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Linear programming models to solve fully fuzzy two person zero sum matrix game

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Abstract

This paper presents a new method for solving the two person zero sum matrix game with fuzzy payoffs and fuzzy strategies for the players. Ranking function of triangular fuzzy numbers is used to develop a pair of crisp linear programming models corresponding to each player. Every established model for each player is illustrated through a numerical example. Sensitivity analysis with respect to different parameters on value of game and strategies of players are demonstrated by graphs.

Keywords

Two person zero sum matrix game, fuzzy number, fuzzy ranking, defuzzification.

AMS Subject Classification

91A05, 03E72, 94D05, 03B52, 90C70.

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Contents

1. Introduction

Game theory was founded by American mathematicians Neumann and Morgenstern [21]. In many real world situations when there is some lack of information it is useful to apply concepts of fuzziness introduced by Zadeh [31]. Sharma and Kumar [27] applied fuzzy numbers in game theory to forecast elections. Campos [7] proposed some auxiliary models to solve two person zero sum matrix games using linear programming models. Multi-objective situations in such games handled by Nishizaki and Sakawa [22]. Bector et. al [6] used defuzzification method to solve matrix games with fuzzy payoffs. Kumar and Keerthana [15] used defuzzification approach to convert fuzzy payoffs into crisp and solved using classical approaches. A solution procedure developed by Maeda [19] for fuzzy matrix games. Campos et. al [8] proposed a direct method using ranking function approach. Max-min strategy with respect to degree of attainment was achieved by Sakawa and Nishizaki [25]. Chen and Larbani [11] solved matrix game by using the degree of attainment of fuzzy goal. Vijay et. al [29] applied primal dual fuzzy LP Models to solve such games with fuzzy objective and payoffs. Cevikel and Ahlatcioglu [9] also presented models to solve the same category of games. A new approach was established by Krishnaveni and Ganesan [14]. Number of researchers [4, 5, 20, 26, 28] developed different models to solve fuzzy matrix games. Intuitionistic fuzzy set theory was used by [2, 3] for payoffs and goals. Genetic algorithm was applied by Roy and Mula [24] used rough variables as payoffs. Roy and Mondal [23] proposed multi-objective LP method to solve matrix games with fuzzy intervals as payoffs. Jana and Roy [13] used generalized trapezoidal fuzzy numbers as payoffs. Triangular fuzzy numbers were used as payoffs by Li [16] to solve constrained games. Li and Cheng [17] investigated constrained matrix games with fuzziness using multi-objective programming problems. Chen and Hsieh [10] established a defuzzification function corresponding to generalized fuzzy numbers.

Most of the work listed above deals with fuzziness in either objective function or in payoffs. Some of them deals with fuzzy goals and fuzzy payoffs. There is no work in the available literature which deals with fully fuzzy matrix games i.e. where strategies of players considered fuzzy numbers.

We propose models to obtain the solution of fully fuzzy two person zero sum matrix games.

This paper is organized as follows: preliminaries used in this paper are presented in section 2. In section 3, a new method is suggested to solve the game. Our proposed method is illustrated by a numerical example in section 4. In section 5, sensitivity analysis and graphical representations of value of game and strategies are discussed. The conclusion of the paper is given in section 6.

2. Preliminaries

This section concern some definitions and notations in fuzzy environment which are used throughout this paper.

Definition 2.1. *The set* $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\}$ *is known as fuzzy set, Where* $\mu_{\tilde{A}} : X \to [0,1]$ *is membership function.*

Definition 2.2. *The* α*-level of fuzzy set* $\tilde{A}$ *is defined as* $\tilde{A}_\alpha = \{x|\mu_{\tilde{A}}(x) \geq \alpha\}$ *for* $\alpha \in [0,1]$.

Definition 2.3. $\tilde{A}$ *in R is called fuzzy number if*
(i) $\mu_{\tilde{A}}(x_0) = 1$ *for at least one* $x_0 \in R$
(ii) $\mu_{\tilde{A}}(x)$ *is piecewise continuous.*
(iii) $\tilde{A}$ *is normal and convex.*

Definition 2.4. *The triplet* $\tilde{A} = (\xi_1, \xi_2, \xi_3)$ *is called a triangular fuzzy number (TFN) if its membership function* $\mu_{\tilde{A}} : X \to [0,1]$ *is given by*
$\mu_{\tilde{A}}(x) =$

$$\begin{cases} 0 & x < \xi_1 \\ \frac{x-\xi_1}{\xi_2-\xi_1} & \xi_1 \leq x \leq \xi_2 \\ \frac{\xi_3-x}{\xi_3-\xi_2} & \xi_2 \leq x \leq \xi_3 \\ 0 & x > \xi_3 \end{cases} \tag{2.1}$$

Definition 2.5. *The* α*-cut* $\tilde{A}_\alpha$ *of TFN* $\tilde{A} = (\xi_1, \xi_2, \xi_3)$ *is defined as* $\tilde{A}_\alpha = [\xi_1 + \alpha(\xi_2 - \xi_1), \xi_3 - \alpha(\xi_3 - \xi_2)] \equiv [A^L_\alpha, A^R_\alpha]$ *for* $\alpha \in [0,1]$.

