# A rivers basic edge weight connectivity of steam network based on fixed point diffusion graph theory

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**Abstract**
In fixed point diffusion analysis, many river distinguished such as the length, width, and depth have been un weeded. Some characteristics have crystal clear effects on the inter connectivity. Thus, the edge weight index is mature and can be used to energizing the actual forms. Populations are growing in a flash and exodus to inner city areas in progressing countries has out turned in a paramount need for the inauguration of centralized water systems to promulgate drink worthy water to occupants. Ripening, accented or gravely kept up apportionment systems can causes the quality of piped drinking water to dip below agreeable levels and lay out grievous health risks. Many aspects can affect the river discharge volume, such as the flow section area, flow section area, flow velocity, geometrical shape, length, slope, roughness, and evaporation. However, because the research area is flat, the slope seems to be zero, and above all the flow velocity is sedate; other characteristics are ancillary and knotty to smack dab measure in actual situations.

**Keywords**
Fixed point, Distribution system, River network, Graph representation learning, Averaging principle, Diffusion.

**AMS Subject Classification**
60H10, 58G32.

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## 1. Introduction

Fixed point theory is a note worthy area of functional examination. This section handle with the scrutiny of literature, coupled to postulation of fixed point theorems. Fixed point theory has tantalize chiliad of analyst since 1922 with the celebrated Banach's fixed point theorem. There endure enormous literature on this topic and it is a extremely nimble field of indignation at present. A self map $T$ of a metric space $X$ is said have a fixed point $x$ if $Tx = x$. Theorems anent the existence and properties of fixed points are known as fixed point theorems. Such theorems are very salient tools for manifesting the existence and individuality of the solutions to copious mathematical models stand for phenomena transpire in discrete fields, such as steady state temperature distribution, chemical equation, economic theories and flow of fluids. They are also used to study the problems of optimal control linked to these systems.

The most cardinal result in the fixed point theory is the

well known theorem of Brouwer, which says that every on-going self mapping of the closed unit interval in $R^n$, the $n$-dimensional Euclidean space possesses a fixed point. This result was published by Brouwer [2]. Brouwer ratified his notable theorem in 1910, where the space are subsets of $R^n$ are not of much use in functional analysis where one is generally involve with an fathomless dimensional subset of some function spaces. There exist many contraction mapping theorems in different spaces. In 1930, Schauder [10] extended Brouwer fixed point theorem to the result that every compact convex set in a Banach space has the fixed point property for contentious mapping, as well as that every weakly compact convex set in a separable Banach space has the fixed point property for weakly continuous mappings. The condition of denseness in Schauder fixed point theorem was a very strong condition. As many problems in the analysis do not have compact setting, it is natural to modify this theorem by relaxing the condition of compactness.

The interconnection of a stream system is in particularly the chief one, which set the seal on that the stream network drop ship water, nutrients, and sediments via the basin and hasten the self cleaning faculty of the system. There are sundry studies that scrutinize the numerous dearth of spasmodic supply, since it causes a hornets' nest in the system infrastructure itself, produces health menace for uses [1, 4, 7] and give rise to the inequity.

On one hand, the natural sedimentation, chunks and dry spell retard the flow and cause water exchange ability to turn down. On the other hand, because of the pursuit of high acquittal rate of land in the urbanization contrive process, many rivers were filled or diminish to issue more contrive land, which feigned reinforce the uncouple the situation of the stream network and causes drainage strain. Water idiosyncrasy deterioration, poor ecological environment and many types of water hurdle. Howbei, water is currently delivered to millions of people around the world under fitful supply conditions.

