i) \vee \delta]$ (where $\delta \in T$) is the fuzzy residual set in $(X,T)$.*

**Theorem 2.12.** *[27] If $\lambda$ is the fuzzy nowhere dense set in the fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy closed set in $(X,T)$.*

**Theorem 2.13.** *[27] If $\lambda$ is the fuzzy semi-closed set in the fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy closed set in $(X,T)$.*

**Theorem 2.14.** *[27] If $\lambda$ is the fuzzy $\alpha$-open set in the fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy open set in $(X,T)$.*

**Theorem 2.15.** *[27] If $\lambda$ is the fuzzy dense and fuzzy $G_\delta$-set in the fuzzy strongly irresolvable and fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy open set in $(X,T)$.*

**Theorem 2.16.** *[27] If $\lambda$ is the fuzzy first category set in the fuzzy Baire, fuzzy strongly irresolvable and fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy closed set in $(X,T)$.*

**Theorem 2.17.** *[27] If Int $(\lambda) \neq 0$, for the fuzzy set $\lambda$ defined on X in the fuzzy globally disconnected and fuzzy hyperconnected space $(X,T)$, then $\lambda$ is the fuzzy open set in $(X,T)$.*

**Theorem 2.18.** *[9] If $\lambda$ is the fuzzy Baire set in a fuzzy globally disconnected and fuzzy P- space $(X,T)$, then $\lambda$ is the fuzzy open set in $(X,T)$.*

**Theorem 2.19.** *[24] If $\lambda$ is the fuzzy semi-open and fuzzy dense set in the fuzzy globally disconnected space $(X,T)$, then $\lambda$ is the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.20.** *[24] If $\lambda$ is the fuzzy set such that Int $(\lambda) \neq 0$ in the fuzzy globally disconnected and fuzzy hyperconnected space $(X,T)$, then $\lambda$ is the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.21.** *[20] If $\lambda$ is the fuzzy simply open set in the fuzzy topological space $(X,T)$, then $\lambda$ is the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.22.** *[18] If $f:(X,T) \to (Y,S)$ is the fuzzy simply $\star$ continuous function from the fuzzy hyper-connected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f$ is a fuzzy simply continuous function.*

**Theorem 2.23.** *[18] If $f:(X,T) \to (Y,S)$ is the fuzzy simply $\star$ continuous function from the fuzzy strongly irresolvable space $(X,T)$ into the fuzzy topological space $(Y,S)$ and each fuzzy simply $\star$ open set is fuzzy dense in $(X,T)$, then $f$ is a fuzzy simply continuous function.*

**Theorem 2.24.** *[19] If $\lambda$ is a fuzzy pseudo -open set in the fuzzy hyperconnected space $(X,T)$, then $\lambda$ is a fuzzy resolvable set in $(X,T)$.*

**Theorem 2.25.** *[9] If $f:(X,T) \to (Y,S)$ is a fuzzy continuous function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$ such that Int $[f^{-1}\{Bd(\lambda)\}] = 0$, for the fuzzy open set $\lambda$ in $(Y,S)$, then the function $f$ is the fuzzy simply continuous function.*

**Theorem 2.26.** *[26] If $\lambda$ is the fuzzy nowhere dense set in the fuzzy perfectly disconnected space $(X,T)$, then $\lambda = 0$, in $(X,T)$.*

**Theorem 2.27.** *[9] If $\lambda$ is the fuzzy Baire set in the fuzzy submaximal and fuzzy P-space $(X,T)$, then $\lambda$ is the fuzzy open set in $(X,T)$.*

**Theorem 2.28.** *[19] If $\lambda$ is the fuzzy pseudo -open set in the fuzzy hyperconnected space $(X,T)$, then $\lambda$ is the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.29.** *[18] If $\lambda$ is the fuzzy simply $\star$ open set in the fuzzy hyperconnected space $(X,T)$, then $\lambda$ is the fuzzy simply open set in $(X,T)$.*

**Theorem 2.30.** *[24] If $\lambda$ is the fuzzy Baire set in the fuzzy hyperconnected, fuzzy submaximal and fuzzy P-space $(X,T)$, then $\lambda$ is the fuzzy resolvable set in $(X,T)$.*

**Theorem 2.31.** *[25] If $\lambda$ is the fuzzy somewhere dense set in the fuzzy topological space $(X,T)$ in which fuzzy open sets are fuzzy resolvable sets, cl $[\lambda \vee (1-\lambda)] = 1$, in $(X,T)$.*

**Theorem 2.32.** *[6] Let $(X,T)$ be the fuzzy topological space. Then the following properties are equivalent :*

*(i) $(X,T)$ is fuzzy hyperconnected.*

*(ii) $1_X$ and $0_X$ are the only fuzzy regular open sets in $X$.*

**Theorem 2.33.** *[27] If $\lambda$ is the fuzzy nowhere dense set in the fuzzy globally disconnected space $(X,T)$, then $1-\lambda$ is the fuzzy dense and fuzzy open set in $(X,T)$.*

**Theorem 2.34.** *[28] If the fuzzy topological space $(X,T)$ is the fuzzy hyperconnected space, then $(X,T)$ is the fuzzy irresolvable space.*

**Theorem 2.35.** *[14] If $(X,T)$ is the fuzzy Baire space, then $(X,T)$ is the fuzzy second category space.*

**Theorem 2.36.** *[27] If $(X,T)$ is the fuzzy strongly irresolvable and fuzzy globally disconnected space, then $(X,T)$ is the fuzzy submaximal space.*

**Theorem 2.37.** *[9] If the function $f:(X,T)\to(Y,S)$ is the fuzzy continuous and fuzzy skeletal function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f$ is the fuzzy simply continuous function.*

## 3. Fuzzy resolvable functions

**Definition 3.1.** *[20] Let $(X,T)$ and $(Y,S)$ be any two fuzzy topological spaces. The function $f:(X,T)\to(Y,S)$ is called the fuzzy resolvable function, if for each fuzzy open set $\lambda$ in $(Y,S)$, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

**Example 3.1.** *Consider the set $X=\{a,b,c\}$. Let $I=[0,1]$ . The fuzzy sets $\alpha,\beta,\lambda$ and $\mu$ are defined on $X$ as follows:*

$$\alpha: X\to I \quad \text{is defined by} \quad \alpha(a)=0;\alpha(b)=0;\alpha(c)=0.6,$$
$$\beta: X\to I \quad \text{is defined by} \quad \beta(a)=1;\beta(b)=0.8;\beta(c)=0,$$
$$\lambda: X\to I \quad \text{is defined by} \quad \lambda(a)=0;\lambda(b)=0;\lambda(c)=0.7,$$
$$\mu: X\to I \quad \text{is defined by} \quad \mu(a)=0.7;\mu(b)=0;\mu(c)=0.$$

*Now $T=\{0,\alpha,\beta,\alpha\vee\beta,1\}$ and $S=\{0,\mu,1\}$ are fuzzy topologies for the set $X$. By computation, one can find that*

$$Int\,Cl([Cl\{(1-\alpha)\wedge\lambda\}]\wedge[Cl\{(1-\alpha)\wedge(1-\lambda)\}])=0,$$
$$Int\,Cl([Cl\{(1-\beta)\wedge\lambda\}]\wedge[Cl\{(1-\beta)\wedge(1-\lambda)\}])=0,$$
$$Int\,Cl([Cl\{(1-[\alpha\vee\beta])\wedge\lambda\}]\wedge[Cl\{(1-[\alpha\vee\beta])\wedge(1-\lambda)\}])=0.$$