Definition 2.6. *Let* $\tilde{A} = (\xi_1, \xi_2, \xi_3)$ *and* $\tilde{B} = (\eta_1, \eta_2, \eta_3)$ *then*
(i) *Addition:* $\tilde{A}(+)\tilde{B} = (\xi_1 + \eta_1, \xi_2 + \eta_2, \xi_3 + \eta_3)$
(ii) *Symmetric Image:* $-\tilde{A} = -(\xi_1, \xi_2, \xi_3) \equiv (-\xi_3, -\xi_2, -\xi_1)$
(iii) *Scalar Product:* $k\tilde{A} = \begin{cases} (k\xi_1, k\xi_2, k\xi_3) & k \geq 0 \\ (k\xi_3, k\xi_2, k\xi_1) & k < 0 \end{cases}$
(iv) *Subtraction:* $\tilde{A}(-)\tilde{B} = (\xi_1 - \eta_3, \xi_2 - \eta_2, \xi_3 - \eta_1)$
(v) *Multiplication:*
$\tilde{A}(\times)\tilde{B} = (min(\xi_1\eta_1, \xi_2\eta_2, \xi_3\eta_3), \xi_2\eta_2, max(\xi_1\eta_1, \xi_2\eta_2, \xi_3\eta_3))$

Definition 2.7. $F : N(R) \to R$ *is defined as ranking function for which* $F(\tilde{A}) \leq F(\tilde{B})$ *for* $\tilde{A}, \tilde{B} \in N(R)$ *which is equivalent to* $\tilde{A} \leq \tilde{B}$. *Ranking approaches proposed by Yager [30]:*

$$(i)\ F_1(\tilde{A}) = \frac{\int_{\xi_L}^{\xi_U} x\mu_{\tilde{A}}(x)dx}{\int_{\xi_L}^{\xi_U} \mu_{\tilde{A}}(x)dx}$$

$$(ii)\ F_1(\tilde{A}) = \int_0^{\alpha_{max}} M[A^L_\alpha, A^R_\alpha]d\alpha$$

Approach proposed by Adamo [1]: For a given index $k \in [0,1]$

$$F_k(\tilde{A}) = max\{x|\mu_{\tilde{A}}(x) \geq k\}.$$

Approach proposed by Liou and Wang [18]: For $\tilde{A} = (\xi_1, \xi_2, \xi_3)$ *membership function is*
$\mu_{\tilde{A}}(x) =$

$$\begin{cases} 0 & x < \xi_1 \\ f^L_{\tilde{A}}(x) & \xi_1 \leq x \leq \xi_2 \\ f^R_{\tilde{A}}(x) & \xi_2 \leq x \leq \xi_3 \\ 0 & x > \xi_3 \end{cases} \tag{2.2}$$

is given as

$$I^\alpha_T(\tilde{A}) = \alpha I_R(\tilde{A}) + (1-\alpha)I_L(\tilde{A})$$

where $I_R(\tilde{A}) = \int_0^1 g^R_{\tilde{A}}(y)dy$ *and* $I_L(\tilde{A}) = \int_0^1 g^L_{\tilde{A}}(y)dy$. $g^R_{\tilde{A}}(y)$ *and* $g^L_{\tilde{A}}(y)$ *are the inverse functions of* $f^R_{\tilde{A}}(x)$ *and* $f^L_{\tilde{A}}(x)$ *respectively having the property that are integrable in* $[0,1]$ *and* $\alpha \in [0,1]$.
Approach proposed by Garcia and Lamata [12]:

$$I_{\beta,\alpha}(\tilde{A}) = \beta S_M(\tilde{A}) + (1-\beta)I^\alpha_T(\tilde{A})$$

where $S_M(\tilde{A})$ *is area of the core,* $\beta \in [0,1]$, $\alpha \in [0,1]$ *and* $I^\alpha_T(\tilde{A})$ *is the total integral value.*