Euler (1707-1782) became the father of graph theory as well as topology when in 1736 he settled a prominent unsolved problem of his day called the Konigsberg bridge problem. Another puzzle approach to the graphs was proffer by Hamilton. The psychologist Lewin proposed in 1936 that the "life space" of an individual be stand in by a planar map (Lewin used only planar maps because he always drew his figures in the plane). In such a map the regions would stands for various activities of a person, such as his work environment, his digs and his sideline. The psychologists at the Research center for group dynamics to another psychological exegesis of a graph, in which people are stand in by points and communal tie–up by lines. Such tie-ups include doting, abhor, divulgence, and power. In fact, it was precisely this approach which led the author to a personal breakthrough of graph theory, aided and abetted by psychologists L. Festinger and D. Cartwright. The study of Markov chains in probability theory take account of directed graphs in the sense that events are stand in by

points and a directed line from one point to another indicates a positive probability of direct line-up of these dyad events. This is made clear-cut in which a Markov chains elucidate as a network with the sum of the values the directed lines from each point equal to 1. A similar simulacrum of a directed graph arise in that part of numerical analysis entail matrix inversion and the calculation of eigen values. A square matrix is given, preferably "sparse," and a directed graph is cognate with it in the following way. The points denote the index of the rows and columns of the given matrix, and there is a directed line from point $i$ to point $j$ whenever $i, j$ entry of the matrix is nonzero. The alikeness betwixt this approach and that for Markov chains is whirlwind Graph theory is a theoretical model that uses an abstract method to stand in by the mutual relations among the study objects. The graphs in graph theory do not have exclusive such as size, shape or mass except objects (stand in by nodes) and their tie-ups(stand in by edges). The edge node structure in graph theory system is highly kindred to actual stream network. The rivers in an actual stream network can be simplified to edges, and the lakes and junctions (points where dyad channels coalesce into one) can be simplified to nodes. According to the structure characteristic of stream network and the graph theory method, the stream network can be gauged by some analysis methods of graph theory where the connectivity of the stream network can be quantitatively analyzed. Many quondam studies scrutinize the stream network analysis using graph theory. Although plunk flow meters at the incoming pipes of each sector is common for leak control, sectors without measurement can exit in fitful supply networks , since their main goal is to deliver water at differentiated agenda.

However, most researchers used single indicators to analyze the connectivity of stream networks, which introduced many problems caused by the limitations of individual indices. Nevertheless, different indicators of graph theory are sporadically unify to make a comprehensive analysis of stream systems. For district metered area implementation in networks with continuous water supply, there is a general drift to use optimization techniques to achieve an adequate service level. Sundry authors also suggest graph theory for sectorization process. In both supply system improvement perspectives, networks sectorization is a root step. Sectors are also important in change over processes to unbroken supply, and pivotal for intermittent supply system management that aims to improve fairness in supply. Moreover, sectorization under an intermittent supply based on slant may be useful for vulnerable unceasing supply systems. In 2016, for cite, the unbroken supply network of Lapaz (Bolivia) had to become ephemeral intermittent due to paltry water in its supply sources.

If an intermittent supply network is not sectorized, the peak flow exigency during supply hours is very high, since water exigency occurs concomitantly for the total network. Thus, high water exigency results in low service level conditions and may produce deficient pressure areas, which then produces supply inequity. Network sectorization and supply

agenda setting help taper this high peak exigency.

In this paper, an approach based on the theoretical pinnacle of flow concept, which uses soft computing tools from graph theory and cluster analysis, is burgeon to explicate sectors to produce scrupulous water supply. For node clustering, this process also includes water company expert opinions, from the individuals who best know network details. Moreover, this approach helps opt the supply time for each sector based on their hydraulic characteristics.

Thamirabarani river is flowing continuously for 120 km.It passes via many villages, town and Trinelveli corporation. It is a perennial river and monsoon based catchment. Many toxic waste are added to this river at all points. It has solid, liquid and gaseous toxics such as BOD, COD, TSS and deliquesces oxygen [5, 12]. Many moat water and night soil toxics are also added to this point. Textile toxic waste and paper industrial waste are also added to this point.These are the main sources of poison of Thamirabarani river. Thamirabarani is a main source of water supply to many towns which include Tirunelveli corporation. In Tirunelveli municipal area the drinking water is not treated properly. Entire flow on the river on the river has turbidity. The main aim of this paper is to analyze the copious hazardous waste and its removal process.

For more details on this theory, we suggest the reader to refer [3, 6, 8, 11, 13].

## 2. Preliminaries

In this section, we recall some basic definitions, notations and results which are very useful to our work.

