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# Tolerance sensitivity analysis of objective functions coefficients in multiobjective transportation problem

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**Abstract**

We have proposed a method to obtain the tolerance ranges and a symmetric tolerance range for objective functions coefficients of the multiobjective transportation problem in this paper. This method allows to change objective functions coefficients simultaneously and independently preserving the same optimal basis. We have also obtained symmetric tolerance percentage range within which objective functions coefficients of each objective function can vary in either direction. We have obtained a compromise solution using additive fuzzy programming approach as it is not possible to obtain the unique optimal solution of the multiobjective transportation problem due to conflicting nature of objective functions. This compromise solution is used for post-optimality tolerance analysis. The method is illustrated by a numerical example.

**Keywords**

Multiobjective transportation problem, compromise solution, sensitivity analysis, tolerance analysis, additive fuzzy programming, membership function.

**AMS Subject Classification**

90C31, 90C70, 90B06, 90C05, 65K05.

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## Contents



## 1. Introduction

Multiobjective transportation problem is widely used nowadays in management sciences. But the objective functions in the multiobjective transportation problem are non commensurable and conflicting in nature. In this case the study of sensitivity analysis will help the decision maker in proper decision making. Sensitivity analysis of single objective linear programming problem and the multiobjective linear programming problem is studied by many researchers. Wendell [13, 14] has presented tolerance approach for a single objective linear programming problem which allows simultaneous and independent changes in objective functions coefficients and right hand values without affecting the optimal basis. He has given the method to obtain symmetric tolerance limits of objective functions coefficients. Arsham and Oblak [2] gave another approach to obtain tolerance ranges of objective functions coefficients and right hand side values of a single objective linear programming problem. Hansen et al [6] used the concept of Wendell to obtain sensitivity and tolerance analysis ranges of parameters in the multiple objective linear problem. Deshpande [5] proposed the method of solving a pair of problems of maximization and minimization to find ranges for objective functions coefficients. Hladik and Sitarz [8] computed maximal tolerance of the multiple objective linear programming. Sitarz [12] has given a different approach

to obtain sensitivity and tolerance analysis of the multiple objective linear programming problem. Wendell and Chen [15] have reviewed all approaches proposed by researchers and extension in original results of tolerance analysis. Hladik [7] has improved Wendell's results and also applied his approach to several sensitivity invariances.

Arsham [1] used the revised simplex method to obtain ordinary sensitivity and tolerance analysis in a single objective transportation problem. He also obtained symmetric tolerance range for right hand side values and objective functions coefficients of a single objective transportation problem. Kavitha and Pandian [9] discussed sensitivity analysis of cost coefficients of an interval single objective transportation problem. King Ting Ma et al [10] has given sensitivity analysis of a single objective transportation problem for change in one cost coefficient at one time. Badra [**?** ] extended the approach of Wendell to obtain tolerance analysis of the multiobjective transportation problem. He used the weighted sum approach to solve the multiobjective transportation problem. We have derived the method to obtain the ranges for the supply and demand values of the multiobjective transportation problem in Paratane and Bit [11]. We have also obtained symmetric tolerance limit for the supply and demand values.

In this paper, we are proposing a method to obtain ordinary sensitivity analysis and tolerance analysis for objective functions coefficients of the multiobjective transportation problem using approach of Arsham and Oblak [2]. We have used the additive fuzzy linear programming (Bit et al [4]) to obtain a compromise solution of the multiobjective transportation problem. The feasibility conditions of the multiobjective transportation problem are also taken into consideration.

## 2. Mathematical model for the multiobjective transportation problem

The mathematical model for the multiobjective transportation problem [P1] is given as follows:

$$\text{Minimize } Z^{(p)}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(p)} x_{ij}, \quad p = 1,2,\cdots,P \tag{2.1}$$

subject to,

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,\cdots,m \tag{2.2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2,\cdots,n \tag{2.3}$$

$$x_{ij} \geq 0, \tag{2.4}$$

where,

$Z^{(p)}(x) =$ $p^{th}$ objective function.

$c_{ij}^{(p)} = p^{th}$ penalty criterion delivered per unit from $i^{th}$ source to $j^{th}$destination.

