Malaya Journal of Matematik, Vol. 8, No. 3, 815-821, 2020

https://doi.org/10.26637/MJM0803/0014

M-Fuzzy hyponormal operators

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Abstract

In this paper, We introduce the definition of M-fuzzy hyponormal operator and also explored about some important properties of M-Fuzzy hyponormal operator from Fuzzy hyponormal operators in fuzzy Hilbert space. For a fuzzy continuous linear operator $\mathcal{T}$ on a Fuzzy Hilbert space $\mathcal{H}$ there exists a real number $\mathrm{M} \ni$ if $\left\|(\mathcal{T}-zI)^{*} u\right\| \leq \mathrm{M}\|(\mathcal{T}-zI) u\|$ for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$ (field of complex numbers).We have given some definitions which are related to M-fuzzy hyponormal operator in fuzzy Hilbert space.

Keywords

Adjoint fuzzy operator, Fuzzy Hilbert space(FH-space), Fuzzy Hyponormal operator, Fuzzy Normal operator, M-Fuzzy Hyponormal operator (MFHO), Self-Adjoint fuzzy operator.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

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Article History: Received **24** January **2020**; Accepted **21** April **2020**

Contents

1. Introduction

In very first, Katsaras[4] introduced the notation of Fuzzy Norm on a linear space in 1984. In 1991, First Biswas[10] introduced the definition of Fuzzy Inner Product space[FIP space].Some other mathematicians like Kohil and Kumar also gave an alternate definitions of Fuzzy Inner Product space[FIP-space] in[6][8][9][10]. Youngfusu[13] gave Riesz theorem using fuzzy concept in 2007.The definition of a Fuzzy Hilbert space has been first introduced by Goudarzi and Vaezpour [8] in 2009. The concept and properties of Adjoint Fuzzy operator and Self-adjoint fuzzy operators using the triplet , $(\mathcal{H}, \mathcal{F}, *)$ where $\mathcal{H}$ is a real vector space, $\mathcal{F}$ is the Fuzzy Set on $\mathcal{H}^{2} \times \mathrm{R}$ and $*$ is continuous t- norm in a Fuzzy Hilbert space which was first introduced by Sudad M Rasheed [3].
An operator $\mathcal{T} \in \mathrm{FB}(\mathcal{H})$ be a $\tau_{\mathcal{F}}$ Continuous linear functional $\exists\ \mathcal{T}^{*} \in \mathrm{FB}(\mathcal{H})$ such that $\langle\mathcal{T} u, v\rangle=\langle\mathcal{T}^{*} u, v\rangle$,$\forall u, v \in \mathcal{T}$.Also $\mathcal{T}$ is a Self - Adjoint Fuzzy operator if $\mathcal{T}=\mathcal{T}^{*}$.Fuzzy Normal operator (FN operator) was introduced by Radharamani et al.[2] in 2008 . If $\mathcal{T}$ is said to be Fuzzy Normal operator then it commutes with its Ajoint Fuzzy operator. i.e. $\mathcal{T}^{*} \mathcal{T}=\mathcal{T} \mathcal{T}^{*}$. In 2019, Fuzzy Hyponormal operators and their properties were studied by A.Radharamani et al.[1, 18] and investigate many interesting properties of some Fuzzy Hyponormal operators similar to these of Fuzzy Normal operators.An operator $\mathcal{T} \in$ FB $(\mathcal{H})$ is said to be Fuzzy Hyponormal if $\mathcal{T}^{*} \mathcal{T} \geq \mathcal{T} \mathcal{T}^{*}$.Also Fuzzy class of N operators were defined if $\left\|\mathcal{T}^{2} u\right\| \geq\|\mathcal{T} u\|^{2} \forall u \in \mathcal{H},\|u\|=1$.
Here we introduced M-Fuzzy hyponormal operator if

$$\left\|(\mathcal{T}-zI)^{*} u\right\| \leq \mathrm{M}\|(\mathcal{T}-zI) u\|$$

for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$ (field of complex numbers).We have given some lemmas and properties of M-Fuzzy Hyponormal operator. An operator $\mathcal{T} \in$ FB $(\mathcal{H})$ is M-Fuzzy Hyponormal operator then $\left\|(\mathcal{T}-\bar{z} I)^{-1} u\right\| \leq \mathrm{M}\left\|(\mathcal{T}-zI)^{-1} u\right\|$ for each $u \in \mathcal{H}$.An operator $\mathcal{T} \in$ FB $(\mathcal{H})$ is M-Fuzzy hyponormal operator then their powers also M-Fuzzy hyponormal operator.We will also discuss these in detail. If an operator $\mathcal{T}$ is a M -Fuzzy hyponormal operator then $\mathrm{M} \geq 1$ and $\mathcal{T}$ is fuzzy hyponormal operator iff $\mathrm{M}=1$.