*By the definition 2.4 (v), $\lambda$ is the fuzzy resolvable set in $(X,T)$. Now let us define a function $f:(X,T)\to(X,S)$ by $f(a)=b; f(b)=c; f(c)=a$. By computation, for the non-zero fuzzy open set $\mu$ in $(Y,S)$, $f^{-1}(\mu)=\lambda$, which is the fuzzy resolvable set in $(X,T)$. Hence $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function.*

**Remark 3.2.** *It is to be noted that fuzzy resolvable function need not be the fuzzy continuous function. For, in Example 3.1, for the non-zero fuzzy open set $\mu$ in $(Y,S)$, $f^{-1}(\mu)=\lambda$ and since $\lambda$ is not fuzzy open in $(X,T)$, the function $f:(X,T)\to(Y,S)$ is not the fuzzy continuous function from $(X,T)$ into $(Y,S)$.*

**Example 3.2.** *Consider the set $X=\{a,b,c\}$. Let $I=[0,1]$. The fuzzy sets $\alpha,\beta,\lambda$ and $\mu$ are defined on $X$ as follows :*

$$\alpha: X\to I \quad \text{is defined by} \quad \alpha(a)=0;\alpha(b)=0;\alpha(c)=0.6,$$
$$\beta: X\to I \quad \text{is defined by} \quad \beta(a)=1;\beta(b)=0.8;\beta(c)=0,$$
$$\lambda: X\to I \quad \text{is defined by} \quad \lambda(a)=0;\lambda(b)=0;\lambda(c)=0.7,$$
$$\mu: X\to I \quad \text{is defined by} \quad \mu(a)=0.7;\mu(b)=0.2;\mu(c)=0.$$
$$\delta: X\to I \quad \text{is defined by} \quad \delta(a)=0.2;\delta(b)=0;\delta(c)=0.7.$$

*Now $T=\{0,\alpha,\beta,\alpha\vee\beta,1\}$ and $S=\{0,\mu,1\}$ are fuzzy topologies for the set $X$. By computation, one can find that Cl $(\alpha)=1-\beta$, Cl $(\beta)=1-\alpha$ and Cl $(\alpha\vee\beta)=1$, Int$(1-\alpha)=\beta$, Int $(1-\beta)=\alpha$ and Int $(1-[\alpha\vee\beta])=0$, in $(X,T)$.*

$$Int\,Cl([Cl\{(1-\alpha)\wedge\lambda\}]\wedge[Cl\{(1-\alpha)\wedge(1-\lambda)\}])=0,$$
$$Int\,Cl([Cl\{(1-\beta)\wedge\lambda\}]\wedge[Cl\{(1-\beta)\wedge(1-\lambda)\}])=0,$$
$$Int\,Cl([Cl\{(1-[\alpha\vee\beta])\wedge\lambda\}]\wedge[Cl\{(1-[\alpha\vee\beta])\wedge(1-\lambda)\}])=0.$$

*By the definition 2.4 (v), $\lambda$ is the fuzzy resolvable set in $(X,T)$. Also by computation Int Cl ([cl $\{(1-\alpha)\wedge\delta\}]\wedge$ [cl $\{(1-\alpha)\wedge(1-\delta)\}]) = \beta\neq 0_X$, implies that $\delta$ is not the fuzzy resolvable set in $(X,T)$. Define the function $f:(X,T)\to(Y,S)$ by $f(a)=b; f(b)=c; f(c)=a$. On computation, for the non-zero fuzzy open set $\mu$ in $(Y,S)$, $f^{1-}(\mu)=\delta\neq\lambda$. Hence the function $f:(X,T)\to(Y,S)$ is not the fuzzy resolvable function.*

**Proposition 3.3.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S)$,*

*(i) $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))$ is the fuzzy nowhere dense set in $(X,T)$.*

*(ii) int [ $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))]=0$, where $\mu$ is fuzzy closed in $(X,T)$.*

*Proof.* (i) Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function. Then, for the non-zero fuzzy open set $\lambda$ in $(Y,S)$, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. This implies that for each fuzzy closed set $\mu$ in $(X,T)$, Int Cl [Cl $(\mu\wedge f^{-1}(\lambda))\wedge Cl(\mu\wedge(1-f^{-1})(\lambda))]=0$ in $(X,T)$. Now Cl $(\mu\wedge f^{-1}(\lambda))\wedge Cl(\mu\wedge(1-f^{-1}(\lambda))\geq Cl[(\mu\wedge f^{-1}(\lambda))\wedge(\mu\wedge(1-f^{-1}(\lambda))]$ and then, Cl $(\mu\wedge f^{-1}(\lambda))\wedge Cl(\mu\wedge(1-f^{-1}(\lambda))\geq Cl[(\mu\wedge f^{-1}(\lambda)\wedge(1-f^{-1}(\lambda))]$. Now Int Cl $\{$Cl $(\mu\wedge f^{-1}(\lambda))\wedge Cl(\mu\wedge(1-f^{-1}(\lambda))\}\geq IntCl\{Cl[(\mu\wedge f^{-1}(\lambda)\wedge(1-f^{-1}(\lambda))]\}=IntCl\{[(\mu\wedge f^{-1}(\lambda)\wedge(1-f^{-1}(\lambda))]\}$ and thus $0\geq intcl[(\mu\wedge f^{-1}(\lambda)\wedge(1-f^{-1}(\lambda))]$. That is, Int Cl $[(\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))]=0$. Hence $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))$ is the fuzzy nowhere dense set in $(X,T)$.

(ii). From (i), for the fuzzy open set $\lambda$ in $(Y,S)$, $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))$ is the fuzzy nowhere dense set in $(X,T)$.

Then , Int Cl $[\mu \wedge f^{-1}(\lambda \wedge (1-\lambda))] = 0$. But Int $[\mu \wedge f^{-1}(\lambda \wedge (1-\lambda))] \leq IntCl[\mu \wedge f^{-1}(\lambda \wedge (1-\lambda))]$ implies that Int $[\mu \wedge f^{-1}(\lambda \wedge (1-\lambda))] = 0$, in $(X,T)$. □

**Proposition 3.4.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S), \delta \vee f^{-1}[(1-\lambda) \vee \lambda]$ is a fuzzy dense set in $(X,T)$, where $\delta \in T$.*

*Proof.* Let $f:(X,T) \to (Y,S)$ be the fuzzy resolvable function. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by proposition 3.3, Int Cl $[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 0$, where $\mu$ is the fuzzy closed set in $(X,T)$. Then $1 - IntCl[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 1$. Thus, Cl Int $\{1-[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))]\} = 1$. This implies that Cl Int $[(1-\mu) \vee ((1-f)^{-1}(\lambda)) \vee f^{-1}(\lambda)] = 1$. But Cl Int $[(1-\mu) \vee ((1-f^{-1}(\lambda)) \vee f^{-1}(\lambda)] \leq Cl[(1-\mu) \vee ((1-f^{-1}(\lambda)) \vee f^{-1}(\lambda)]$. Then $1 \leq Cl[(1-\mu) \vee ((1-f^{-1}(\lambda)) \vee f^{-1}(\lambda)]$ and hence Cl$[(1-\mu \vee ((1-f^{-1}(\lambda) \vee f^{-1}(\lambda)] = 1$. Let $1-\mu = \delta$. Then, Cl $[\delta \vee ((1-f^{-1}(\lambda)) \vee f^{-1}(\lambda)] = 1$, where $\delta$ is the fuzzy open set in $(X,T)$. This means that cl $\{\delta \vee (f^{-1}[(1-\lambda) \vee \lambda])\} = 1$. Thus, for the fuzzy open set $\lambda$ in $(Y,S), \delta \vee f^{-1}[(1-\lambda) \vee \lambda]$ is the fuzzy dense set in $(X,T)$, where $\delta$ is the fuzzy open set in $(X,T)$. □