Definition 2.8. *The process of to transforms the fuzzy inference results into an appropriate precise value i.e. an inverse approach of fuzzification is referred as the defuzzification. Several defuzzification techniques have been investigated, here we will use the graded mean integral value as a defuzzification function of a fuzzy number proposed by Chen and Hsieh [10]. Let* $\tilde{A} = (\xi_1, \xi_2, \xi_3)$ *then*
$\mu_{\tilde{A}}(x) =$

$$\begin{cases} 0 & x < \xi_1 \\ f^L_{\tilde{A}}(x) & \xi_1 \leq x \leq \xi_2 \\ f^R_{\tilde{A}}(x) & \xi_2 \leq x \leq \xi_3 \\ 0 & x > \xi_3 \end{cases} \tag{2.3}$$

then the graded mean integral value of $\tilde{A}$ *is*

$$P(\tilde{A}) = \frac{\int_0^1 x\{(1-k)I_L(\tilde{A}) + kI_R(\tilde{A})\}dx}{\int_0^1 xdx}$$

$$= 2\int_0^1 x\{(1-k)I_L(\tilde{A}) + kI_R(\tilde{A})\}dx$$

where $I_R(\tilde{A})$ *and* $I_L(\tilde{A})$ *are integral values of functions* $f^R_{\tilde{A}}(x)$ *and* $f^L_{\tilde{A}}(x)$ *respectively which are integrable in* $[0,1]$ *and* $k \in [0,1]$.

Definition 2.9. *The triplet* $G = (S^m, S^n, A)$, *where* S^m *and* S^n *and A is* $m \times n$ *matrix with usual meaning . The triplet* $(x^*, y^*, v^*) \in S^m \times S^n \times R$ *is solution G if* ${x^*}^T Ay \geq v^* \quad \forall y \in S^n$ *and* $x^T Ay^* \leq v^* \quad \forall x \in S^m$.

Definition 2.10. *The triplet $FG = (S^m, S^n, \tilde{A})$ where S^m and S^n and $m \times n$ have usual meaning then $(x^*, y^*, \tilde{v}^*, \tilde{w}^*)$ is solution of FG (fuzzy game) if there is $x^* \in S^m$, $y^* \in S^n$, $\tilde{v}^* \in \Gamma_1$, $\tilde{w}^* \in \Gamma_2$ such that*
(i) $x^{*T}\tilde{A}y \succcurlyeq \tilde{v} \quad \forall y \in S^n$
(ii) $x^T\tilde{A}y^* \preccurlyeq \tilde{w} \quad \forall x \in S^m$
(iii) $F(\tilde{v}^*) \geq F(\tilde{v}) \quad \forall \tilde{v} \in \Gamma_1$
(iv) $F(\tilde{w}^*) \leq F(\tilde{w}) \quad \forall \tilde{w} \in \Gamma_2$
where Γ_1, Γ_2 are the set of fuzzy values of $\tilde{v}$ and $\tilde{w}$.

Definition 2.11. *For fully fuzzy case the triplet $FMG = (S^m, S^n, \tilde{A})$ where S^m and S^n have usual meaning and $\tilde{A} = [\tilde{a}_{ij}]$ be an $m \times n$ matrix with fuzzy numbers. Then $(\tilde{x}^*, \tilde{y}^*, \tilde{v}^*, \tilde{w}^*)$ is the solution of FMG if there is $\tilde{x}^* \in S^m$, $\tilde{y}^* \in S^n$, $\tilde{v}^* \in \Gamma_1$, $\tilde{w}^* \in \Gamma_2$ such that*
(i) $(\tilde{x}^*)^T\tilde{A}\tilde{y} \succcurlyeq \tilde{v} \quad \forall \tilde{y} \in S^n$
(ii) $\tilde{x}^T\tilde{A}\tilde{y}^* \preccurlyeq \tilde{w} \quad \forall \tilde{x} \in S^m$
(iii) $F(\tilde{v}^*) \geq F(\tilde{v}) \quad \forall \tilde{v} \in \Gamma_1$
(iv) $F(\tilde{w}^*) \leq F(\tilde{w}) \quad \forall \tilde{w} \in \Gamma_2$
where Γ_1, Γ_2 are the collections of reasonable fuzzy values of $\tilde{v}$ and $\tilde{w}$, $\tilde{x}^*$ and $\tilde{y}^*$ are the optimal fuzzy strategy for players, $\tilde{v}^*$ and $\tilde{w}^*$ are the fuzzy values of the game for players.