2. Preliminaries

In this section , we recollect some basic definitions and theorems of fuzzy operators which will be used in this paper.

Definition 2.1. ***[FIP-Space][8]*** *A Fuzzy Inner Product space (FIP- Space) is a triplet $(\mathcal{H},\mathcal{F},*)$, where $\mathcal{H}$ is a real vector space, $*$ is a continuous t-norm, $\mathcal{F}$ is a fuzzy set on $\mathcal{H}^2 \times \mathrm{R}$ satisfying the following conditions for every $u,v,w \in \mathcal{H}$ and $s,r,t \in \mathrm{R}$.*

FI-1: $\mathcal{F}(u,u,0) = 0$ *and* $\mathcal{F}(u,u,t) > 0$*, for each* $t > 0$

FI-2: $\mathcal{F}(u,u,t) \neq H(t)$ *for some* $t \in \mathrm{R}$ *if and only if* $u \neq 0$*, where*

$$H(t) = \begin{cases} 1 & if\ t > 0 \\ 0 & if\ t \geq 0 \end{cases}$$

FI-3: $\mathcal{F}(u,v,t) = \mathcal{F}(v,u,t)$

FI-4: *For all* $\alpha \in R$,

$$\mathcal{F}(\alpha u,v,t) = \begin{cases} \mathscr{F}(u,v,\frac{t}{\alpha}) & : \alpha > 0 \\ H(t) & : \alpha = 0 \\ 1-\mathcal{F}(x,y,\frac{t}{\alpha}) & : \alpha < 0 \end{cases}$$

FI-5: $\mathcal{F}(u,u,t) * \mathcal{F}(v,v,s) \leqslant \mathcal{F}(u+v,u+v,t+s)$

FI-6: $Sup_{s+r=t}[\mathcal{F}(u,w,s) * \mathcal{F}(v,w,r)] = F(u+v,w,t)$

FI-7: $\mathcal{F}(u,v,.) : \mathrm{R} \to [0,1]$ *is continuous on* $\mathrm{R} \setminus \{0\}$

FI-8: $\lim_{t\to\infty} \mathcal{F}(u,v,t) = 1$

Definition 2.2. *[13] Let $(\mathcal{H},\mathcal{F},*)$ be probabilistic inner product space.*

1. *A sequence $\{u_n\} \in \mathcal{H}$ is called τ- converges to $u \in \mathcal{H}$, if for any $\in > 0$ and $\lambda > 0$, $\exists\ N \in Z^+$, $N = N(\varepsilon,\lambda)$ such that $\mathcal{F}(u_n - u, u_n - u, \varepsilon) > 1-\lambda$, whenever $n > N$.*
2. *A linear Functional $f(u)$ defined on $\mathcal{H}$ is called $\tau_{\mathcal{F}}$ - continuous, if $u_n \overset{\tau_{\mathcal{F}}}{\to} u$ implies $f(u_n) \overset{\tau_{\mathcal{F}}}{\to} f(u)$ for any $\{u_n\}, u \in \mathcal{H}$.*

Definition 2.3. *[8] Let $(\mathcal{H},\mathcal{F},*$ be a FIP-space, where $*$ is strong t - norm, and for each $u,v \in \mathcal{H}$, $\sup\{t \in R : \mathcal{F}(u,v,t) < 1\} < \infty$. Define $\langle .,. \rangle : \mathcal{H} \times \mathcal{H} \to \mathrm{R}$ by*

$$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}.$$

Then $(\mathcal{H},\langle .,. \rangle)$ is an IP-space(Inner Product space), so that $(\mathcal{H},\|.\|)$ is a N-space(Normed space), where $\|.\| = \langle u,u \rangle^{1/2}$, $\forall u \in \mathcal{H}$.

Theorem 2.4. *[8] Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$, $\forall\ u,v \in \mathcal{H}$ for $u_n \in \mathcal{H}$ and $u_n \overset{\|.\|}{\to} u$ then $u_n \overset{\tau_{\mathcal{F}}}{\to} u$.*

Theorem 2.5. ***[Riesz Theorem][8][13]*** *Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space. For any $\tau_{\mathcal{F}}$ -continuous functional, $\exists$ unique $v \in \mathcal{H}$ such that for all $u \in \mathcal{H}$, then we have*

$$g(x) = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$$

Theorem 2.6. *[3] Let $(\mathcal{H},\mathcal{F},*)$ be a FIP-space, where $*$ is strong t - norm , and $\sup\{u \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\} < \infty$ for all $u,v \in \mathcal{H}$, then*

$$\sup\{t \in \mathrm{R} : \mathcal{F}(u+v,w,t) < 1\} = \sup\{t \in \mathrm{R} : \mathcal{F}(u,w,t) < 1\} + \sup\{t \in \mathrm{R} : \mathcal{F}(v,w,t) < 1\}, \quad \forall u,v,w \in \mathcal{H}$$