$x_{ij} =$ number of units to be transported from $i^{th}$ source to $j^{th}$destination.

$a_i =$ supply at $i^{th}$source, $(i = 1,2,\cdots,m)$

$b_j =$ demand at $j^{th}$destination, $(j = 1,2,\cdots,n)$

The penalty criterion could represent transportation cost, delivery time, under used capacity, quantity of goods etc. The supply and demand values must satisfy the feasibility condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The unique optimal solution of the multiobjective transportation problem [P1] can not be obtained due to conflicting nature of objective functions. We are using the additive fuzzy programming approach given by Bit et al [4] using linear membership function to obtain the compromise solution of the multiobjective transportation problem [P1] which is near to the best compromise solution. The linear membership function for each objective function $Z^{(p)}(x)$ is given as,

$$\mu^{(p)}[Z^{(p)}(x)] = \begin{cases} 1 & \text{if } Z^{(p)}(x) \leq L^{(p)} \\ \dfrac{U^{(p)} - Z^{(p)}(x)}{U^{(p)} - L^{(p)}} & \text{if } L^{(p)} < Z^{(p)}(x) < U^{(p)} \\ 0 & \text{if } Z^{(p)}(x) \geq U^{(p)} \end{cases}$$

The values of upper bound $U^{(p)}$ and lower bound $L^{(p)}$, $\forall\ p = 1,\ 2,\ \cdots, P$ can be obtained from the following payoff matrix.

| | $x^{(1)^*}$ | $x^{(2)^*}$ | $\cdots$ | $x^{(P)^*}$ |
|---|---|---|---|---|
| $Z^{(1)}(x)$ | $Z^{(1)}(x^{(1)^*})$ | $Z^{(1)}(x^{(2)^*})$ | $\cdots$ | $Z^{(1)}(x^{(P)^*})$ |
| $Z^{(2)}(x)$ | $Z^{(2)}(x^{(1)^*})$ | $Z^{(2)}(x^{(2)^*})$ | $\cdots$ | $Z^{(2)}(x^{(P)^*})$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $Z^{(P)}(x)$ | $Z^{(P)}(x^{(1)^*})$ | $Z^{(P)}(x^{(2)^*})$ | $\cdots$ | $Z^{(P)}(x^{(P)^*})$ |

Then, Lower bound $L^{(p)} = \left\{Z^{(p)}\left(x^{(p)^*}\right)\right\}$;
where $x^{(p)^*}$ is the solution of $Z^{(p)}(x)$ obtained by considering each $Z^{(p)}(x)$ one at a time (ignoring others) and
Upper bound
$U^{(p)} = Max\left\{Z^{(p)}\left(x^{(1)^*}\right),\ Z^{(p)}\left(x^{(2)^*}\right),\ \cdots,\ Z^{(p)}\left(x^{(P)^*}\right)\right\}$
$\forall\ p = 1,\ 2,\ \cdots, P.$

Then the single objective additive fuzzy programming model [P2] of the multiobjective transportation problem [P1] is:

$$\text{Maximize } V(\mu(x)) = \sum_{p=1}^{P} \mu^{(p)}[Z^{(p)}(x)] = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij} x_{ij}$$

subject to

constraints (2.2), (2.3), (2.4),

where, $\mu^{(p)}[Z^{(p)}(x)] = \dfrac{U^{(p)} - Z^{(p)}(x)}{U^{(p)} - L^{(p)}}$

$\forall\ p = 1,2,\cdots,P.$

The function $V(\mu(x))$ is called the fuzzy achievement function or the fuzzy decision function. The values $d_{ij}$ are the objective function coefficients of the objective function $V(\mu(x))$. The model [P2] is a single objective linear programming problem and can be solved by simplex method. We solve it here by *LINGO* software. Let the solution of [P2] be $X$. Then substitute $X$ in each $Z^{(p)}(x)$, $p = 1,2,\cdots,P$ to get the compromise value of each objective function. The solution report of linear programming problem when solved by *LINGO* software does not show the final optimal table values of the solution. But we can get these values by computation using the data from example. We consider the matrix form of model [P2] to get final optimal table values. The matrix form is:

$$\text{Maximize } V(\mu(x)) = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij},$$

subject to

$$AX = b,$$
$$X \geq 0, b \geq 0$$

where $A =$ coefficient matrix

$$b = [a_1, a_2, \cdots, a_m, b_1, b_2, \cdots, b_n]^T$$

and $C = [d_{11} \quad d_{12} \quad \cdots \quad d_{mn}]$

We use the following notations to represent the corresponding matrices:

$C_N$ = objective function coefficients matrix of non-basic variables.
$C_B$ = objective function coefficients matrix of basic variables.
$A_B$ = coefficient matrix of basic variables
$A_N$ = coefficient matrix of non-basic variables

The coefficients of non-basic variables of objective function in the final optimal table (i.e. the last row in the final table) can be calculated using $C_N - C_B A_B^{-1} A_N$ and the solution is given by $A_B^{-1} b$.

We will obtain here the post-optimality ranges for objective function coefficients $d_{ij}$ of [P2]. These ranges are applicable to all corresponding objective functions coefficients of the multiobjective transportation problem [P1]. We can also get equal percentage symmetric range using symmetric tolerance limits of $d_{ij}$. Now, we will consider the parametric model for problem [P2] with perturbed objective function coefficients to get the ranges in post-optimality analysis.

## 3. Post-optimality analysis of objective functions coefficients

The parametric model [P3] of linear programming model [P2] with perturbed objective function coefficients is given as:

$$\text{Maximize } V(\mu(x)) = \sum_{i=1}^{m}\sum_{j=1}^{n} [d_{ij} \pm \delta d_{ij}]\, x_{ij},$$

subject to

constraints (2.2), (2.3), (2.4)

where $\delta$ is percentage deviation in objective function coefficients $d_{ij}$. Let $d'_{ij} = \pm\delta d_{ij}$ which represents the sensitivity analysis parameters of the objective function coefficients $d_{ij}$. Thus the perturbed objective function coefficients are $d_{ij} + d'_{ij}$. Note that here $C = [d_{11} + d'_{11} \quad d_{12} + d'_{12} \quad \cdots \quad d_{mn} + d'_{mn}]$. This parametric model can not be solved directly by *LINGO* software. But we can compute the final optimal table values as given in section (2). Let $C_N^* = C_N - C_B A_B^{-1} A_N$ denote the coefficients of non basic variables with perturbed values of objective function coefficients for [P3] in the final optimal table. We have to obtain the post-optimality ranges such that $C_N^* \leq 0$ to maintain the feasibility and optimality conditions. We are using Arsham and Oblak [2] method to obtain ordinary sensitivity ranges, tolerance ranges and symmetric tolerance ranges. The ranges (limits) obtained for $d_{ij}$ by this method can be applied to the corresponding objective functions coefficients $c_{ij}^{(p)}$ of each objective function $Z^{(p)}(x)$, $p = 1,2,\cdots,P$.

### 3.1 Ordinary sensitivity analysis

In ordinary sensitivity analysis, only one objective function coefficient can vary at one time, holding all else unchanged. We can obtain the ordinary sensitivity ranges for objective functions coefficients by solving $C_N^* \leq 0$ directly for each $d'_{ij}$ substituting other $d'_{ij} = 0$; $i = 1,2,\cdots,m$ and $j = 1,2,\cdots,n$. The lower limit $d_{ij}^-$ and upper limit $d_{ij}^+$ for the objective function coefficient $d_{ij}$ for $d'_{ij} \neq 0$ can be determined as:

$$\text{Lower limit} = d_{ij}^- = \max\left\{d'_{ij} \mid d'_{ij} < 0\right\} \tag{3.1}$$

$$\text{Upper limit} = d_{ij}^+ = \min\left\{d'_{ij} \mid d'_{ij} > 0\right\} \tag{3.2}$$

If at least one $d'_{ij} = 0$ and remaining all $d'_{ij} \geq 0$, then define $d_{ij}^- = -\infty$ and $d_{ij}^+$ is as per (3.2). Similarly if at least one $d'_{ij} = 0$ and remaining all $d'_{ij} \leq 0$, then define $d_{ij}^+ = +\infty$ and $d_{ij}^-$ is as per (3.1).