3. Proposed Method

Step I Construct the pair of fully fuzzy LP problems of both players with adequacies $\tilde{p}$ and $\tilde{q}$.

$$\begin{aligned}&\text{Max}\,\tilde{v}\\&\text{subject to the double fuzzy constraint}\\&\tilde{x}^T\tilde{A}\tilde{y} \succcurlyeq_{\tilde{p}} \tilde{v}\ \forall \tilde{y} \in S^n\\&\text{and } \tilde{x} \in S^m \end{aligned} \tag{3.1}$$

And

$$\begin{aligned}&\text{Min}\,\tilde{w}\\&\text{subject to the double fuzzy constraint}\\&\tilde{x}^T\tilde{A}\tilde{y} \preccurlyeq_{\tilde{q}} \tilde{w}\ \forall \tilde{x} \in S^m\\&\text{and } \tilde{y} \in S^n \end{aligned} \tag{3.2}$$

where $\tilde{v}, \tilde{w} \in N(R)$.
Step II Using strategic spaces S^m and S^n in the constraints of the problems (3.1) and (3.2).

$$\begin{aligned}&\text{Max}\,\tilde{v}\\&\text{subject to the double fuzzy constraint}\\&\tilde{x}^T\tilde{A}_j \succcurlyeq_{\tilde{p}} \tilde{v}\ \forall j\\&e^T\tilde{x} \approx \tilde{1}\\&\text{and } \tilde{x} \succcurlyeq \tilde{0}\,\text{with}\,\tilde{v} \succcurlyeq \tilde{0} \end{aligned} \tag{3.3}$$

And

$$\begin{aligned}&\text{Min}\,\tilde{w}\\&\text{subject to the double fuzzy constraint}\\&\tilde{A}_i\tilde{y} \preccurlyeq_{\tilde{q}} \tilde{w}\ \forall i\\&e^T\tilde{y} \approx \tilde{1}\\&\text{and } \tilde{y} \succcurlyeq \tilde{0}\,\text{with}\,\tilde{w} \succcurlyeq \tilde{0} \end{aligned} \tag{3.4}$$

where the symbols $\tilde{A}_i$ and $\tilde{A}_j$ denotes the i^{th} row and the j^{th} column of the fuzzy payoff matrix $\tilde{A}$ for all i and j.
Step III Apply Yager [30] resolution method for the double fuzzy constraints of the problems (3.3) and (3.4).

$$\begin{aligned}&\text{Max}\,\tilde{v}\\&\text{subject to the constraint}\\&\sum_{i=1}^{m}\tilde{a}_{ij}\tilde{x}_i \gtrsim \tilde{v} - \tilde{p}(1-\lambda),\ \forall j\\&e^T\tilde{x} \approx \tilde{1}\\&\lambda \leq 1\\&\text{and } \tilde{x} \succcurlyeq \tilde{0}, \lambda \geq 0\,\text{with}\,\tilde{v} \succcurlyeq \tilde{0} \end{aligned} \tag{3.5}$$

And

$$\begin{aligned}&\text{Min}\,\tilde{w}\\&\text{subject to the constraint}\\&\sum_{j=1}^{n}\tilde{a}_{ij}\tilde{y}_j \lesssim \tilde{w} + \tilde{q}(1-\eta),\ \forall i\\&e^T\tilde{y} \approx \tilde{1}\\&\eta \leq 1\\&\text{and } \tilde{y} \succcurlyeq \tilde{0}, \eta \geq 0\,\text{with}\,\tilde{w} \succcurlyeq \tilde{0} \end{aligned} \tag{3.6}$$

Step IV Using $F : N(R) \to R$ to obtain crisp LPP.

$$\begin{aligned}&\text{Max}\,F(\tilde{v})\\&\text{subject to the constraint}\\&\sum_{i=1}^{m}F(\tilde{a}_{ij}\tilde{x}_i) \geq F(\tilde{v}) - F(\tilde{p})(1-\lambda),\ \forall j\\&F(e^T\tilde{x}) = 1\\&\lambda \leq 1\\&\text{and } \tilde{x} \succcurlyeq \tilde{0}, \lambda \geq 0\,\text{with}\,\tilde{v} \succcurlyeq \tilde{0} \end{aligned} \tag{3.7}$$

And

$$\begin{aligned}&\text{Min}\,F(\tilde{w})\\&\text{subject to the constraint}\\&\sum_{j=1}^{n}F(\tilde{a}_{ij}\tilde{y}_j) \leq F(\tilde{w}) + F(\tilde{q})(1-\eta),\ \forall i\\&F(e^T\tilde{y}) = 1\\&\eta \leq 1\\&\text{and } \tilde{y} \succcurlyeq \tilde{0}, \eta \geq 0\,\text{with}\,\tilde{w} \succcurlyeq \tilde{0} \end{aligned} \tag{3.8}$$

where $\tilde{v}=(v_1,v_2,v_3)$, $\tilde{w}=(w_1,w_2,w_3)$, $\tilde{a}_{ij}=((a_{ij})_1,(a_{ij})_2,(a_{ij})_3)$, $\tilde{x}_i=(x_{i1},x_{i2},x_{i3})$, and $\tilde{y}_j=(y_{j1},y_{j2},y_{j3})$ $\forall i=1,2,...,m$ and $\forall j=1,2,...,n$. So our problem is equivalent to solve two crisp LP problems (3.7) and (3.8) for players.