Remark 2.7. *[3] Let $FB(\mathcal{H})$ be the set of all Fuzzy Bounded linear operators on $\mathcal{H}$.*

Definition 2.8. ***[Fuzzy Hilbert Space][8]*** *Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$,$\forall u,v \in \mathcal{H}$. If $\mathcal{H}$ is complete in the $\|.\|$ then $\mathcal{H}$ is called Fuzzy Hilbert space (FH -space).*

Theorem 2.9. ***[Adjoint Fuzzy operator in FH-space][3]*** *Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space and let $\mathcal{T} \in FB(\mathcal{H})$ be $\tau_{\mathcal{F}}$ - continuous linear functional, then $\exists$ unique $\mathcal{T}^* \in FB(\mathcal{H})$ such that $\langle \mathcal{T}u,v \rangle = \langle u,\mathcal{T}^*v \rangle$,$\forall u,v \in \mathcal{H}$.*

Definition 2.10. ***[Self -Adjoint Fuzzy operator][3]*** *Let $(\mathcal{H},\mathcal{F},*)$ be a Fuzzy Hilbert space with IP:*

$$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\},$$

$\forall u,v \in \mathcal{H}$ and let $\mathcal{T} \in FB(\mathcal{H})$. Then $\mathcal{T}$ is Self -adjoint fuzzy operator if $\mathcal{T}$=$\mathcal{T}^*$, where $\mathcal{T}^*$ is adjoint fuzzy operator of $\mathcal{T}$.

Theorem 2.11. *[3] Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$,$\forall u,v \in \mathcal{H}$. Let $\mathcal{T} \in FB(\mathcal{H})$ .Then $\|\mathcal{T}u\| = \|\mathcal{T}^*u\|$ for all $u \in \mathcal{H}$.*

Theorem 2.12. *[3] Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$ for all $u \in \mathcal{H}$. Let $\mathcal{T} \in FB\ (\mathcal{H})$. Then $\mathcal{T}$ is a Self -adjoint fuzzy operator.*

Definition 2.13. ***[Fuzzy Isometry Operator][2]*** *Let$(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$, $\forall u,v \in \mathcal{H}$. and let $\mathcal{T} \in FB\ (\mathcal{H})$. Then $\mathcal{T}$ is said to be Fuzzy Isometry Operator if $\|\mathcal{T}u\| = \|u\|$ for any $u \in \mathcal{H}$. i.e. $\langle \mathcal{T}u,\mathcal{T}v \rangle = \langle u,v \rangle$.*

Definition 2.14. ***[Fuzzy Hyponormal operator][1]*** *Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$, $\forall u,v \in \mathcal{H}$. Let $\mathcal{T} \in FB\ (\mathcal{H})$. Then $\mathcal{T}$ is a Fuzzy Hyponormal operator if $\|\mathcal{T}^*u\| \leq \|\mathcal{T}u\|$, $\forall u \in \mathcal{H}$ or equivalently $\mathcal{T}^*\mathcal{T} - \mathcal{T}\mathcal{T}^* \geq 0$.*

Theorem 2.15. *[1] Let $\mathcal{T} \in FB\ (\mathcal{H})$ be Fuzzy hyponormal iff $\|\mathcal{T}^*u\| \leq \|\mathcal{T}u\|$, for all $u \in \mathcal{H}$.*

Theorem 2.16. *[1] Let $\mathcal{T} \in FB\ (\mathcal{H})$. Then $\mathcal{T} = 0 \Leftrightarrow \langle \mathcal{T}u,u \rangle = 0$, $u \in \mathcal{H}$.*

Theorem 2.17. *[1] Let $(\mathcal{H},\mathcal{F},*)$ be a FH-space with IP:$\langle u,v \rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$, $\forall u,v \in \mathcal{H}$ and let $\mathcal{T} \in FB(\mathcal{H})$ be a Fuzzy hyponormal operator on $\mathcal{H}$. Then $\|(\mathcal{T} - zI)u\| \geq \|(\mathcal{T}^* - \bar{z}I)u\|$, $\forall u \in \mathcal{H}$, i.e. $\mathcal{T} - zI$ is Fuzzy hyponormal operator.*

3. Main Results of M- Fuzzy Hyponormal Operator

In this section we introduced the definition of M- Fuzzy hyponormal operator and some theorems are discussed in detail.