**Proposition 3.5.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S)$, Int $[f^{-1}(\lambda \wedge [1-\lambda])] \leq Cl(1-\mu)$, where $\mu$ is the fuzzy closed set in $(X,T)$.*

*Proof.* Let $f:(X,T) \to (Y,S)$ be the fuzzy resolvable function. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by the proposition 3.3 (ii), Int $[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 0$, where $\mu$ is the fuzzy closed set in $(X,T)$. Since Int $[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = Int(\mu) \wedge Int[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda)]$, $Int(\mu) \wedge Int[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 0$. Then, Int $[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] \leq 1 - Int(\mu)$ and this means that Int $[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] \leq Cl(1-\mu)$, for the fuzzy closed set $\mu$ in $(X,T)$. □

**Proposition 3.6.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S)$, there exists a fuzzy regular closed set $\gamma$ in $(X,T)$ such that Int $[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] \leq \gamma$.*

*Proof.* Let $f:(X,T) \to (Y,S)$ be the fuzzy resolvable function. By proposition 3.5, for the fuzzy open set $\lambda$ in $(Y,S)$, Int $[f^{-1}(\lambda \wedge [1-\lambda)] \leq Cl(1-\mu)$, where $\mu$ is the fuzzy closed set in $(X,T)$. Because $\mu$ is the fuzzy closed set, $1-\mu$ is the fuzzy open set in $(X,T)$. Let $1-\mu = \gamma$. By the theorem 2.9, $Cl(\gamma)$ is the fuzzy regular closed set in $(X,T)$. Thus, for the fuzzy open set $\lambda$ in $(Y,S)$, there exists a fuzzy regular closed set $\gamma$ in $(X,T)$ such that Int $[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] \leq \gamma$. □

**Proposition 3.7.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S)$,*

*(i) there exists a fuzzy regular open set $\delta$ in $(X,T)$ such that $\delta \leq Cl[f^{-1}\{(1-\lambda) \vee \lambda\}]$.*

*(i) $f^{-1}\{(1-\lambda) \vee \lambda\}$ is the fuzzy somewhere dense set in $(X,T)$.*

*Proof.* (i). Let $f:(X,T) \to (Y,S)$ be the fuzzy resolvable function. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by proposition 3.3 (ii), Int $[\mu \wedge f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 0_X$, in $(X,T)$. This implies that Int $(\mu) \wedge Int[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))] = 0$. Then Int $(\mu) \leq 1 - Int[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))]$, in $(X,T)$. This implies that Int $(\mu) \leq Cl\{1-[f^{-1}(\lambda) \wedge (1-f^{-1}(\lambda))]\}$. Hence Int $(\mu) \leq Cl\{[1-f^{-}(\lambda)] \vee f^{-1}(\lambda)\}$ in $(X,T)$. Since $\mu$ is the fuzzy closed set, by theorem 2.9, Int $(\mu)$ is the fuzzy regular open set in $(X,T)$. Let int $(\mu) = \delta$. Then, $\delta \leq Cl[(1-f^{-1}(\lambda)) \vee f^{-1}(\lambda)] = Cl[\{f^{-1}(1-\lambda)\} \vee f^{-1}(\lambda)] = cl[f^{-1}\{(1-\lambda) \vee \lambda\}]$. Hence if $f$ is a fuzzy resolvable function, there exists a fuzzy regular open set $\delta$ in $(X,T)$ such that $\delta \leq Cl[f^{-1}\{(1-\lambda) \vee \lambda\}]$.

(ii). Since each fuzzy regular open set is the fuzzy open set in the fuzzy topological space, the fuzzy regular open set $\delta$ is the fuzzy open set in $(X,T)$. Then, Int (Cl $[f^{-1}\{(1-\lambda) \vee \lambda\}]) \neq 0$ and hence $f^{-1}\{(1-\lambda) \vee \lambda\}$ is the fuzzy somewhere dense set in $(X,T)$. □

**Proposition 3.8.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$ and if $\lambda$ is the fuzzy open set in $(Y,S)$, then $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy open set in $(Y,S)$. Since $f:(X,T) \to (Y,S)$ is a fuzzy resolvable function, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Then, by the theorem 2.10, $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 3.9.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$ and if $(\lambda_i)'s$ are fuzzy open sets in $(Y,S)$, then $\wedge_{i=1}^{\infty}[f^{-1}\{\lambda_i \vee (1-\lambda_i)\} \vee \delta]$ is the fuzzy residual set in $(X,T)$, where $\delta \in T$.*

*Proof.* Let $(\lambda_i)'$s $(i = 1 to \infty)$ be fuzzy open sets in $(Y,S)$. Since the function $f:(X,T) \to (Y,S)$ is fuzzy resolvable, $f^{-1}(\lambda_i)]'$s are fuzzy resolvable sets in $(X,T)$. By the theorem 2.11, $\wedge_{i=1}^{\infty}[f^{-1}(\lambda_i) \vee (1-f^{-1}\{\lambda_i\}) \vee \delta]$ is the fuzzy residual set in $(X,T)$. Now $f^{-1}(\lambda_i) \vee (1-f^{-1}\{\lambda_i\}) = f^{-1}\{\lambda_i \vee (1-\lambda_i)\}$. Hence $\wedge_{i=1}^{\infty}[f^{-1}(\lambda_i) \vee (1-f^{-1}\{\lambda_i\}) \vee \delta]$ is the fuzzy residual set in $(X,T)$, where $\delta \in T$. □

**Proposition 3.10.** *If $f:(X,T) \to (Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$ and $g:(Y,S) \to (Z,W)$ is the* *fuzzy continuous function from $(Y,S)$ into the fuzzy topological space $(Z,W)$, then $g \circ f : (X,T) \to (Z,W)$ is the fuzzy resolvable function from $(X,T)$ into $(Z,W)$.*

*Proof.* Let $\lambda$ be the non-zero fuzzy open set in $(Z,W)$. Since $g$ is the fuzzy continuous function from $(Y,S)$ into $(Z,W), g^{-1}(\lambda)$ is the fuzzy open set in $(Y,S)$. Since $f$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S), f^{-1}(g^{-1}(\lambda))$ is the fuzzy resolvable set in $(X,T)$. Hence $(g \circ f)^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$, for the fuzzy open set $\lambda$ in $(Z,W)$. Therefore $g \circ f$ is the fuzzy resolvable function from $(X,T)$ into $(Z,W)$. □

It is to be analyzed that whether the fuzzy continuous function from the fuzzy topological space into another fuzzy topological space need necessarily be the fuzzy resolvable function. For, consider the following example:

**Example 3.3.** *Consider the set $X = \{a,b,c\}$. Let $I = [0,1]$. The fuzzy sets $\lambda$ and $\mu$ are defined on the set X as follows:*

$$\lambda : X \to I \quad \text{is defined by} \quad \lambda(a) = 0; \lambda(b) = 0.2; \lambda(c) = 0.2,$$
$$\mu : X \to I \quad \text{is defined by} \quad \mu(a) = 0; \mu(b) = 0.2; \mu(c) = 0.7.$$