**Results:** Denote

$$C_n = \frac{1}{n!} \sum_{\sigma \in S_n} x_i^{a_i(\sigma)}.$$

Then we defined

$$C(t) = \sum_{n=0}^{\infty} t^n C_n$$

$$= e^{\sum_{n=0}^{\infty} x_i \frac{t^i}{i}}.$$

This has a lot of information in it.

**Example 2.1.** *Suppose $C(\sigma)$ is the number of cycle of $\sigma$.*
*Then,*

$$C(\sigma) = \sum_{i=1}^{n} a_i(\sigma).$$

*Setting all $x_i = x$, we have*

$$C_n(x) = \frac{1}{n!} \sum_{\sigma \in s_n} x_0^{C(\sigma)}.$$

*Then*

$$C(t) = \frac{1}{(1-t)^x}$$

$$= \sum_{j=0}^{\infty} \binom{x}{j} (-t)^j$$

$$= \sum_{j} \frac{t^j}{j!} x(x+1)\ldots.(x+j-1).$$

*Since $\sum_i \frac{t^i}{i}$ is the power series expansion for $-\log(1-t)$.*
*Therefore,*

$$C_n = \frac{1}{n!} x(x+1)\ldots.(x+n-1)$$

$$= x \left(\frac{x+1}{2}\right) \left(\frac{x+2}{3}\right) \ldots. \left(\frac{x+n-1}{n}\right)$$

$$= E(x^{S_n}) = \pi E_x^{x_i}.$$

*Here $S_n$ denotes the sum, not the symmetric group. Here $P(x_i = 0) = \frac{i-1}{i}$ and $P(x_i = 1) = \frac{1}{i}$. Thus, we have*

$$E(x^{S_n}) = \sum_{j=0}^{n} x^j P(x_n = j)$$

$$E(fS_n) = \sum f(j) P(S_n = j).$$

*We took $f(j) = x^j$, we have*

$$AV(S_n) = 1 + \frac{1}{2} + \cdots + \frac{1}{n} \sim \log n.$$

$$VAR(S_n) = \sum_{n=1}^{\infty} \frac{1}{i} \left(1 - \frac{1}{i}\right) \sim \log n.$$

*That is*

$$P \left\{ \frac{C(\sigma) - \log n}{\sqrt{\log n}} \leq x \right\} \to \varphi(x).$$

*The coefficient of $x_j$ is the number of permutations with $j$ cycles. These happen to be called sterling numbers of the first kind.*

### 2.1 Quizzing

Who cares about all this stuff with fixed points?

There was a fixture where someone took duo decks of cards up to n. People take part in this fixture and you get a dollar if the same number comes up. The canvass is a question of the number of fixed points Monmort in 1708 evince the number of fixed points has a poisson distribution as we evince last time. Note that we may as well call the cards on the mono deck $1, 2, \ldots, n$. So the number of matches is just the number of fixed points in a random permutation.

We also have a metric,

$$D(\pi, \sigma) = \{i : \pi(i) \neq \sigma(i)\}.$$

See, Persi Diaconis et al. [9] on fixed points of permutations for a classification of possible fixed points of transitive primitive actions of the symmetric group.

**Definition 2.1.** *The Cayley distance between duo permutations*

$d_c(\sigma, \pi) = $ *minimum number of transpositions needed to express* $\pi\sigma^{-1}$.

*i.e., this is the distance in the Cayley graph where the vertices are permutations and the edges affix duo elements differing by a permutations.*

## 2.2 Perceive

The above two distance measures are the only duo bi-invariant distances that Persi knows of.

**Definition 2.2.** *Graph diffusion, which is equivalent to linear weighting for nodes by mega-scale random walk on graph. However, graph diffusion has a center of attention on node-level transformations rather than content-level transformations. To reveal the relationship between node features, we consider looking for a function $f(A) = \sigma(AC + x)$ to non linearly map A from the input space to the representation space, where $A = A_0$ is original node features and $C \in R^{(dh)}$ is the transformation matrix. Combined with graph diffusion, single diffusion can be expressed as follows:*

$$f(B) = \sigma(BC + x)$$
$$= \sigma((I - \delta S)^{-1}AC + x).$$

## 2.3 Diffusion

A specific important issue in harmonic analysis is to connect the smoothness of a function with the speed of convergence of its diffused version to itself, in the limit as time goes to zero. In order to consider the smoothness of diffusing functions in more general settings, a distance defined in terms of the diffusion itself seems particularly appropriate.