### 3.2 Tolerance analysis

In tolerance analysis, the change in one or more than one objective functions coefficients can be done simultaneously and independently. We have to find the percentage deviation $\delta$ such that the feasibility conditions $C_N^* \leq 0$ should be preserved. As there may be variation in either direction in the sensitivity analysis parameters $d'_{ij}$, it takes two values for each $d'_{ij}$ viz. $d'_{ij} = \delta d_{ij}$ and $d'_{ij} = -\delta d_{ij}$. We have to consider here all combinations of $d'_{ij}$ present in all components of $C_N^*$. We will obtain value of $\delta$ by substituting $d'_{ij}$ in equations $C_N^* = 0$ for each combination. Then we get values of corresponding $d'_{ij}$. Now if coefficient of $d'_{ij}$ is positive in any $C_N^* = 0$ and value of $d'_{ij}$ is negative, then any decrease in corresponding objective function coefficient will not affect the feasibility and optimality condition $C_N^* \leq 0$ and vice versa. So we will eliminate such values of $d'_{ij}$. The rationale of eliminating $r_i$ are as follows as per given by Arsham [1] and Arsham and Oblak [2]:

1. If $d'_{ij} \neq 0$ and coefficient of $d'_{ij} \neq 0$, then any $d'_{ij}$ such that $\left(d'_{ij}\right) \times \left(\text{coefficient of } d'_{ij} \text{in } C^*_N\right) \leq 0$ does not affect feasibility. Therefore we eliminate such $d'_{ij}$.

2. If $d'_{ij} = 0$ then product is zero and the boundary point must be one of the limits for the right hand side range. Thus, although we eliminate this point, we will replace it by the allowable increase/decrease for the right hand side range.

Then the lower and upper limit for $d_{ij}$ is given by (3.1) and (3.2) respectively.

### 3.3 Symmetric tolerance limit

We get different ranges for each objective function coefficient in tolerance analysis. Wendell [13], Arsham and Oblak [2] has given one common range as symmetric tolerance limit $d^s_{ij}$ using tolerance ranges of $d_{ij}$ by which we can vary each $d_{ij}$ without affecting current basis. We can also obtain the equal percentage symmetric tolerance range (maximum allowable percentage change) using $d^s_{ij}$ for each $d_{ij}$ which is applicable to the corresponding objective functions coefficients of $Z^{(p)}(x),\ p = 1,2,\cdots,P$. Then variations in objective functions coefficients of each objective function of [P1] can be done by equal percentage symmetric range. The symmetric tolerance range $d^s_{ij}$ for each of $d_{ij}$ is given as: $d^s_{ij} = \min\left\{d^+_{ij},\ -d^-_{ij}\right\}$.
We illustrate our method in the following example.

## 4. Numerical Example

Consider the multiobjective transportation problem [E1] from Badra[3]

$$Min\ Z^{(1)}(x) = 4x_{11} + x_{12} + 3x_{13} + x_{21} + 4x_{22} + 2x_{23}$$
$$Min\ Z^{(2)}(x) = 4x_{11} + 6x_{12} + 0.5x_{13} + x_{21} + 1.5x_{22} + 7x_{23}$$

subject to

$$x_{11} + x_{12} + x_{13} = 90 \quad (4.1)$$
$$x_{21} + x_{22} + x_{23} = 90 \quad (4.2)$$
$$x_{11} + x_{21} = 60 \quad (4.3)$$
$$x_{12} + x_{22} = 60 \quad (4.4)$$
$$x_{13} + x_{23} = 60 \quad (4.5)$$
$$x_{ij} \geq 0, i = 1,2; \quad j = 1,2,3 \quad (4.6)$$