*Now $T = \{0,\lambda,1\}$ and $S = \{0,\mu,1\}$ are fuzzy topologies on $X$. By computation, one can find that Cl $(\lambda) = 1-\lambda$ in $(X,T)$. Now for the fuzzy set $\lambda, Cl\{(1-\lambda)\wedge\lambda\} \wedge Cl\{(1-\lambda)\wedge(1-\lambda)\} = Cl(\lambda)\wedge Cl(1-\lambda) = (1-\lambda)\wedge(1-\lambda) = (1-\lambda)$ and Int Cl (Cl $\{(1-\lambda)\wedge\lambda\}] \wedge [Cl\{(1-\lambda)\wedge(1-\lambda)\}]) = IntCl(1-\lambda) = Int(1-\lambda) = 1-Cl(\lambda) = 1-[1-\lambda] = \lambda \neq 0$, in $(X,T)$. This implies that $\lambda$ is not the fuzzy resolvable set in $(X,T)$.*

*Now define the function $f : (X,T) \to (X,S)$ by $f(a) = a; f(b) = b; f(c) = b$. On computation, for the non-zero fuzzy open set $\mu$ in $(Y,S), f^{-1}(\mu) = \lambda$, the fuzzy open set in $(X,T)$. Hence $f : (X,T) \to (Y,S)$ is the fuzzy continuous function. But $f : (X,T) \to (Y,S)$ is not the fuzzy resolvable function since $f^{-1}(\mu)$ is not the fuzzy resolvable set in $(X,T)$. Thus, the fuzzy continuous function need not be the fuzzy resolvable function.*

**Proposition 3.11.** *If $f : (X,T) \to (Y,S)$ is the fuzzy simply continuous function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f : (X,T) \to (Y,S)$ is the fuzzy resolvable function.*

*Proof.* Let $f : (X,T) \to (Y,S)$ be the fuzzy simply continuous function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$. Now for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy simply open set in $(X,T)$. By theorem 2.21, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$ and hence $f : (X,T) \to (Y,S)$ is the fuzzy resolvable function. □

**Remark 3.12.** *It is to be noted that the fuzzy resolvable function between fuzzy topological spaces need not be the fuzzy simply continuous function. For, consider the following example:*

**Example 3.4.** *Consider the set $X = \{a,b,c\}$. Let $I = [0,1]$. The fuzzy sets $\alpha,\beta,\lambda$ and $\mu$ are defined on X as follows :*

$$\alpha : X \to I \text{ is defined by} \quad \alpha(a) = 1; \alpha(b) = 1; \alpha(c) = 0.4,$$
$$\gamma : X \to I \text{ is defined by} \quad \gamma(a) = 0; \gamma(b) = 0.2; \gamma(c) = 1.$$
$$\lambda : X \to I \text{ is defined by} \quad \lambda(a) = 0.3; \lambda(b) = 0; \lambda(c) = 0.3,$$
$$\mu : X \to I \text{ is defined by} \quad \mu(a) = 0.7; \mu(b) = 0.5; \mu(c) = 0.6.$$
$$\beta : X \to I \text{ is defined by} \quad \beta(a) = 0.5; \beta(b) = 0.5; \beta(c) = 0.5.$$

*Now $T = \{0,\alpha,\gamma,\alpha\wedge\gamma,1\}$ and $S = \{0,\lambda,\mu,1\}$ are fuzzy topologies for $X$. By computation , one can find that*

$$Int\,Cl([Cl\{(1-\alpha)\wedge\beta\}] \wedge [Cl\{(1-\alpha)\wedge(1-\beta)\}]) = IntCl(1-\alpha) = Int(1-\alpha) = 0,$$
$$Int\,Cl([Cl\{(1-\gamma)\wedge\beta\}] \wedge [Cl\{(1-\gamma)\wedge(1-\beta)\}])) = IntCl(1-\gamma) = Int(1-\gamma) = 0.$$

*This means that $\beta$ is the fuzzy resolvable set in $(X,T)$. Define the function $f : (X,T) \to (X,S)$ by $f(a) = b; f(b) = b; f(c) = b$. By computation, for the non-zero fuzzy open sets $\lambda,\mu$ in $(X,S), f^{-1}(\lambda) = 0, f^{-1}(\mu) = \beta$ which are fuzzy resolvable sets in $(X,T)$. Hence $f : (X,T) \to (X,S)$ is the fuzzy resolvable function. Now Int Cl (Bd $[\beta]) = IntCl(cl[\beta]\wedge cl[1-\beta] = IntCl([1-(\alpha\wedge\gamma)])\wedge[1-(\alpha\wedge\gamma)]) = IntCl([1-(\alpha\wedge\gamma)]) = \alpha\wedge\gamma \neq 0_X$ and then $\beta$ is not the fuzzy simply open set in $(X,T)$. Hence $f : (X,T) \to (X,S)$ is not the fuzzy simply continuous function.*

The conditions for fuzzy functions between fuzzy topological spaces to become fuzzy resolvable functions are given in the following propositions.

**Proposition 3.13.** *If $f : (X,T) \to (Y,S)$ is the fuzzy function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$ and if each fuzzy set is the fuzzy simply open set in $(X,T)$, then $f : (X,T) \to (Y,S)$ is the fuzzy resolvable function.*

*Proof.* Let $f : (X,T) \to (Y,S)$ be the function from $(X,T)$ into $(Y,S)$ and each fuzzy set defined on $X$ be the fuzzy simply open set in $(X,T)$. Then, for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy set in $(X,T)$. By hypothesis, $f^{-1}(\lambda)$ is the fuzzy simply open set in $(X,T)$ and thus Int Cl [ Bd $f^{-1}(\lambda)] = 0$. This means that Int Cl $\{Cl[f^{-1}(\lambda)] \wedge Cl[1-f^{-1}(\lambda)]\} = 0$ in $(X,T)$. Now , for the fuzzy closed set $\mu$ in $(X,T), Cl[f^{-1}(\lambda)\wedge\mu] \wedge Cl\{[1-f^{-1}(\lambda)]\wedge\mu\} \leq Cl[f^{-1}(\lambda)]\wedge Cl[1-f^{-1}(\lambda)]$. Hence Int Cl $\{Cl[f^{-1}(\lambda)\wedge\mu] \wedge Cl[1-f^{-1}(\lambda)\wedge\mu]\} \leq IntCl\{cl[f^{-1}(\lambda)]\wedge Cl[1-f^{-1}(\lambda)]\}$ in $(X,T)$. This means that Int Cl $\{Cl[f^{-1}(\lambda)\wedge\mu]\wedge Cl([1-f^{-1}(\lambda)])\wedge\mu]\} \leq 0$. Thus, Int Cl $\{Cl[f^{-1}(\lambda)\wedge\mu]\wedge Cl([1-f^{-1}(\lambda)]\wedge\mu]\} = 0$ and hence $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Thus the function $f : (X,T) \to (Y,S)$ is the fuzzy resolvable function. □