Step V Use the graded mean integration value as a defuzzification function for triangular fuzzy number keeping in view their operations which are considered in this paper, the problems (3.7) and (3.8) reforms as

$$\begin{aligned}
&\text{Max}\ \frac{(1-k)v_1+2v_2+kv_3}{3}\\
&\text{subject to the constraint}\\
&\sum_{i=1}^{m}\{(1-k)(a_{ij})_1x_{i1}+2(a_{ij})_2x_{i2}+k(a_{ij})_3x_{i3}\}\geq\\
&(1-k)v_1+2v_2+kv_3-(1-\lambda)\{(1-k)p_1+2p_2+kp_3\}\\
&\forall j=1,2,...,n\\
&\sum_{i=1}^{m}\{(1-k)x_{i1}+2x_{i2}+kx_{i3}\}=3\\
&\lambda\leq 1\\
&x_{i1},x_{i2},x_{i3},\lambda\geq 0\ \forall i=1,2,...,m
\end{aligned}\tag{3.9}$$

And

$$\begin{aligned}
&\text{Min}\ \frac{(1-k)w_1+2w_2+kw_3}{3}\\
&\text{subject to the constraint}\\
&\sum_{j=1}^{n}\{(1-k)(a_{ij})_1y_{j1}+2(a_{ij})_2y_{j2}+k(a_{ij})_3y_{j3}\}\leq\\
&(1-k)w_1+2w_2+kw_3+(1-\eta)\{(1-k)q_1+2q_2+kq_3\}\\
&\forall i=1,2,...,m\\
&\sum_{j=1}^{n}\{(1-k)y_{j1}+2y_{j2}+ky_{j3}\}=3\\
&\eta\leq 1\\
&y_{j1},y_{j2},y_{j3},\eta\geq 0\ \forall j=1,2,...,n
\end{aligned}\tag{3.10}$$

Step VI If we take $\frac{(1-k)v_1+2v_2+kv_3}{3}$ and $\frac{(1-k)w_1+2w_2+kw_3}{3}$ as V and W respectively for then the modify problems (3.9) and (3.10) are

$$\begin{aligned}
&\text{Max}\ V\\
&\text{subject to the constraint}\\
&\sum_{i=1}^{m}\{(1-k)(a_{ij})_1x_{i1}+2(a_{ij})_2x_{i2}+k(a_{ij})_3x_{i3}\}\geq\\
&3V-(1-\lambda)\{(1-k)p_1+2p_2+kp_3\}\ \forall j=1,2,...,n\\
&\sum_{i=1}^{m}\{(1-k)x_{i1}+2x_{i2}+kx_{i3}\}=3\\
&\lambda\leq 1\\
&x_{i1},x_{i2},x_{i3},\lambda\geq 0\ \forall i=1,2,...,m
\end{aligned}\tag{3.11}$$

And

$$\begin{aligned}
&\text{Min}\ W\\
&\text{subject to the constraint}\\
&\sum_{j=1}^{n}\{(1-k)(a_{ij})_1y_{j1}+2(a_{ij})_2y_{j2}+k(a_{ij})_3y_{j3}\}\leq\\
&3W+(1-\eta)\{(1-k)q_1+2q_2+kq_3\}\ \forall i=1,2,...,m\\
&\sum_{j=1}^{n}\{(1-k)y_{j1}+2y_{j2}+ky_{j3}\}=3\\
&\eta\leq 1\\
&y_{j1},y_{j2},y_{j3},\eta\geq 0\ \forall j=1,2,...,n
\end{aligned}\tag{3.12}$$

In contrast with crisp case The problems (3.11) and (3.12) are not primal-dual pair. It provides only the numerical values of V and W for the players.