Definition 3.1. ***[M-Fuzzy hyponormal operator]*** *Let* $(\mathcal{H},\mathcal{F},*)$ *be a FH-space with IP :* $\langle u,v\rangle = \sup\{t \in \mathrm{R} : \mathcal{F}(u,v,t) < 1\}$, $\forall u,v \in \mathcal{H}$ *and let* $\mathcal{T} \in FB(\mathcal{H})$ *is called a* M*-Fuzzy hyponormal operator if there exist a real number* M, $\ni \|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}\|(\mathcal{T}-zI)u\|$ *for all* $u \in \mathcal{H}$ *and for all* $z \in \mathrm{C}$.

Theorem 3.2. $\mathcal{T}$ *is a* M*-Fuzzy hyponormal operator iff* $\mathrm{M}^2 (\mathcal{T}-zI)^*(\mathcal{T}-zI) \ -(\mathcal{T}-zI)(\mathcal{T}-zI)^* \geq 0$ *for all* $z \in \mathrm{C}$.

Proof. Given $\mathcal{T}$ is a M-Fuzzy hyponormal operator,

i.e. $\|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}\|(\mathcal{T}-zI)u\|$

Consider,

$$\begin{aligned}
&\mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI)-(\mathcal{T}-zI)(\mathcal{T}-zI)^* \geq 0\\
\Leftrightarrow&\langle \mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI)-(\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u\rangle \geq 0\\
\Leftrightarrow&\langle \mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI)u,u\rangle-\langle(\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u\rangle \geq 0\\
\Leftrightarrow&\mathrm{M}^2\sup\{t\in\mathrm{R}:\mathcal{F}((\mathcal{T}-zI)^*(\mathcal{T}-zI)u,u,t)<1\}\\
&\quad-\sup\{t\in\mathrm{R}:\mathcal{F}((\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u,t)<1\}\geq 0\\
\Leftrightarrow&\mathrm{M}^2\sup\{t\in\mathrm{R}:\mathcal{F}((\mathcal{T}-zI)u,(\mathcal{T}-zI)u,t)<1\}\\
&\quad-\sup\{t\in\mathrm{R}:\mathcal{F}((\mathcal{T}-zI)^*u,(\mathcal{T}-zI)^*u,t)<1\}\geq 0\\
\Leftrightarrow&\mathrm{M}^2\langle(\mathcal{T}-zI)u,(\mathcal{T}-zI)u\rangle-\langle(\mathcal{T}-zI)^*u,(\mathcal{T}-zI)^*u\rangle\geq 0\\
\Leftrightarrow&\mathrm{M}^2\|(\mathcal{T}-zI)u\|^2-\|(\mathcal{T}-zI)^*u\|^2\geq 0\\
\Leftrightarrow&\mathrm{M}^2\|(\mathcal{T}-zI)u\|^2\geq\|(\mathcal{T}-zI)^*u\|^2\\
\Leftrightarrow&\mathrm{M}\|(\mathcal{T}-zI)u\|\geq\|(\mathcal{T}-zI)^*u\|
\end{aligned}$$

$\Leftrightarrow\mathcal{T}$ is M- Fuzzy hyponormal operator

□

Theorem 3.3. *Let* $\mathcal{T}$ *is a* M *- Fuzzy hyponormal operator then* $\mathrm{M} \geq 1$.$\mathcal{T}$ *is Fuzzy hyponormal operator iff* $\mathrm{M} = 1$.

Proof. Given $\mathcal{T}$ is a M− Fuzzy hyponormal operator then $\mathrm{M} \geq 1$.
i.e $\|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}\|(\mathcal{T}-zI)u\|$ for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$.
If $\mathrm{M} = 1$, then
$\|(\mathcal{T}-zI)^*u\| \leq 1.\|(\mathcal{T}-zI)u\|$ for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$ by the definition
$\Leftrightarrow$ $\mathcal{T}$ is M Fuzzy hyponormal operator □

Theorem 3.4. *Let* $\mathcal{T}$ *is a* M*-Fuzzy hyponormal operator then for any complex number* λ, $\mathcal{T}-\lambda I$ *and* $\lambda\mathcal{T}$ *are also* M *- Fuzzy hyponormal operator*

Proof. Given $\mathcal{T}$ is an M-Fuzzy hyponormal operator,

i). For any complex number λ,we consider

$$\begin{aligned}
\|((\mathcal{T}-\lambda I)-zI)^*u\| &= \|(\mathcal{T}-\lambda I-zI)^*u\|\\
&= \|(\mathcal{T}-(\lambda+z)I)^*u\|
\end{aligned}$$