**Proposition 3.14.** *If $f : (X,T) \to (Y,S)$ is the fuzzy contra-continuous function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$ and if the interior of each fuzzy closed set is zero in $(X,T)$, then $f : (X,T) \to (Y,S)$ is the fuzzy resolvable function.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy contra –continuous function and $\lambda$ be the fuzzy open set in $(Y,S)$. Then, $f^{-1}(\lambda)$ is the fuzzy closed set in $(X,T)$. By hypothesis, Int $[f^{-1}(\lambda)]=0$, in $(X,T)$. Now, for the fuzzy closed set $\mu$ in $(X,T), \{Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))\}\leq Cl[f^{-1}(\lambda)]\wedge Cl[1-f^{-1}(\lambda)]$. Then $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge[\mu\wedge(1-f^{-1}(\lambda))\}\leq Cl[f^{-1}(\lambda)]\wedge\{1-Int(f^{-1}(\lambda))\}$ in $(X,T)$. Since Int $(f^{-1}(\lambda))=0, Cl[f^{-1}(\lambda)]\wedge\{1-Int(f^{-1}(\lambda))\}=Cl[f^{-1}(\lambda)]\wedge[1-0]=Cl[f^{-1}(\lambda)]$. Since $f^{-1}(\lambda)$ is the fuzzy closed set, Cl $[f^{-1}(\lambda)]=f^{-1}(\lambda)$ and then $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))\}\leq f^{-1}(\lambda)$. Now Int Cl $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge[\mu\wedge(1-f^{-1}(\lambda))\}\leq IntCl[f^{-1}(\lambda)]=Int[f^{-1}(\lambda)]=0$. Thus, for each fuzzy closed set $\mu$ in $(X,T)$, Int Cl $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))\}=0$, implies that $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$ and hence $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function. □

**Remark 3.15.** *The fuzzy resolvable function between fuzzy topological spaces need not be the fuzzy contra –continuous function.*

For, in example 3.1, $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, but $f$ is not the fuzzy contra–continuous function, since for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\mu)=\lambda$, which is not the fuzzy closed set in $(X,T)$.

**Proposition 3.16.** *If $f:(X,T)\to(Y,S)$ is the fuzzy continuous function from the fuzzy topological space $(X,T)$ into the fuzzy topological space $(Y,S)$ such that Int $[f^{-1}\{Bd(\lambda)\}]=0$, for the fuzzy open set $\lambda$ in $(Y,S)$, then $f$ is the fuzzy resolvable function.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy continuous function from $(X,T)$ into $(Y,S)$ such that Int $[f^{-1}\{Bd(\lambda)\}]=0$, for the fuzzy open set $\lambda$ in $(Y,S)$. Then, by the theorem 2.25, $f$ is the fuzzy simply continuous function and by the proposition 3.11, $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function. □

**Proposition 3.17.** *If $f:(X,T)\to(Y,S)$ is the fuzzy simply $\star$ continuous function from the fuzzy strongly irresolvable space $(X,T)$ into the fuzzy topological space $(Y,S)$ and each fuzzy simply $\star$ open set is fuzzy dense in $(X,T)$, then $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy simply $\star$ continuous function from the fuzzy strongly irresolvable space $(X,T)$ into the fuzzy topological space $(Y,S)$. By hypothesis, each fuzzy simply $\star$ open set is fuzzy dense in $(X,T)$. Then, by the theorem 2.23, $f$ is the fuzzy simply continuous function. and by the proposition 3.11, $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function. □

The conditions for the fuzzy topological spaces to become fuzzy Baire spaces under fuzzy resolvable functions are given in the following propositions:

**Proposition 3.18.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ onto another fuzzy topological space $(Y,S)$ and if Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)])=0$, where $(\lambda_i)'$ s are fuzzy open sets in $(Y,S)$, then*

*(i) Int $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]))=0$ in $(X,T)$.*

*(ii) $(X,T)$ is the fuzzy Baire space.*

*Proof.* (i). Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. Then, by the proposition 3.3, for fuzzy open sets $(\lambda_i)'$ s in $(Y,S), (\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])$'s are fuzzy nowhere dense sets in $(X,T)$, where $\mu$ is the fuzzy closed set in $(X,T)$. Now $\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]\leq f^{-1}[\lambda_i\wedge(1-\lambda_i)]$, means that $\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])\leq\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)]$. Then Int $\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])\leq int\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)]$. By hypothesis, Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)])=0$ and this implies that Int $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]))=0$ in $(X,T)$.

(ii). From (i), $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]))=0$, in $(X,T)$. Since $(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])'$s are fuzzy nowhere dense sets in $(X,T), (X,T)$ is the fuzzy Baire space. □

**Proposition 3.19.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$ and if Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)])=0$, where $(\lambda_i)'$s are fuzzy open sets in $(Y,S)$, then $(X,T)$ is a fuzzy second category space.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$ and $(\lambda_i)'$s be fuzzy open sets in $(Y,S)$. By hypothesis, Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i\wedge(1-\lambda_i)])=0$. Then, by proposition 3.18, $(X,T)$ is the fuzzy Baire space. Since each fuzzy Baire space is the fuzzy second category space [by theorem 2.35 ], $(X,T)$ is the fuzzy second category space. □

**Proposition 3.20.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$ and if Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i])=0$, where $(\lambda_i)'$s are fuzzy open sets in $(Y,S)$, then $(X,T)$ is the fuzzy Baire space.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. Then, by the proposition 3.3, for fuzzy open sets $(\lambda_i)'$s in $(Y,S), (\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])'$ s are fuzzy nowhere dense sets in $(X,T)$, where $\mu$ is the fuzzy closed set in $(X,T)$. Now $\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]\leq\mu\wedge f^{-1}[\lambda_i]\wedge f^{-1}[1-\lambda_i]\leq f^{-1}[\lambda_i]$ and then, Int $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])\leq Int(\vee_{i=1}^{\infty}f^{-1}[\lambda_i])$. By hypothesis, Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i])=0$ and this means that Int $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])=0$, where $(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)])$'s are fuzzy nowhere dense sets in $(X,T)$. Hence $(X,T)$ is the fuzzy Baire space. □

**Proposition 3.21.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into another fuzzy topological space $(Y,S)$ and if Int $(\vee_{i=1}^{\infty}f^{-1}[\lambda_i])=0$, where $(\lambda_i)'$s are fuzzy open sets in $(Y,S)$, then $(X,T)$ is the fuzzy second category space.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. By hypothesis, Int $(\vee_{i=1}^{\infty} f^{-1}[\lambda_i]) = 0$, where $(\lambda_i)'$s are fuzzy open sets in $(Y,S)$. Then, by the proposition 3.20, $(X,T)$ is the fuzzy Baire space. By the theorem 2.35, each fuzzy Baire space is the fuzzy second category space and hence $(X,T)$ is the fuzzy second category space. □

**Proposition 3.22.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ onto another fuzzy topological space $(Y,S)$ and if $(\lambda_i)'$s are fuzzy open sets in $(Y,S)$, $Cl(\wedge_{i=1}^{\infty}[\delta\vee f^{-1}[\lambda_i\vee(1-\lambda_i)]]) = 1$, where $\delta\in T$.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ onto $(Y,S)$ and $(\lambda_i)'$s be fuzzy open sets in $(Y,S)$. Then, by proposition 3.18 (i), Int $(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]) = 0$. This gives that $1-Int(\vee_{i=1}^{\infty}(\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)] = 1-0 = 1$. Then, Cl $(1-(\vee_{i=}^{\infty}(\mu\wedge f^{-1}\lambda_i\wedge(1-\lambda_i)]) = 1$ and Cl $(\wedge_{i=1}^{\infty}(1-\{\mu\wedge f^{-1}[\lambda_i\wedge(1-\lambda_i)]\}) = 1$. This gives that Cl $(\wedge_{i=1}^{\infty}[(1-\mu)]\vee(1-f^{-1}[\lambda_i\wedge(1-\lambda_i)]))] = 1$ in $(X,T)$ and this implies that Cl $(\wedge_{i=1}^{\infty}[(1-\mu)]\vee f^{-1}[\lambda_i\vee(1-\lambda_i)]]) = 1$. Let $\delta = 1-\mu$. Hence Cl $(\wedge_{i=1}^{\infty}\delta\vee f^{-1}[\lambda_i\vee(1-\lambda_i)]]) = 1$, where $\delta\in T$. □