4. Numerical Example

Consider the two person zero sum game having the payoff matrix

$$\tilde{A}=\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12}\\ \tilde{a}_{21} & \tilde{a}_{22}\end{bmatrix}\tag{4.1}$$

where $\tilde{a}_{11}=(175,180,190)$, $\tilde{a}_{12}=(150,156,158)$, $\tilde{a}_{21}=(80,90,100)$, $\tilde{a}_{22}=(175,180,190)$. Assuming $\tilde{p}_1\equiv\tilde{p}_2=(0.08,0.10,0.11)$ for first player and $\tilde{q}_1\equiv\tilde{q}_2=(0.14,0.15,0.17)$ for second player. To solve this payoff matrix game by the graded mean integral value as a defuzzification function for the triangular fuzzy number $\tilde{A}=(\xi_1,\xi_2,\xi_3)$ as defined as $P(\tilde{A})=\frac{(1-k)\xi_1+2\xi_2+k\xi_3}{3}$ we have to solve the following two crisp LP problems.

$$\begin{aligned}
&\text{Max}\ \frac{(1-k)v_1+2v_2+kv_3}{3}\\
&\text{subject to the constraint}\\
&175(1-k)x_{11}+360x_{12}+190kx_{13}+80(1-k)x_{21}\\
&+180x_{22}+100kx_{23}\geq(1-k)v_1+2v_2+kv_3\\
&-(1-\lambda)\{0.08(1-k)+0.20+0.11k\}\\
&150(1-k)x_{11}+312x_{12}+158kx_{13}+175(1-k)x_{21}\\
&+360x_{22}+190kx_{23}\geq(1-k)v_1+2v_2+kv_3\\
&-(1-\lambda)\{0.08(1-k)+0.20+0.11k\}\\
&(1-k)(x_{11}+x_{21})+2(x_{12}+x_{22})+k(x_{13}+x_{23})=3\\
&\lambda\leq 1\\
&\text{and}\ x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},\lambda\geq 0
\end{aligned}\tag{4.2}$$

And

$$\begin{aligned}
&\text{Min}\,\frac{(1-k)w_1+2w_2+kw_3}{3}\\
&\text{subject to the constraint}\\
&175(1-k)y_{11}+360y_{12}+190ky_{13}+150(1-k)y_{21}\\
&+312y_{22}+158ky_{23}\le(1-k)w_1+2w_2+kw_3\\
&+(1-\eta)\{0.14(1-k)+0.30+0.17k\}\\
&80(1-k)y_{11}+180y_{12}+100ky_{13}+175(1-k)y_{21}\\
&+360y_{22}+190ky_{23}\le(1-k)w_1+2w_2+kw_3\\
&+(1-\eta)\{0.14(1-k)+0.30+0.17k\}\\
&(1-k)(y_{11}+y_{21})+2(y_{12}+y_{22})+k(y_{13}+y_{23})=3\\
&\eta\le 1\\
&\text{and } y_{11},y_{12},y_{13},y_{21},y_{22},y_{23},\eta\ge 0 \qquad (4.3)
\end{aligned}$$

where $\tilde{v}=(v_1,v_2,v_3)$, $\tilde{w}=(w_1,w_2,w_3)$, $\tilde{x}_1=(x_{11},x_{12},x_{13})$, $\tilde{x}_2=(x_{21},x_{22},x_{23})$, $\tilde{y}_1=(y_{11},y_{12},y_{13})$ and $\tilde{y}_2=(y_{21},y_{22},y_{23})$. Now to obtain the full membership representation of the fuzzy value for both the players we consider v_1,v_2,v_3,w_1,w_2,w_3 be non negative such that they behave like the basic variables of the above linear programming problems. And taking $V=\frac{(1-k)v_1+2v_2+kv_3}{3}$ and $W=\frac{(1-k)w_1+2w_2+kw_3}{3}$, then we get

$$\begin{aligned}
&\text{Max}\,V\\
&\text{subject to the constraint}\\
&175(1-k)x_{11}+360x_{12}+190kx_{13}+80(1-k)x_{21}\\
&+180x_{22}+100kx_{23}\ge 3V-(1-\lambda)\{0.08(1-k)\\
&+0.20+0.11k\}\\
&150(1-k)x_{11}+312x_{12}+158kx_{13}+175(1-k)x_{21}\\
&+360x_{22}+190kx_{23}\ge 3V-(1-\lambda)\{0.08(1-k)\\
&+0.20+0.11k\}\\
&(1-k)(x_{11}+x_{21})+2(x_{12}+x_{22})+k(x_{13}+x_{23})=3\\
&\lambda\le 1\\
&\text{and } x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},\lambda\ge 0 \qquad (4.4)
\end{aligned}$$