### 4.1 Compromise solution

Solving [E1] for $Z^{(1)}(x)$ separately (ignoring $Z^{(2)}(x)$), the solution is: $x^{(1)^*} : x_{12} = 60,\ x_{13} = 30,\ x_{21} = 60,\ x_{23} = 30$, and $Z^{(1)}(x^{(1)^*}) = 270,\ Z^{(2)}(x^{(1)^*}) = 645$. Similarly, solving [E1] for $Z^{(2)}(x)$ separately (ignoring $Z^{(1)}(x)$), the solution is: $x^{(2)^*} : x_{11} = 30,\ x_{13} = 60,\ x_{21} = 30,\ x_{22} = 60$, and $Z^{(2)}(x^{(2)^*}) = 270,\ Z^{(1)}(x^{(2)^*}) = 570$.
Then the pay off matrix is:

| | $x^{(1)^*}$ | $x^{(2)^*}$ |
|---|---|---|
| $Z^{(1)}$ | 270 | 570 |
| $Z^{(2)}$ | 645 | 270 |

$$U^{(1)} = 570, \quad L^{(1)} = 270; \quad U^{(1)} - L^{(1)} = 300$$
$$U^{(2)} = 645, \quad L^{(2)} = 270; \quad U^{(2)} - L^{(2)} = 375$$

Then the single objective additive fuzzy programming model [E2] for [E1] is given as:

$$\text{Maximize } Z(\mu(x)) = \frac{570 - Z^{(1)}(x)}{300} + \frac{645 - Z^{(2)}(x)}{375} = \sum_{i=1}^{2}\sum_{j=1}^{3} d_{ij}x_{ij}$$

subject to

constraints (4.1), (4.2), (4.3), (4.4), (4.5), (4.6)

Solving [E2] by *LINGO* software, we get the compromise solution of [E1] as $X : x_{12} = 30,\ x_{13} = 60,\ x_{21} = 60,\ x_{22} = 30$ and $Z^{(1)}(X) = 390,\ Z^{(2)}(X) = 315$.

### 4.2 Ranges of objective functions coefficients:

The perturbed objective function coefficients of [E2] will be:
$C = \left[-0.024 + d'_{11} \quad -0.01933 + d'_{12} \quad -0.01133 + d'_{13} \quad -0.006 + d'_{21} \quad -0.01733 + d'_{22} \quad -0.02533 + d'_{23}\right]$
Here, the basic variables are $(x_{12}, x_{13}, x_{21}, x_{22})$ and non basic variables are $(x_{11}, x_{23})$ as per solution obtained for [E1]. The compromise solution of the multiobjective transportation problem contains a set of 4 basic variables. One of the 5 constraints is redundant, due to the feasibility condition of the transportation problem. We have considered here the constraint (4.5) as a redundant constraint.(verified). The coefficients of non basic variables with perturbed values of objective function coefficients for [E2] in the final optimal table can be obtained using $C^*_N = C_N - C_B A_B^{-1} A_N$. Here
$C_B = \left[-0.01933 + d'_{12} \quad -0.01133 + d'_{13} \quad -0.006 + d'_{21} \quad -0.01733 + d'_{22}\right]$
and $C_N = \left[-0.024 + d'_{11} \quad -0.02533 + d'_{23}\right]$. Also,

$$A_B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \text{ and } A_N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_B^{-1} = \begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Then the matrix $C^*_N$ for coefficients of non basic variables $x_{11}$ and $x_{23}$ in the final table of solution of parametric model of [E2] is given as:

$$C^*_N = \left[-0.016 + d'_{11} - d'_{12} - d'_{21} + d'_{22} \quad -0.016 + d'_{12} - d'_{13} - d'_{22} + d'_{23}\right]$$

Thus, we have two expressions viz.

$$C^*_{N_1} = -0.016 + d'_{11} - d'_{12} - d'_{21} + d'_{22} \quad (4.7)$$
$$C^*_{N_2} = -0.016 + d'_{12} - d'_{13} - d'_{22} + d'_{23} \quad (4.8)$$

We will obtain the post-optimality ranges of objective function coefficients $d_{ij}$ of problem [E2] such that $C^*_{N_1} \leq 0$ and $C^*_{N_2} \leq 0$ to maintain the feasibility.