The condition for the fuzzy continuous function between fuzzy topological spaces to be fuzzy resolvable function , is given in the following proposition:

**Proposition 3.23.** *If $f:(X,T)\to(Y,S)$ is the fuzzy continuous and fuzzy skeletal function from the fuzzy topological space $(X,T)$ into a fuzzy topological space $(Y,S)$, then $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function.*

*Proof.* The proof follows from the theorem 2.37 and the proposition 3.11. □

## 4. Fuzzy resolvable functions and fuzzy globally disconnected spaces

**Proposition 4.1.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy globally disconnected space $(Y,S)$. Then,*

*(i) If $\lambda$ is the fuzzy nowhere dense set in $(Y,S)$, then $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*(ii) If $\lambda$ is the fuzzy semi-closed set in $(Y,S)$, then $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* (i). Let $\lambda$ be the fuzzy nowhere dense set in $(Y,S)$. Because $(X,T)$ is the fuzzy globally disconnected space, by the theorem 2.12, $\lambda$ is the fuzzy closed set in $(X,T)$ and hence $1-\lambda$ is the fuzzy open set in $(X,T)$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function , $f^{-1}(1-\lambda)$ is the fuzzy resolvable set in $(X,T)$. Now $f^{-1}(1-\lambda) = 1-f^{-1}(\lambda)$, implies that $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.

(ii). Let $\lambda$ be the fuzzy semi-closed set in $(Y,S)$. Because $(X,T)$ is the fuzzy globally disconnected space, by the theorem 2.13, $\lambda$ is the fuzzy closed set in $(X,T)$ and hence $1-\lambda$ is the fuzzy open set in $(X,T)$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, $f^{-1}(1-\lambda)$ is the fuzzy resolvable set in $(X,T)$ and hence $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.2.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy globally disconnected space $(Y,S)$ and $\lambda$ is the fuzzy $\alpha$-open set in the fuzzy globally disconnected space $(Y,S)$, then $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy $\alpha$-open set in $(Y,S)$. Since $(Y,S)$ is the fuzzy globally disconnected space, by the theorem 2.14, $\lambda$ is the fuzzy open set in $(Y,S)$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.3.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy strongly irresolvable and fuzzy globally disconnected space $(Y,S)$ and $\lambda$ is the fuzzy dense and fuzzy $G_\delta$-set in $(Y,S)$, then $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy dense and fuzzy $G_\delta$-set in $(Y,S)$. Since $(Y,S)$ is the fuzzy strongly irresolvable and fuzzy globally disconnected space, by the theorem 2.15, $\lambda$ is the fuzzy open set in $(Y,S)$. Because $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.4.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy Baire, fuzzy strongly irresolvable and fuzzy globally disconnected space $(Y,S)$ and $\lambda$ is the fuzzy first category set in $(Y,S)$, then $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Proof :*

*Proof.* Let $\lambda$ be the fuzzy first category set in $(Y,S)$. Because $(X,T)$ is the fuzzy Baire, fuzzy strongly irresolvable and fuzzy globally disconnected space, by the theorem 2.16, $\lambda$ is the fuzzy closed set in $(X,T)$ and hence $1-\lambda$ is the fuzzy open set in $(X,T)$. Because $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, $f^{-1}(1-\lambda)$ is the fuzzy resolvable set in $(X,T)$. This implies $1-f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.5.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy globally disconnected and fuzzy hyper-connected space $(Y,S)$ and for the fuzzy set $\lambda$ defined on $Y$ Int $(\lambda)\neq 0$ in $(Y,S)$, then $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy set defined on $Y$ such that Int $(\lambda)\neq 0$ in $(Y,S)$. Because $(Y,S)$ is the fuzzy globally disconnected and fuzzy hyper-connected space, by the theorem 2.17, $\lambda$ is the fuzzy open set in $(Y,S)$. Because $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function , $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.6.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy globally disconnected and fuzzy P-space $(Y,S)$, then for the fuzzy Baire set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy Baire set in the fuzzy globally disconnected and fuzzy P-space $(X,T)$. Then, by the theorem 2.18, $\lambda$ is the fuzzy open set in $(X,T)$. Because $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 4.7.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy semi-continuous function from the fuzzy globally disconnected space into the fuzzy topological space $(Y,S)$ and each fuzzy semi-open set is fuzzy dense in $(X,T)$, then $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy semi-continuous function from the fuzzy globally disconnected space $(X,T)$ into a fuzzy topological space $(Y,S)$. Now, for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy semi-open set in $(X,T)$. By hypothesis the fuzzy semi-open set is fuzzy dense in $(X,T)$ and hence $f^{-1}(\lambda)$ is the fuzzy semi-open and fuzzy dense set in the fuzzy globally disconnected space $(X,T)$. Then, by the theorem 2. 19, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Hence, for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$, means that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. □

**Proposition 4.8.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from the fuzzy globally disconnected space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S),[\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))]$ is the fuzzy closed set in $(X,T)$.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by the proposition 3.3, $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))$ is the fuzzy nowhere dense set in $(X,T)$. Because $(X,T)$ is the fuzzy globally disconnected space, by the theorem 2.33, $1-[\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))]$ is the fuzzy dense and fuzzy open set in $(X,T)$. Then, $[\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))]$ is the fuzzy closed set in $(X,T)$. □

**Proposition 4.9.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy globally disconnected space $(Y,S)$, then,*

*(i) If $\lambda$ is the fuzzy nowhere dense set in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*(ii) If $\lambda$ is the fuzzy semi-closed set in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* The result follows from the proposition 4.1 and the theorem 2.10. □

## 5. Fuzzy resolvable functions and fuzzy hyperconnected spaces

The condition for fuzzy continuous functions between fuzzy topological spaces to become fuzzy resolvable functions, is given in the following proposition :

**Proposition 5.1.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$. Now, for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy open set in $(X,T)$. Because $(X,T)$ is the fuzzy hyperconnected space, the fuzzy open set $f^{-1}(\lambda)$ is the fuzzy dense set in $(X,T)$ an thus Cl $(f^{-1}(\lambda))=1$, in $(X,T)$. Now for the fuzzy closed set $\mu$ in $(X,T), Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))]\leq[Cl(\mu)\wedge Cl(f^{-1}(\lambda))]\wedge[Cl(\mu)\wedge Cl(1-f^{-1}(\lambda))]=[\mu\wedge 1]\wedge[\mu\wedge Cl(1-f^{-1}(\lambda))]=\mu\wedge[\mu\wedge Cl(1-f^{-1}(\lambda))]=\mu\wedge Cl(1-f^{-1}(\lambda))]=\mu\wedge[1-f^{-1}(\lambda)]$, in $(X,T)$. Then, Int Cl $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))\}=IntCl\{\mu\wedge[1-f^{-1}(\lambda)]\}\leq Int\{Cl(\mu)\wedge Cl[1-f^{-1}(\lambda)]\}=Int\{\mu\wedge[1-f^{-1}(\lambda)]\}=Int(\mu)\wedge Int[1-f^{-1}(\lambda)]\}=Int(\mu)\wedge\{1-Cl[f^{-1}(\lambda)]\}=Int(\mu)\wedge(1-1)=Int(\mu)\wedge 0=0$. That is, Int Cl $\{Cl[\mu\wedge f^{-1}(\lambda)]\wedge Cl[\mu\wedge(1-f^{-1}(\lambda))\}=0$. Thus $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Hence the function $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. □