And

$$\begin{aligned}
&\text{Min}\,W\\
&\text{subject to the constraint}\\
&175(1-k)y_{11}+360y_{12}+190ky_{13}+150(1-k)y_{21}\\
&+312y_{22}+158ky_{23}\le 3W+(1-\eta)\{0.14(1-k)\\
&+0.30+0.17k\}\\
&80(1-k)y_{11}+180y_{12}+100ky_{13}+175(1-k)y_{21}\\
&+360y_{22}+190ky_{23}\le 3W+(1-\eta)\{0.14(1-k)\\
&+0.30+0.17k\}\\
&(1-k)(y_{11}+y_{21})+2(y_{12}+y_{22})+k(y_{13}+y_{23})=3\\
&\eta\le 1\\
&\text{and } y_{11},y_{12},y_{13},y_{21},y_{22},y_{23},\eta\ge 0 \qquad (4.5)
\end{aligned}$$

5. Sensitivity Analysis and Observation

Variation of the strategies and the value of game for both the players are plotted by the solutions of the above crisp linear programming problems for different values of k,λ and η which are tabulated as follows:

Table 1. Strategy and value of game for player I

k	λ	$F(\tilde{x}_1)$	$F(\tilde{x}_2)$	V
0.00	0.00	0.7895	0.2105	161.1460
	0.25	0.7895	0.2105	161.1226
	0.50	0.7895	0.2105	161.0993
	0.75	0.7895	0.2105	161.0760
	1.00	0.7895	0.2105	161.0526
0.25	0.00	0.7377	0.2623	166.4893
	0.25	0.7377	0.2623	166.4653
	0.50	0.7377	0.2623	166.4414
	0.75	0.7377	0.2623	166.4174
	1.00	0.7377	0.2623	166.3934
0.50	0.00	0.7377	0.2623	166.4918
	0.25	0.7377	0.2623	166.4672
	0.50	0.7377	0.2623	166.4426
	0.75	0.7377	0.2623	166.4180
	1.00	0.7377	0.2623	166.3934
0.75	0.00	0.7377	0.2623	166.4943
	0.25	0.7377	0.2623	166.4691
	0.50	0.7377	0.2623	166.4439
	0.75	0.7377	0.2623	166.4187
	1.00	0.7377	0.2623	166.3934
1.00	0.00	0.7377	0.2623	166.4968
	0.25	0.7377	0.2623	166.4709
	0.50	0.7377	0.2623	166.4451
	0.75	0.7377	0.2623	166.4193
	1.00	0.7377	0.2623	166.3934

Table 2. Strategy and value of game for player II

k	η	$F(\tilde{y}_1)$	$F(\tilde{y}_2)$	W
0.00	0.00	0.2083	0.7917	155.0617
	0.25	0.2083	0.7917	155.0983
	0.50	0.2083	0.7917	155.1350
	0.75	0.2083	0.7917	155.1717
	1.00	0.2083	0.7917	155.2083
0.25	0.00	0.2083	0.7917	155.0592
	0.25	0.2083	0.7917	155.0965
	0.50	0.2083	0.7917	155.1338
	0.75	0.2083	0.7917	155.1710
	1.00	0.2083	0.7917	155.2083
0.50	0.00	0.2083	0.7917	155.0567
	0.25	0.2083	0.7917	155.0946
	0.50	0.2083	0.7917	155.1325
	0.75	0.2083	0.7917	155.1704
	1.00	0.2083	0.7917	155.2083
0.75	0.00	0.2083	0.7917	155.0542
	0.25	0.2083	0.7917	155.0927
	0.50	0.2083	0.7917	155.1313
	0.75	0.2083	0.7917	155.1698
	1.00	0.2083	0.7917	155.2083
1.00	0.00	0.2105	0.7895	160.8960
	0.25	0.2105	0.7895	160.9351
	0.50	0.2105	0.7895	160.9743
	0.75	0.2105	0.7895	161.0135
	1.00	0.2105	0.7895	161.0526

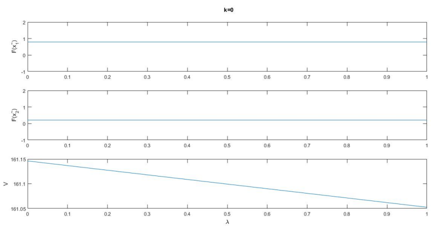

Figure 1. Variation of $F(\tilde{x}_1)$, $F(\tilde{x}_2)$, V with respect to λ for $k = 0$

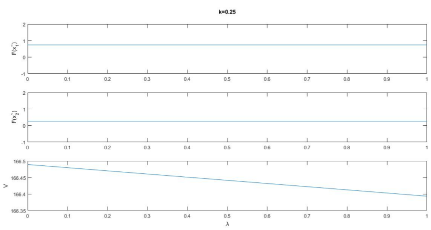

Figure 2. Variation of $F(\tilde{x}_1)$, $F(\tilde{x}_2)$, V with respect to λ for $k = 0.25$