#### 4.2.1 Ordinary Sensitivity ranges

Ordinary sensitivity ranges for objective function coefficients can be obtained by solving $C_N^* \leq 0$ for each $d'_{ij}$, $i = 1,2$ and $j = 1,2,3$ substituting zero for rest of $d'_{ij}$ (Subsection(3.1)). For example, we put $d'_{12} = d'_{13} = d'_{21} = d'_{22} = d'_{23} = 0$ in $C^*_{N_1} \leq 0$ and $C^*_{N_2} \leq 0$ to obtain limits for $d'_{11}$. We obtain limits for $d'_{12}$, $d'_{13}$, $d'_{21}$, $d'_{22}$ and $d'_{23}$ by similar manner. Thus the ranges for $c_{ij}^{(1)}$ and $c_{ij}^{(2)}$, $i = 1,2$; $j = 1,2,3$ obtained are as follows: (Table 1).

| $d'_{ij}$ | Lower limit | Upper limit | Range for $c_{ij}^{(1)}$ in $Z^{(1)}$ | Range for $c_{ij}^{(2)}$ in $Z^{(2)}$ |
|---|---|---|---|---|
| $d'_{11}$ | $-\infty$ | 0.016 | $-\infty \leq c_{11}^{(1)} \leq 4.016$ | $-\infty \leq c_{11}^{(2)} \leq 4.016$ |
| $d'_{12}$ | $-0.016$ | 0.016 | $0.984 \leq c_{12}^{(1)} \leq 1.016$ | $5.984 \leq c_{12}^{(2)} \leq 6.016$ |
| $d'_{13}$ | $-0.016$ | $\infty$ | $2.984 \leq c_{13}^{(1)} \leq \infty$ | $0.484 \leq c_{13}^{(2)} \leq \infty$ |
| $d'_{21}$ | $-0.016$ | $\infty$ | $0.984 \leq c_{21}^{(1)} \leq 1.016$ | $0.984 \leq c_{21}^{(2)} \leq 1.016$ |
| $d'_{22}$ | $-0.016$ | 0.016 | $3.984 \leq c_{22}^{(1)} \leq 4.016$ | $1.484 \leq c_{22}^{(2)} \leq 1.516$ |
| $d'_{23}$ | $-\infty$ | 0.016 | $-\infty \leq c_{23}^{(1)} \leq 2.016$ | $-\infty \leq c_{23}^{(2)} \leq 7.016$ |

**Table 1.** Ordinary sensitivity ranges for $c_{ij}^{(1)}$ and $c_{ij}^{(2)}$

#### 4.2.2 Tolerance Ranges

We have to obtain the acceptable changes in objective function coefficients of [E2] such that $C_N^* \leq 0$ viz. $C^*_{N_1} \leq 0$ and $C^*_{N_2} \leq 0$. The components of $C_N^*$ contains sensitivity analysis parameters of all objective function coefficients and each $d'_{ij} = \pm\delta d_{ij}$, $i = 1,2$; $j = 1,2,3$. Thus we have $d'_{11} = \pm 0.024\delta$, $d'_{12} = \pm 0.01933\delta$, $d'_{13} = \pm 0.01133\delta$, $d'_{21} = \pm 0.006\delta$, $d'_{22} = \pm 0.01733\delta$ and $d'_{23} = \pm 0.02533\delta$. We have total $2^6$ vectors of values of $d'_{ij}$. However, two of these vectors are with opposite signs and represents two lines in opposite directions along same line. So we consider only 32 cases here only for calculation purpose. We will get the same ranges although we consider all $2^6$ combinations. It is difficult to obtain all $2^6$ combinations manually so we have obtained it by using *python programming*. Now we will obtain values of $d'_{ij}$ as given in subsection (3.2). Some of 32 combinations of values of $d'_{ij}$ are given in the table (2).