Since $\mathcal{T}$ is M -Fuzzy hyponormal operator, we have

$$\begin{aligned}
\|(\mathcal{T}-zI)^*u\| &\leq \mathrm{M}\|(\mathcal{T}-zI)u\|\\
\|((\mathcal{T}-\lambda I)-zI)^*u\| &\leq \mathrm{M}\|(\mathcal{T}-(\lambda+z)I)u\|\\
&\leq \mathrm{M}\|((\mathcal{T}-\lambda I)-zI)u\|,\\
&\qquad \forall u\in\mathcal{H}
\end{aligned}$$

Therefore, $\mathcal{T}-\lambda I$ is M-Fuzzy hyponormal operator.

ii). To prove $\lambda\mathcal{T}$ is M - Fuzzy hyponormal operator
If λ=0, then $\lambda\mathcal{T}=0$.
Since $\mathcal{T}$ is M - Fuzzy hyponormal operator then

$$\|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}\|(\mathcal{T}-zI)u\|$$

If $\lambda \neq 0$, consider

$$\begin{aligned}
\|(\lambda\mathcal{T}-zI)^*u\| &\leq |\lambda|\left\|\left(\mathcal{T}-\left(\frac{z}{\lambda}\right)I\right)^*u\right\|\\
&\leq |\lambda|\mathrm{M}\left\|\left(\mathcal{T}-\left(\frac{z}{\lambda}\right)I\right)u\right\|\\
&\leq \mathrm{M}\|(\lambda\mathcal{T}-zI)u\|
\end{aligned}$$

Therefore $\lambda\mathcal{T}$ is M-fuzzy hyponormal operator.

□

Theorem 3.5. *If* $\mathcal{T}$ *is* M *- Fuzzy hyponormal operator* $\mathcal{T}u = zu$ *then* $\mathcal{T}^*u = \bar{z}u$.

Proof. Given $\mathcal{T}$ is M -Fuzzy hyponormal operator.
Let 'u' be an eigen vector of $\mathcal{T}$ corresponding to the eigen value z.
Since, $\mathcal{T}u = zu$

$$\begin{aligned}
&\Rightarrow\mathcal{T}u-zu=0\\
&\Rightarrow\mathcal{T}u-zIu=0\\
&\Rightarrow(\mathcal{T}-zI)u=0\\
&\Rightarrow(\mathcal{T}-zI)=0
\end{aligned}$$

We know that, $\mathcal{T}=0$ iff $\langle\mathcal{T}u,\mathcal{T}u\rangle=0$.

$$\begin{aligned}
&\Rightarrow\sup\{t\in\mathrm{R}:\mathcal{F}((\mathcal{T}-zI)^*u,(\mathcal{T}-zI)^*u,t)\}=0\\
&\Rightarrow(\mathcal{T}-zI)^*u=0 \text{ then for each } u\in\mathcal{H}\\
&\Rightarrow(\mathcal{T}^*-\bar{z}I)u=0\\
&\Rightarrow\mathcal{T}^*-\bar{z}Iu=0\\
&\therefore\mathcal{T}^*u=\bar{z}u
\end{aligned}$$

Thus, u is an eigen vector of $\mathcal{T}^*$ corresponding to a eigen value $\bar{z}$. □

Theorem 3.6. *$\mathcal{T}$ is M -Fuzzy hyponormal operator. If $\mathcal{T}u = z_1u$ and $\mathcal{T}u = z_2u$, $z_1 \neq z_2$ then $\langle u,v\rangle = 0$.*

Proof. Since $\mathcal{T}$ is M -Fuzzy hyponormal operator.
If $\mathcal{T}u = z_1u$ and $\mathcal{T}u = z_2u$, $z_1 \neq z_2$
then by the theorem, $\mathcal{T}^*u = \bar{z}_1u$ and $\mathcal{T}^*v = \bar{z}_2v$

$$\begin{aligned}
\text{Let } z_1\langle u,v\rangle =& \langle z_1u,v\rangle\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}(z_1u,v,t) < 1\}\\
=& \sup\{t \in \mathrm{R}; \mathcal{F}(\mathcal{T}u,v,t) < 1\}\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}(u,\mathcal{T}^*v,t) < 1\}\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}(u,\bar{z}_2v,t) < 1\}\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}(z_2u,v,t) < 1\}\\
=& \langle z_2u,v\rangle\\
z_1\langle u,v\rangle =& z_2\langle u,v\rangle
\end{aligned}$$

Hence if $z_1 \neq z_2$, then $\langle u,v\rangle = 0$ □

Theorem 3.7. *If $\mathcal{T}$ is M -Fuzzy hyponormal operator then $\|(\mathcal{T}^* - \bar{z}I)^{-1}u\| \leq \mathrm{M}\|(\mathcal{T} - zI)^{-1}u\|$ for each $u \in \mathrm{H}$.*