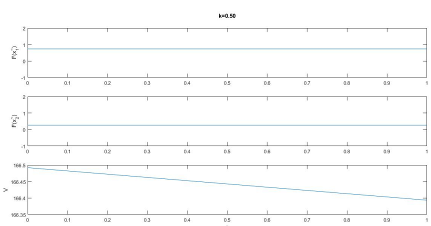

Figure 3. Variation of $F(\tilde{x}_1)$, $F(\tilde{x}_2)$, V with respect to λ for $k = 0.50$

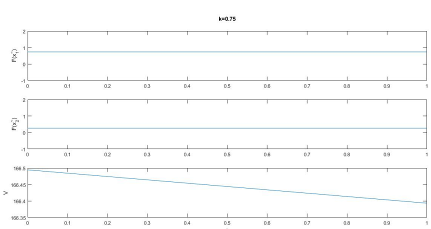

Figure 4. Variation of $F(\tilde{x}_1)$, $F(\tilde{x}_2)$, V with respect to λ for $k = 0.75$

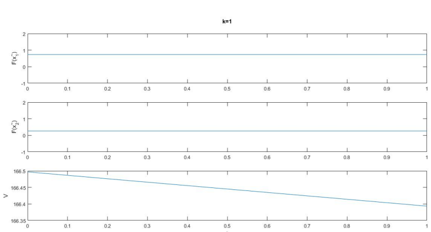

Figure 5. Variation of $F(\tilde{x}_1)$, $F(\tilde{x}_2)$, V with respect to λ for $k = 1$

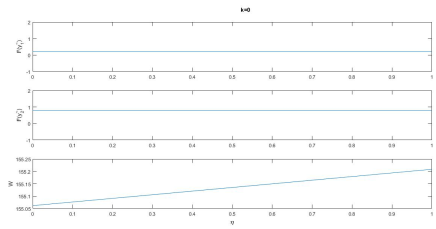

Figure 6. Variation of $F(\tilde{y}_1)$, $F(\tilde{y}_2)$, W with respect to λ for $k = 0$

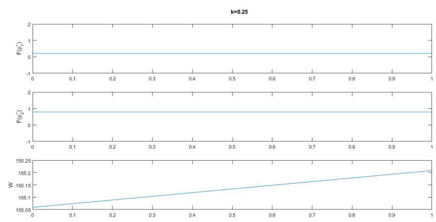

Figure 7. Variation of $F(\tilde{y}_1)$, $F(\tilde{y}_2)$, W with respect to λ for $k = 0.25$

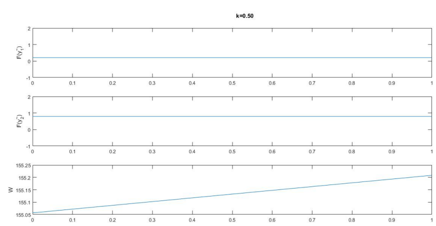

Figure 8. Variation of $F(\tilde{y}_1)$, $F(\tilde{y}_2)$, W with respect to λ for $k = 0.50$

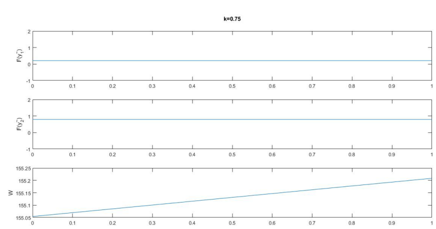

Figure 9. Variation of $F(\tilde{y}_1)$, $F(\tilde{y}_2)$, W with respect to λ for $k = 0.75$

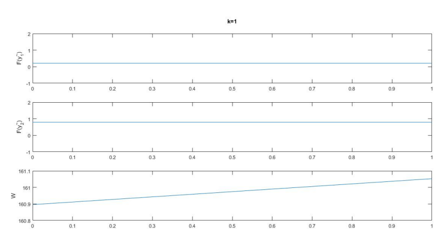

Figure 10. Variation of $F(\tilde{y}_1)$, $F(\tilde{y}_2)$, W with respect to λ for $k = 1$

6. Conclusion

In this paper, we proposed a method which provides the optimal solution of fully fuzzy two person zero sum matrix games. Triangular fuzzy numbers are considered as fuzzy goals, fuzzy payoffs and fuzzy strategies of each player in game. It is observed through sensitivity analysis of numerical example that for increasing values of λ, the value of game for player I decreases linearly and for increasing values of η, the value of game for player II increases linearly around a suitable value. There is no effect on strategies of players of parameter λ and η. The proposed method can be extended for trapezoidal, generalized triangular, intuitionistic fuzzy numbers etc. Moreover, proposed method is applicable to solve matrix games with multiple goals.

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