| Case | $d'_{11}$ | $d'_{12}$ | $d'_{13}$ | $d'_{21}$ | $d'_{22}$ | $d'_{23}$ |
|---|---|---|---|---|---|---|
| 1 | $0.024\delta$ | $0.01933\delta$ | $0.01133\delta$ | $0.006\delta$ | $0.01733\delta$ | $0.02533\delta$ |
| 2 | $0.024\delta$ | $0.01933\delta$ | $0.01133\delta$ | $0.006\delta$ | $0.01733\delta$ | $-0.02533\delta$ |
| 3 | $0.024\delta$ | $0.01933\delta$ | $0.01133\delta$ | $0.006\delta$ | $-0.01733\delta$ | $0.02533\delta$ |
| 4 | $0.024\delta$ | $0.01933\delta$ | $0.01133\delta$ | $0.006\delta$ | $-0.01733\delta$ | $-0.02533\delta$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 31 | $-0.024\delta$ | $0.01933\delta$ | $-0.01133\delta$ | $-0.006\delta$ | $-0.01733\delta$ | $0.02533\delta$ |
| 32 | $-0.024\delta$ | $0.01933\delta$ | $-0.01133\delta$ | $-0.006\delta$ | $-0.01733\delta$ | $-0.02533\delta$ |

**Table 2.** Values of $d'_{ij}$

Now, we consider equations $C^*_{N_1} = 0$ and $C^*_{N_2} = 0$ to get value of $d'_{ij}$ as given in subsection (3.2). The values of $d'_{ij}$ obtained are as follows:(given in Table 3 and 4).

| | | $C^*_{N_1} = 0$ | | | | | |
|---|---|---|---|---|---|---|---|
| | Coeff. of $d'_{ij}$ | 1 | $-1$ | 0 | $-1$ | 1 | 0 |
| Case | **value of** $\delta\downarrow$ | $d'_{11}$ | $d'_{12}$ | $d'_{13}$ | $d'_{21}$ | $d'_{22}$ | $d'_{23}$ |
| 1 | 1 | 0.024 | 0.019 | 0.011 | 0.006 | 0.017 | 0.025 |
| 2 | 1 | 0.024 | 0.019 | 0.011 | 0.006 | 0.017 | -0.025 |
| 3 | -0.88 | -0.021 | -0.017 | -0.01 | -0.005 | 0.015 | -0.022 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | -0.29 | 0.007 | -0.006 | 0.003 | 0.0018 | 0.005 | -0.007 |
| | -0.29 | 0.007 | -0.006 | 0.003 | 0.0018 | 0.005 | 0.007 |

**Table 3.** Values of $d'_{ij}$ using $C^*_{N_1} = 0$

| | | $C^*_{N_2} = 0$ | | | | | |
|---|---|---|---|---|---|---|---|
| | Coeff. of $d'_{ij}$ | 0 | 1 | $-1$ | 0 | $-1$ | 1 |
| Case | **value of** $\delta\downarrow$ | $d'_{11}$ | $d'_{12}$ | $d'_{13}$ | $d'_{21}$ | $d'_{22}$ | $d'_{23}$ |
| 1 | 1 | 0.024 | 0.019 | 0.011 | 0.006 | 0.017 | 0.025 |
| 2 | -0.46 | -0.01 | -0.009 | -0.005 | -0.003 | -0.008 | 0.012 |
| 3 | 0.32 | 0.008 | 0.006 | 0.004 | 0.002 | -0.005 | 0.008 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 31 | 0.22 | -0.005 | 0.004 | -0.002 | -0.001 | -0.004 | 0.006 |
| 32 | 0.70 | -0.02 | 0.01 | -0.008 | -0.004 | -0.01 | -0.02 |

**Table 4.** Values of $d'_{ij}$ using $C^*_{N_2} = 0$

We will eliminate those values of $d'_{ij}$ for which $d'_{ij} * \left(\text{coefficient of } \ d'_{ij} \text{in } \ C_N^*\right) \leq 0$ for any $i$, $j$ as they do not affect the optimality and feasibility conditions. (refer subsection (3.2)). After eliminating such $d'_{ij}$'s, the lower and upper limits for $d'_{ij}$ as well as symmetric tolerance limits (obtained as given in subsection (3.3)) and equal symmetric tolerance percentage is calculated in the following table (5):

| $d'_{ij}$ | Lower limit $d_{ij}^-$ | Upper limit $d_{ij}^+$ | Symmetric tolerance limit $d_{ij}^s$ | Equal Percentage Symmetric Tolerance Range |
|---|---|---|---|---|
| $d'_{11}$ | -0.0052 | 0.0052 | 0.0052 | 21.82% |
| $d'_{12}$ | -0.0046 | 0.0042 | 0.0042 | 21.82% |
| $d'_{13}$ | -0.0025 | 0.0027 | 0.0025 | 21.82% |
| $d'_{21}$ | -0.0013 | 0.0013 | 0.0013 | 21.82% |
| $d'_{22}$ | -0.0039 | 0.0042 | 0.0038 | 21.82% |
| $d'_{23}$ | -0.0061 | 0.0055 | 0.0055 | 21.82% |