Defining diffusion distances is of interest in applications as well. As we already mooted in [3, 4, 7, 8], dimensionality reduction of data and the concomitant issue of finding structures in data are highly prime objectives in the fields of information theory, statistics, machine learning, sampling theory, etc. It is often useful to organize the given data as nodes in a weighted graph, where the weights reflect local interaction between data points. Random walks, or diffusion, on graphs may then help understand the interactions among the data points at expanding distance scales. To even consider different distance scales, it is mandatory to define an appropriate diffusion distance on the constructed data graph.

## 3. Main Results

We consider a general symmetric diffusion semigroup $T_t f_{(t \geq 0)}$ on a topological space $X$ with a efficacious $\sigma-$ finite measure (i.e., $X$ is a countable union of measurable sets with finite measure), given, for $t > 0$, by an integral kernel operator: $T_f f(x) \triangleq \int_x \rho_t(x, y) f(y) dy$. Coifman and Leeb initiated a family of multi scale diffusion distances and manifest quantitative results about the equivalence of a bounded function $f$ being Lipschitz, and the rate of convergence of $T_t f$ to $f$ as

$t \to 0^+$ (we are discussing some of their results using a continuous time for $t$ convenience; most of Coifman's and Leeb's derivations are done for radically discretized times. Moreover, most of the authors' results are in fact manifested without the assumption of symmetry and under the weaker condition than positivity of the kernel, namely, an appropriate $L_1$ integrability statement. To prove the implication that Lipschitz implies an appropriate estimate on the rate of convergence, Coifman and Leeb make a quantitative assumption about the decay of

$$\sup \int_x |\rho_t(x, y)| d(x, y) dy, \quad \text{as} \quad t \to 0^+ \tag{3.1}$$

for their distances d, namely, that

$$\sup \int_x |\rho_t(x, y)| d(x, y) dy \leq Ct^\alpha. \tag{3.2}$$

for some $\alpha > 0$. Coifman and Leeb also manifest that (3.2) above, in the case of efficacious diffusion kernels, is in fact equivalent to their termination about the rate of convergence of $T_t f$ to $f$ as $t \to 0^+$, for a Lipschitz function $f$. Additionally, Coifman and Leeb show that, in some of the settings they consider (with decay and continuity conjectures on the diffusion kernels relative to an intrinsic metric), their multi scale diffusion distance is equivalent to (localized)$D(x, y)^\alpha$ where $D(x, y)$ is the intrinsic metric of the underlying space and $\alpha$ is a efficacious number strictly less than 1. The authors emphasize that $\alpha$ cannot be taken to equal 1.

The focal reason is that we wish to avoid making any gauges about the decay of (3.1) and still manifest a correspondence between some version of smoothness of a function $f$ and convergence of $T_t f$ to $f$ as $t \to 0^+$. Our foremost contribution is to manifest, under almost no presumes, that local equi-continuity (in $t$) is equivalent to local convergence; i.e., local control of the differences $T_t f(x) - T_t f(y)$ for all $t$ small is equivalent to local control of the differences $T_t f(x) - f(x)$ for all small $t$. Here "local" is defined relative to a representative of our family of proffer diffusion distances.

**Theorem 3.1.** *For $A > 0, Ax = \delta_{\max}x$ are strictly positive.*

*Proof.* The key idea is to look at all number $t$ such that $Ax \geq tx$ for some non negative vector $x$ (other than $x = 0$). We are allowing inequality in $Ax \geq tx$ in order to have many positive candidates $t$. For the immense value $t_{\max}$ (which attained), we will show that equality holds $Ax = t_{\max}x$.