Proof. For each $u \in \mathrm{H}$,

$$\begin{aligned}
\|(\mathcal{T}^* - \bar{z}I)^{-1}u\|^2 \leq& \langle(\mathcal{T}^* - \bar{z}I)^{-1}u, (\mathcal{T}^* - \bar{z}I)^{-1}u\rangle\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}^* - \bar{z}I)^{-1}u,\\
& (\mathcal{T}^* - \bar{z}I)^{-1}u,t) < 1\}\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}(((\mathcal{T} - zI)^*)^{-1}u,\\
& ((\mathcal{T} - zI)^*)^{-1}u,t) < 1\}\\
=& \langle((\mathcal{T} - zI)^*)^{-1}u, ((\mathcal{T} - zI)^*)^{-1}u\rangle\\
=& \|((\mathcal{T} - zI)^*)^{-1}u\|^2\\
=& \|((\mathcal{T} - zI)^*)^{-1}\|^2\|u\|^2\\
=& \|(\mathcal{T} - zI)^*\|^{-2}\|u\|^2\\
\leq& (\mathrm{M}\|\mathcal{T} - zI\|)^{-2}\|u\|^2\\
\leq& \mathrm{M}^{-2}\|\mathcal{T} - zI\|^{-2}\|u\|^2\\
\leq& \frac{1}{\mathrm{M}^2}\|\mathcal{T} - zI\|^{-2}\|u\|^2\\
\leq& \frac{1}{\mathrm{M}^2}\|(\mathcal{T} - zI)^{-1}\|^2\|u\|^2\\
\|(\mathcal{T}^* - \bar{z}I)^{-1}u\|^2 \leq& \frac{1}{\mathrm{M}^2}\|(\mathcal{T} - zI)^{-1}u\|^2\\
\|(\mathcal{T}^* - \bar{z}I)^{-1}u\| \leq& \frac{1}{\mathrm{M}}\|(\mathcal{T} - zI)^{-1}u\| \leq \mathrm{M}\|(\mathcal{T} - zI)^{-1}u\|
\end{aligned}$$

This implies $\|(\mathcal{T}^* - \bar{z}I)^{-1}u\| \leq \mathrm{M}\|(\mathcal{T} - zI)^{-1}u\|$ for any real number M. □

Theorem 3.8. *If $\mathcal{T}$ is M -Fuzzy hyponormal operator then $\|(\mathcal{T} - zI)u\|^{n+1} \leq \mathrm{M}^{n(n+1)/2}\|(\mathcal{T} - zI)^{n+1}u\|$ for each $u \in \mathrm{H}$*

Proof. Given $\mathcal{T}$ is M -Fuzzy hyponormal operator. by the definition , $\exists$ a real number M such that
$\|(\mathcal{T} - zI)^*u\| \leq \mathrm{M}\|(\mathcal{T} - zI)u\|$ for all $u \in \mathcal{H}$ and $z \in \mathrm{C}$
To prove that

$$\|(\mathcal{T} - zI)u\|^{n+1} \leq \mathrm{M}^{n(n+1)/2}\|(\mathcal{T} - zI)^{n+1}u\|$$

By using the mathematical induction,
For $n = 1$, $\|(\mathcal{T} - zI)u\|^2 \leq \mathrm{M}\|(\mathcal{T} - zI)^2u\|$
Let us assume that it is true $n = k$,
i.e. $\|(\mathcal{T} - zI)u\|^{k+1} \leq \mathrm{M}^{k(k+1)/2}\|(\mathcal{T} - zI)^{k+1}u\|$
Now, we have to prove that it is true for $n = k+1$.

$$\begin{aligned}
\|(\mathcal{T} - zI)u\|^{(k+1)+1} =& \|(\mathcal{T} - zI)u\|^{(k+1)}\|(\mathcal{T} - zI)u\|\\
\leq& \mathrm{M}^{k(k+1)/2}\|(\mathcal{T} - zI)^{k+1}u\|\|(\mathcal{T} - zI)u\|\\
\leq& \mathrm{M}^{k(k+1)/2}\|(\mathcal{T} - zI)^{k}u\|\|(\mathcal{T} - zI)^2u\|\\
\leq& \mathrm{M}^{k(k+1)/2}\|(\mathcal{T} - zI)^{k+2}u\|\\
\|(\mathcal{T} - zI)u\|^{(k+1)+1} \leq& \mathrm{M}^{\frac{k(k+1)}{2}+1}\|(\mathcal{T} - zI)^{(k+1)+1}u\|
\end{aligned}$$

Hence proved. □

Theorem 3.9. *If $\mathcal{T}$ is M -Fuzzy hyponormal operator then if $(\mathcal{T} - zI)u^* = 0$ then $(\mathcal{T} - zI)u = 0$.*

Proof. Given $\mathcal{T}$ is M -Fuzzy hyponormal operator. Then we have $\|(\mathcal{T} - zI)^*u\| \leq \mathrm{M}\|(\mathcal{T} - zI)u\|$ for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$.