**Example 5.1.** *Consider the set $X=\{a,b,c\}$. Let $I=[0,1]$. The fuzzy sets $\lambda,\mu,\alpha$ and $\beta$ are defined on $X$ as follows :*

$$\lambda:X\rightarrow I \text{ is defined by} \lambda(a)=0.5;\lambda(b)=0.6;\lambda(c)=0.6,$$
$$\mu:X\rightarrow I \text{ is defined by} \mu(a)=0.5;\mu(b)=0.7;\mu(c)=0.6,$$
$$\alpha:X\rightarrow I \text{ is defined by} \alpha(a)=0.6;\alpha(b)=0.5;\alpha(c)=0.6,$$
$$\beta:X\rightarrow I \text{ is defined by} \beta(a)=0.6;\beta(b)=0.5;\beta(c)=0.7.$$

*Now $T=\{0,\lambda,\mu,1\}$ and $S=\{0,\alpha,\beta,1\}$ are fuzzy topologies for $X$. By computation, one can find that $Cl\ (\lambda)=1$ and $Cl\ (\mu)=1$. Hence $(X,T)$ is the fuzzy hyperconnected space. Also by computation, one can find that*

$$IntCl([cl\{(1-\lambda)\wedge\lambda\}]\wedge[cl\{(1-\lambda)\wedge(1-\lambda)\}])=0,$$
$$IntCl([cl\{(1-\mu)\wedge\lambda\}]\wedge[cl\{(1-\mu)\wedge(1-\lambda)\}])=0,$$
$$IntCl([cl\{(1-\lambda)\wedge\mu\}]\wedge[cl\{(1-\lambda)\wedge(1-\mu)\}])=0,$$
$$IntCl([cl\{(1-\mu)\wedge\mu\}]\wedge[cl\{(1-\mu)\wedge(1-\mu)\}])=0.$$

*This means that $\lambda$ and $\mu$ are fuzzy resolvable sets in $(X,T)$. Now define the function $f:(X,T)\rightarrow(Y,S)$ by $f(a)=b; f(b)=c; f(c)=a$. By computation, $f^{-1}(\alpha)=\lambda$ and $f^{-1}(\beta)=\mu$ and $\lambda$ and $\mu$ are fuzzy open sets in $(X,T)$, implies that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function. Also $\lambda$ and*

*$\mu$ are fuzzy resolvable sets in $(X,T)$. Thus $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(X,S)$.*

**Proposition 5.2.** *If $f:(X,T)\rightarrow(Y,S)$ is the somewhat fuzzy continuous function from the fuzzy globally disconnected and fuzzy hyperconnected space $(X,T)$ into a fuzzy topological space $(Y,S)$, then $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Let $f:(X,T)\rightarrow(Y,S)$ be the somewhat fuzzy continuous function from $(X,T)$ into $(Y,S)$. Then, for the non-zero fuzzy open set $\lambda$ in $(Y,S), Int[f^{-1}(\lambda)]\neq 0$, in $(X,T)$. Because $(X,T)$ is the fuzzy globally disconnected and fuzzy hyperconnected space, by the theorem 2.20, $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Hence, for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. Thus $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. □

**Proposition 5.3.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy simply $\star$ continuous function from the fuzzy hyper-connected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy simply $\star$ continuous function from the fuzzy hyper-connected space $(X,T)$ into the fuzzy topological space $(Y,S)$. Then, by the theorem 2.22, $f$ is the fuzzy simply continuous function. By the proposition 3.11, $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. □

**Proposition 5.4.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy pseudo-continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$ and then $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy pseudo-continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$. Now for the non-zero fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy pseudo-open set in $(X,T)$. Because $(X,T)$ is the fuzzy hyperconnected space, by the theorem 2.24, the fuzzy pseudo-open set $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$ and thus $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. □

**Proposition 5.5.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function from the fuzzy hyperconnected space $(X,T)$ into another fuzzy topological space $(Y,S)$, then either there exists no non-zero fuzzy regular open set $\delta$ in $(X,T)$ such that $\delta\leq Cl[(1-f^{-1}(\lambda))\vee f^{-1}(\lambda)]$ or Cl $(f^{-1}[(1-\lambda)\vee\lambda])=1$, in $(X,T)$.*

*Proof.* Let $f:(X,T)\rightarrow(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by the proposition 3.7, there exists a fuzzy regular open set $\delta$ in $(X,T)$ such that $\delta\leq cl[(1-f^{-1}(\lambda))\vee f^{-1}(\lambda)]$. Since $(X,T)$ is the fuzzy hyperconnected space, by the theorem 2.32, $1_X$ and $0_X$ are the only fuzzy regular open sets in $(X,T)$ and hence either $\delta=0_X$ or $\delta=1_X$, in $(X,T)$. If $\delta=0_X$, then there exists no non-zero fuzzy regular open set $\delta$ in $(X,T)$ such that $\delta\leq Cl[(1-f^{-1}(\lambda))\vee f^{-1}(\lambda)]$. If $\delta=1_X$, then $1\leq Cl[(1-f^{-1}(\lambda))\vee f^{-1}(\lambda)]$. That is, Cl $[(1-f^{-1}(\lambda))\vee f^{-1}(\lambda)]=1$ and thus Cl $(f^{-1}\{(1-\lambda)\vee\lambda\})=1$, in $(X,T)$. □

**Proposition 5.6.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f^{-1}\{(1-\lambda)\vee\lambda\}$ is the fuzzy somewhere dense set in $(X,T)$.*

*Proof.* Suppose that $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$. Then, by the proposition 5.1, $f:(X,T)\rightarrow(Y,S)$ is the fuzzy resolvable function. Now, for the fuzzy open set $\lambda$ in $(Y,S)$, by the proposition 3.7, $f^{-1}\{(1-\lambda)\vee\lambda\}$ is the fuzzy somewhere dense set in $(X,T)$. □

**Proposition 5.7.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then there exists a fuzzy resolvable set $\mu$ in $(X,T)$ such that $\mu\leq Cl[f^{-1}\{(1-\lambda)\vee\lambda\}]$.*

*Proof.* Let $f:(X,T)\rightarrow(Y,S)$ be the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then by the proposition 5.6, $f^{-1}\{(1-\lambda)\vee\lambda\}$ is the fuzzy somewhere dense set in $(X,T)$. This implies that Int Cl $[f^{-1}\{(1-\lambda)\vee\lambda\}]\neq 0$ and then there exists a fuzzy open set $\mu$ in $(X,T)$ such that $\mu\leq Cl[f^{-1}\{(1-\lambda)\vee\lambda\}]$. Because $(X,T)$ is the fuzzy hyperconnected space, by the proposition 5.1 , the fuzzy open set $\mu$ is the fuzzy resolvable set in $(X,T)$. Thus, there exists a fuzzy resolvable set $\mu$ in $(X,T)$ such that $\mu\leq Cl[f^{-1}\{(1-\lambda)\vee\lambda\}]$. □

**Proposition 5.8.** *If $f:(X,T)\rightarrow(Y,S)$ is the fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into the fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)\vee\mu$ is the fuzzy resolvable set in $(X,T)$, where $\mu$ is the fuzzy nowhere dense set in $(X,T)$.*