Otherwise if $Ax \geq t_{\max}x$ is not a equality, multiply by $A$. Because $A$ is positive that produces a strict inequality $A^2x > t_{\max}Ax$. Therefore the positive vector $y = Ax$ satisfies $Ay > t_{\max}y$ and $t_{\max}$ could be increased. This contradiction forces the equality $Ax = t_{\max}x$ and we have an eigen value. Its eigen vector $x$ is positive because on the left side of that quality, $Ax$ is sure to be positive.

To see that no eigen value can be larger than $t_{\max}$, suppose $Az = \delta z$. Since $\delta$ and $z$ may involve negative or complex number, we take absolute values $|\delta||z| = |Az| \leq A|z|$ by the triangle inequality. This $|z|$ is a non negative vector, so $|\delta|$ is

one of the possible candidate $t$. Therefore $|\delta|$ cannot exceed $t_{\max}$ which must be $\delta_{\max}$.

$\square$

**Theorem 3.2.** *Let $\sum_A$ be topologically mixing $\varphi \in \gamma_A \cap C(\sum_A^+)$ and $\mu = \mu_\varphi$ as above. There are $\delta > 0, h \in C(\sum_A^+)$ with $h > 0$ and $v \in M(\sum_A^+)$ for which $\mu h = \delta h, \mu^* v = \delta v, vh = 1$ and $\lim_{m \to \infty} \|\delta^{(-m)} \mu^m g - v(g)h\| = 0$ for all $g \in C(\sum_A^+)$.*

## 4. Study area and future work

### 4.1 Thamirabarani

The Thamirabarani River is a river in India. It starts seeping from the peak Agastyarkoodam in the western Ghats hills in Papanasam of Tirunelveli district, Tamil Nadu. The river had the name of porunai in ancient Tamil history. The name of Thamirabarani is presumed from the high content of copper in this river [12, 13]. This copper content also makes the water of this river honeyed or luscious.

## 5. Location

| Locale | |
| --- | --- |
| Country | India |
| State | Tamil Nadu |
| District | Tirunelveli |
| | Thoothukudi |
| Cities | Tirunelveli |
| | Ambasamudram |
| | Palayamkottai |

### 5.1 Physical Characteristics

| Source | Pothigai hills |
| --- | --- |
| Coordinates | 8.601962°N |
| | 77.264131°E |
| Mouth | |
| Location | Gulf of Mannar |
| Coordinates | 8.6413316°N |
| | 78.127298°E |
| Length | 128km (80 mi) |
| Discharge | |
| Location | Srivaikundam |
| Average | 32 $m^3/s$ |
| | (1,100 $cu\ ft/s$) |

### 5.2 Basin Features

| Tributaries | |
| --- | --- |
| Left | Karaiyar, Servalar, Gandananathi, Chittar River |
| Right | Manimutharu Pachaiyar |

The Thamirabarani River originates from the peak of the Pothigai hills on the eastern slopes of the western Ghats at an elevation of 1,725 meters (5659 ft) above sea-level.

### 5.3 List of dams across Thamirabarani river

1 Kodaimelaalagain anicut, 1,281.67 hectares (3,167.1 acres)

2 Nathiyunni anicut, 1,049.37 hectares (2,593.0 acres)

3 Kannadian anicut, 2,266.69 hectares (5,601.1 acres)

4 Ariyanayagipuram anicut, 4,767.30 hectares (11,780.3 acres)

5 Palavur anicut, 3,557.26 hectares (8,790.2 acres)

6 Suthamalli anicut, 2,559.69 hectares, (6,325.1 acres)

### 5.4 List of Major Tributaries

| Tributaries | Length | Orgin |
| --- | --- | --- |
| Karaiyar | | Mundanthurai reserve forests |
| Servalar river | | Mundanthurai reserve forests |
| Manimuthar River | 9kms(6 mi) | Manjolai hills |
| Gadananathi River | | Agasthyamala Biosphere Reserve |
| Pachaiyar River | 32kms(20 mi) | Kalakkadu Reserve Forests |
| Chittar River | 80kms(50 mi) | Kutralam hills |
| Ramanathi River | | Agasthyamala Biosphere Reserve |