**Table 5.** Equal Percentage Symmetric Tolerance range

We can see that the symmetric tolerance percentage range for $d_{ij}$ is 21.82%, i.e we can decrease or increase the objective functions coefficients of each objective $Z^{(1)}$ and $Z^{(2)}$ by 21.82% without affecting the current basis.

Then the ranges of objective function coefficients of objectives $Z^{(1)}$ and $Z^{(2)}$ of the multiobjective transportation problem [E1] are obtained as follows: (Table (6) and (7))

| Objective Function $Z^{(1)}$ | | |
|---|---|---|
| Coefficients $c_{ij}^{(1)}$ | Lower limit $c_{ij}^{(1)-}$ | Upper limit $c_{ij}^{(1)+}$ |
| $c_{11}^{(1)}$=4 | 3.1272 | 4.8728 |
| $c_{12}^{(1)}$=1 | 0.7818 | 1.2182 |
| $c_{13}^{(1)}$=3 | 2.3454 | 3.6546 |
| $c_{21}^{(1)}$=1 | 0.7818 | 1.2182 |
| $c_{22}^{(1)}$=4 | 3.1272 | 4.8728 |
| $c_{23}^{(1)}$=2 | 1.5636 | 2.4364 |

**Table 6.** Ranges of objective functions coefficients of $Z^{(1)}$

| Objective Function $Z^{(2)}$ | | |
|---|---|---|
| Coefficients $c_{ij}^{(2)}$ | Lower limit $c_{ij}^{(2)-}$ | Upper limit $c_{ij}^{(2)+}$ |
| $c_{11}^{(2)}$=4 | 3.1272 | 4.8728 |
| $c_{12}^{(2)}$=6 | 4.6908 | 7.3092 |
| $c_{13}^{(2)}$=0.5 | 0.3909 | 0.6091 |
| $c_{21}^{(2)}$=1 | 0.7818 | 1.2182 |
| $c_{22}^{(2)}$=1.5 | 1.1727 | 1.8273 |
| $c_{23}^{(2)}$=7 | 5.4726 | 8.5274 |

**Table 7.** Ranges of objective functions coefficients of $Z^{(2)}$

## 5. Conclusion

In this paper, we have proposed a method to obtain the symmetric tolerance ranges and also the equal percentage symmetric range for objective functions coefficients of $Z^{(1)}$ and $Z^{(2)}$ within which it can be changed without affecting the current basis. The equal percentage symmetric range for objective functions coefficients of $Z^{(1)}$ and $Z^{(2)}$ is 21.82% as shown in table(5). Badra [3] has calculated it as 11.11% for the same numerical example [E1]. He used weighted approach to convert the multiobjective transportation problem into single objective transportation problem in which the objective function coefficients are not unit free.

Here, we have applied the additive fuzzy programming approach to obtain the compromise solution of the multiobjective transportation problem. The objective function coefficients in the corresponding single objective linear programming model obtained by this approach are unit free. The compromise solution is obtained by using *LINGO* software. All matrix operations are performed using *Scilab* software for the post-optimality analysis and *Python programming* is used to obtain different cases of $d'_{ij}$ which are present in $C^*_{N_1}=0$ and $C^*_{N_2}=0$. The tolerance ranges and symmetric tolerance ranges are calculated using *Excel Spreadsheet*. The ranges of objective functions coefficients of objectives $Z^{(1)}$ and $Z^{(2)}$ are shown in table (6) and table (7) respectively. Thus our proposed method is more suitable and easy to apply for medium size problems.

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