Given that $(\mathcal{T} - zI)u^* = 0$

Consider,

$$\begin{aligned}
\|(\mathcal{T} - zI)^*u\|^2 =& \langle(\mathcal{T} - zI)^*u, (\mathcal{T} - zI)^*u\rangle\\
=& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)^*u, (\mathcal{T} - zI)^*u,t) < 1\}
\end{aligned}$$

Since$(\mathcal{T} - zI)^*u = 0$

$\Rightarrow 0 = \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)u, (\mathcal{T} - zIt)u,t) < 1\}$

i.e. $\sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)u, (\mathcal{T} - zI)u,t) < 1\} = 0$

$\Rightarrow \langle(\mathcal{T} - zI)u, (\mathcal{T} - zI)u\rangle = 0$

$\Rightarrow \|(\mathcal{T} - zI)u\|^2 = 0$.

$\Rightarrow (\mathcal{T} - zI)u = 0$.

Hence the proof. □

Theorem 3.10. *Let $\mathcal{T}$ be a M -Fuzzy hyponormal operator and let $\mathbb{K} = \{u \in \mathcal{H} : \|(\mathcal{T}^* - \bar{z}I)u\| = \mathrm{M}\|(\mathcal{T} - zI)u\|,\ z \in \mathrm{C}\}$. Then $\mathbb{K}$ is a closed subspace of $\mathcal{H}$.*

Proof. For $u \in \mathcal{H}$, we have

$\|(\mathcal{T}^* - \bar{z}I)u\|^2 = \mathrm{M}^2\|(\mathcal{T} - zI)u\|^2$

i.e. by a known theorem,

$\|(\mathcal{T} - zI)^*u\|^2 = \mathrm{M}^2\|(\mathcal{T} - zI)^*u\|^2$

$\langle(\mathcal{T} - zI)^*u, (\mathcal{T} - zI)^*u\rangle = \mathrm{M}^2\langle(\mathcal{T} - zI)u, (\mathcal{T} - zI)u\rangle$

$$\begin{aligned}
&\sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)^*u, (\mathcal{T} - zI)^*u,t) < 1\} =\\
&\qquad \mathrm{M}^2 \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)u, (\mathcal{T} - zI)u,t) < 1\}\\
&\sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)(\mathcal{T} - zI)^*u,u,t) < 1\} =\\
&\qquad \mathrm{M}^2 \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T} - zI)^*(\mathcal{T} - zI)u,u,t) < 1\}
\end{aligned}$$

$$0 = \mathrm{M}^2 \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-zI)^*(\mathcal{T}-zI)u,u,t) < 1\} - \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u,t) < 1\}$$

$$\mathrm{M}^2\langle(\mathcal{T}-zI)^*(\mathcal{T}-zI)u,u\rangle - \langle(\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u\rangle = 0$$
$$\langle \mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI)u - (\mathcal{T}-zI)(\mathcal{T}-zI)^*u,u\rangle = 0$$
$$\Rightarrow\ [\mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI) - (\mathcal{T}-zI)(\mathcal{T}-zI)^*_u = 0, \text{ for all } z \in \mathrm{C}.$$

From this it follows that $\mathbb{K}$ is the null space of the operator $\mathrm{M}^2(\mathcal{T}-zI)^*(\mathcal{T}-zI-(\mathcal{T}-zI)(\mathcal{T}-zI)^*$.

Hence $\mathbb{K}$ must be a closed subspace of $\mathcal{T}$. □

Theorem 3.11. *Let $\mathcal{T}$ be a* M*-Fuzzy hyponormal operator and let z_1, z_2 be in $\sigma_{ap}(\mathcal{T})$ with $z_1 \neq z_2$. If (u_n) and (v_n) are the sequence of unit vectors of $\mathcal{H}$ such that $\|(\mathcal{T}-z_1I)u_n\| \to 0$ and $\|(\mathcal{T}-z_2I)v_n\| \to 0$ then $\langle u_n, v_n\rangle \to 0$*