*Proof.* Let $f:(X,T)\rightarrow(Y,S)$ be the fuzzy continuous function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy open set in $(X,T)$. Now, for the fuzzy nowhere dense set $\mu$ in $(X,T), f^{-1}(\lambda)\vee\mu$ is the fuzzy simply $\star$ open set in $(X,T)$. Because $(X,T)$ is the fuzzy hyperconnected space, by the theorem 2.29, $f^{-1}(\lambda)\vee\mu$ is the fuzzy simply open set in $(X,T)$ and then, by theorem 2.21, $f^{-1}(\lambda)\vee\mu$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 5.9.** *If $f:(X,T)\rightarrow(Y,S)$ is the somewhere fuzzy continuous function from the fuzzy hyperconnected space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(Y,S)$,*

*(i) $f^{-1}[\lambda\vee(1-\lambda)]$ is the fuzzy dense set in $(X,T)$.*

*(ii) Int $(f^{-1}[\lambda\vee(1-\lambda)])\neq 0$, in $(X,T)$.*

*Proof.* (i) Suppose that $f:(X,T)\to(Y,S)$ is the somewhere fuzzy continuous function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy somewhere dense set in $(X,T)$. Since $(X,T)$ is the fuzzy hyperconnected space, by the theorem 2.31, for the fuzzy somewhere dense set $f^{-1}(\lambda)$ in $(X,T), cl\{f^{-1}(\lambda)\}\vee\{1-[f^{-1}(\lambda)]\}=1$, in $(X,T)$. But $f^{-1}(\lambda)\vee(1-[f^{-1}(\lambda)])=f^{-1}(\lambda)\vee f^{-1}(1-\lambda)\}=f^{-1}[\lambda\vee(1-\lambda)]$ and then Cl $(f^{-1}[\lambda\vee(1-\lambda)])=1$. Hence $f^{-1}[\lambda\vee(1-\lambda)]$ is the fuzzy dense set in $(X,T)$.

(ii). Because $(X,T)$ is the fuzzy hyperconnected space, by the theorem 2.34, $(X,T)$ is the fuzzy irresolvable space. Hence, for the fuzzy dense set $f^{-1}[\lambda\vee(1-\lambda)]$ in $(X,T), cl(1-f^{-1}[\lambda\vee(1-\lambda)])\neq 1$. This implies that $1-Int(f^{-1}[\lambda\vee(1-\lambda)])\neq 1$. Hence, $Int(f^{-1}[\lambda\vee(1-\lambda)])\neq 0$, in $(X,T)$. □

## 6. Fuzzy resolvable functions and other fuzzy topological spaces

**Proposition 6.1.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy perfectly disconnected space $(X,T)$ into another fuzzy topological space $(Y,S)$, then for the fuzzy open set $\lambda$ in $(X,T), \mu\leq f^{-1}(\lambda\vee(1-\lambda))]$, where $\mu$ is the fuzzy closed set in $(X,T)$.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S)$, by the proposition 3.3, $\mu\wedge f^{-1}[\lambda\wedge(1-\lambda)]$ is the fuzzy nowhere dense set in $(X,T)$. Since $(X,T)$ is the fuzzy perfectly disconnected space $(X,T)$, by the theorem 2.26, the fuzzy nowhere dense set $\mu\wedge f^{-1}(\lambda\wedge(1-\lambda))=0$, in $(X,T)$ and then $\mu\leq 1-[f^{-1}(\lambda\wedge(1-\lambda))]$. This implies that $\mu\leq f^{-1}(\lambda\vee(1-\lambda))$, where $\mu$ is the fuzzy closed set in $(X,T)$. □

**Proposition 6.2.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy submaximal and fuzzy P-space $(Y,S)$, then for the fuzzy Baire set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy Baire set in the fuzzy submaximal and fuzzy P-space $(Y,S)$. Then, by the theorem 2.27, $\lambda$ is the fuzzy open set in $(Y,S)$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Proposition 6.3.** *If $f:(X,T)\to(Y,S)$ is the fuzzy Baire continuous function from the fuzzy hyperconnected, fuzzy submaximal and fuzzy P-space $(X,T)$ into the fuzzy topological space $(Y,S)$, then $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S)$.*

*Proof.* Let $f:(X,T)\to(Y,S)$ be the fuzzy Baire continuous function from $(X,T)$ into $(Y,S)$. Then, for the fuzzy open set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy Baire set in $(X,T)$. Since $(X,T)$ is the fuzzy hyperconnected, fuzzy submaximal and fuzzy P-space, by the theorem 2.30, the fuzzy Baire set $f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$ and hence $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S)$. □

**Proposition 6.4.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy topological space $(X,T)$ into the fuzzy strongly irresolvable, fuzzy globally disconnected space, and fuzzy P-space $(Y,S)$, then for the fuzzy Baire set $\lambda$ in $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$.*

*Proof.* Let $\lambda$ be the fuzzy Baire set in the fuzzy strongly irresolvable and fuzzy globally disconnected space and fuzzy P-space $(Y,S)$. Since $(Y,S)$ is the fuzzy strongly irresolvable and fuzzy globally disconnected space, by theorem 2.36 $(Y,S)$ is the fuzzy submaximal space. Hence $(Y,S)$ is the fuzzy submaximal and fuzzy P-space. Then, by the theorem 2.37, $\lambda$ is the fuzzy open set in $(Y,S)$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from $(X,T)$ into $(Y,S), f^{-1}(\lambda)$ is the fuzzy resolvable set in $(X,T)$. □

**Theorem 6.5.** *[15] If cl $(\wedge_{i=1}^{\infty}(\lambda_i))=1$, where $(\lambda_i)'s$ are fuzzy dense sets in the fuzzy submaximal space $(X,T)$, then $(X,T)$ is the fuzzy Baire space.*

**Theorem 6.6.** *[15] If the fuzzy topological space $(X,T)$ is the fuzzy submaximal Baire space, then $(X,T)$ is the fuzzy D-Baire space.*

The conditions for fuzzy topological spaces to become fuzzy Baire spaces under fuzzy resolvable functions are given by the following proposition:

**Proposition 6.7.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy submaximal space $(X,T)$ onto another fuzzy topological space $(Y,S)$ then $(X,T)$ is the fuzzy Baire space.*

*Proof.* Suppose that $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from $(X,T)$ onto $(Y,S)$ and $(\lambda_i)'$ s be fuzzy open sets in $(Y,S)$. Then, by the proposition 3.22, Cl $(\wedge_{i=1}^{\infty}[\delta\vee f^{-1}[\lambda_i\vee(1-\lambda_i)]]=1$, where $\delta\in T$. Since $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function, for the fuzzy open sets $(\lambda_i)'$ s in $(Y,S)$, by the proposition 3.4, $\{\delta\vee f^{-1}[\lambda_i\vee(1-\lambda_i)\vee\lambda]\}$'s are fuzzy dense sets in $(X,T)$, where $\delta\in T$. Since $(X,T)$ is the fuzzy submaximal space, by the theorem 6.5, $(X,T)$ is the fuzzy Baire space. □

The conditions for fuzzy topological spaces to become fuzzy D- Baire spaces under fuzzy resolvable functions are given in the following proposition:

**Proposition 6.8.** *If $f:(X,T)\to(Y,S)$ is the fuzzy resolvable function from the fuzzy submaximal space $(X,T)$ onto another fuzzy topological space $(Y,S)$ then $(X,T)$ is the fuzzy D- Baire space.*

*Proof.* The result follows from the proposition 6.7 and the theorem 6.6. □

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