### 5.5 List of Channels

1 South kodaimelalagain channel

2 North kodaimelalagain channel

3 Nathiyunni channel

4 Kannadian channel

5 Kodagan channel

6 Palayam channel

7 Trinelveli channel

8 Marudur Melakkal

### 5.6 List of Major Tributaries

| Joins at | Length of course of Thamirabarani | Dams on River |
| --- | --- | --- |
| Karaiyar dam | 6kms(4mi) | Karaiyar dam |
| Papanasam reservoir | 22kms(14mi) | |
| Aladiyoor | 36kms(22mi) | Manimuthuar dam |
| Tiruppudaimaruthur | 43kms(27mi) | Gadananathi river dam |
| Tharuvai | 61kms(38mi) | |
| Sivalaperi | 73kms(43mi) | |
| Kizhaambur | 22kms(14mi) | Ramanathi river dam |

## 5.7 Line value connected of steam structure

The sum of the river flow section areas that are linked with lake $\alpha$ is called the Total River Flow lake $\alpha$, which is denoted by $x_\alpha$. Lakes with larger $x_\alpha$ have greater water exchange capacity and self cleaning ability. A lake with more water commonly requires larger $x_\alpha$. Thus, to analyze whether $x_\alpha$ satisfies the lake connectivity requirement, the total river flow is created and denoted by $x_\alpha$. It's relationship with the water volumes is given in the following formula:

$$x_{\beta\alpha} = \tau V_\alpha,$$

where $x_{\beta\alpha}$ is the total river flow linked with lake $\alpha(m^2)$; $\tau$ is the total river flow section coefficient $(m^2/m^3)$; $V_\alpha$ is the water volume of lake $\alpha(m^3)$, calculated by the ordinary water level; $x_{\beta\alpha}$ is the minimum $x_\alpha$ that the lakes require. When $x_\alpha$ is greater than $x_{\beta\alpha}$ the basic connectivity of the lake satisfies the demand. When $x_\alpha$ is less than $x_{\beta\alpha}$ the system cannot satisfy the basic connected demand and must be improved. Coefficient $\tau$ must be adjusted according to the specific situation of the study area.

The line value is the minimum flow area that the rivers must have to satisfy the connected demand is $y_{\beta\gamma}$. Lakes connect with other lakes through rivers or channels; because of different water transmission quantities, the connection between large river flow. The river flow and size of lakes joined with the river have a strong positive correlation. Thus, their relationship can be built to find the line value as follows:

$$y_{\beta1} = \frac{x_{\beta1}V_2}{\sum V_{t1}}; y_{\beta12} = \frac{x_{\beta2}V_2}{\sum V_{t2}}; y_{\beta\gamma} = \max(y_{\beta1}, y_{\beta2}),$$

where $1,2$ are the numbers of lakes connected by rivers $\gamma$; $y_{\beta1}$ is the line value of river $\gamma$ calculated from lake $1(m^2)$; $y_{\beta2}$ is the line value of river $\gamma$ evaluated from lake $2(m^2)$; $x_{\beta1}$ is the basic total river flow section of lake $1(m^2)$; $x_{\beta2}$ is the total river flow section lake $2(m^2)$; $V_1$ is the volume of lake $1(m^3)$; $V_2$ is the volume of lake $2(m^3)$; $\sum V_t 1$ is the total volume of all lakes connected with lake $1(m^3)$; $\sum V_t 2$ is the total volume of all lakes connected with lake $2(m^3)$; $y_{\beta\gamma}$ is the basic edge weight of river $\gamma(m^2)$. The formula is calculated, respectively, from two lakes linked by river $\gamma$. Firstly, we design $x_{\beta1}$ and $x_{\beta2}$ to river $\gamma$ to obtain $y_{\beta1}$ and $y_{\beta2}$ then we take the maximum of those two as $y_{\beta\gamma}$. Also

| Steps | Data |
|---|---|
| Simplify stream structure to fixed point-line figure | River lake connections |
| Estimate the connectivity level of simplified figure | Number of rivers and lakes river areas lake volumes |
| Improve the connectivity of stream structure | Specific projects |

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