Proof. Let $\mathcal{T}$ be a M-Fuzzy hyponormal operator and let z_1, z_2 be eigen values in $\sigma_{ap}(\mathcal{T})$ with $z_1 \neq z_2$. Then we write

$$\begin{aligned}(z_1-z_2)\langle u_n,v_n\rangle =& \langle (z_1-z_2)u_n, v_n\rangle \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((z_1-z_2)u_n, v_n, t) < 1\} \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}(((\mathcal{T}-z_1I)-(\mathcal{T}-z_2I))u_n, v_n, t) < 1\} \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-z_1I)u_n, v_n, t) < 1\} \\ &- \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-z_2I)u_n, v_n, t) < 1\} \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-z_1I)u_n, v_n, t) < 1\} \\ &- \sup\{t \in \mathrm{R} : \mathcal{F}(u_n, (\mathcal{T}-z_2I)^*v_n, t) < 1\} \\ (z_1-z_2)\langle u_n,v_n\rangle =& \langle (\mathcal{T}-z_1I)u_n, v_n\rangle - \langle u_n(\mathcal{T}-z_2I)^*v_n\rangle\end{aligned}$$

Now,

$$\|(z_1-z_2)\langle u_n,v_n\rangle\| = \|\langle(\mathcal{T}-z_1I)u_n,v_n\rangle - \langle u_n,(\mathcal{T}-z_2I)^*v_n\rangle\|$$
$$\|(z_1-z_2)\langle u_n,v_n\rangle\| \leq \|(\mathcal{T}-z_1I)u_n\| + \|(\mathcal{T}-z_2I)^*v_n\|$$
$$\|(z_1-z_2)\langle u_n,v_n\rangle\| \leq \|(\mathcal{T}-z_1I)u_n\| + \mathrm{M}\|(\mathcal{T}-z_2I)v_n\| \to 0$$

Since $\|(\mathcal{T}-z_1I)u_n\| \to 0$ and $\|(\mathcal{T}-z_2I)v_n\| \to 0$

$\|(z_1-z_2)\langle u_n,v_n\rangle\| \to 0$ as $\langle u_n,v_n\rangle \to 0$

Hence the proof. □

Theorem 3.12. *Suppose that a subspace $\mathbb{K}$ of $\mathcal{H}$ reduces an operator $\mathcal{T}$ on $\mathcal{H}$. Then $\mathcal{T}$ is* M *- Fuzzy hyponormal iff $\mathcal{T}|\mathbb{K}$ and $\mathcal{T}|\mathbb{K}^\perp$ are Fuzzy hyponormal .*

Proof. Let us assume that $\mathcal{T}$ is M - Fuzzy hyponormal
i.e.,$\|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}\|(\mathcal{T}-zI)u\|$ for all $u \in \mathcal{H}$ and for all $z \in \mathrm{C}$
Let $\mathcal{T}_1 = \mathcal{T}|\mathbb{K}$ and $\mathcal{T}_2 = \mathcal{T}|\mathbb{K}^\perp$ for $u \in \mathbb{K}$
by the definition of M-Fuzzy hyponormal,

$$\begin{aligned}\|(\mathcal{T}_1-zI)^*u\| &= \|(\mathcal{T}-zI)^*u\| \\ &\leq \mathrm{M}\|(\mathcal{T}-zI)u\| \\ &\leq \mathrm{M}\|(\mathcal{T}_1-zI)u\| \\ i.e., \|(\mathcal{T}|\mathbb{K}-zI)^*u\| &\leq \mathrm{M}\|(\mathcal{T}|\mathbb{K}-zI)u\|\end{aligned}$$

Therefore $\mathcal{T}|\mathbb{K}$ is M - Fuzzy hyponormal operator
Again, for $u \in \mathbb{K}^*$

$$\begin{aligned}\|(\mathcal{T}_2-zI)^*u\| &= \|(\mathcal{T}-zI)^*u\| \\ &\leq \mathrm{M}\|(\mathcal{T}-zI)u\| \\ &\leq \mathrm{M}\|(\mathcal{T}_2-zI)u\| \\ i.e., \|(\mathcal{T}|\mathbb{K}^\perp-zI)^*u\| &\leq \mathrm{M}\|(\mathcal{T}|\mathbb{K}^\perp-zI)u\|\end{aligned}$$

Therefore $\mathcal{T}|\mathbb{K}^\perp$ is M - Fuzzy hyponormal operator.
Hence $\mathcal{T}|\mathbb{K}$ and $\mathcal{T}|\mathbb{K}^\perp$ is M - Fuzzy hyponormal operator.
Conversely, let us assume that $\mathcal{T}|\mathbb{K}$ and $\mathcal{T}|\mathbb{K}^\perp$ are M - Fuzzy hyponormal operator.
For every $u \in \mathcal{H}$, it can be written as $u = u_1 + u_2$ where $u_1 \in \mathbb{K}$ and $u_2 \in \mathbb{K}^\perp$ for $z \in \mathrm{C}$ and for all vectors of $u \in \mathcal{H}$.
Consider

$$\begin{aligned}\|(\mathcal{T}-zI)^*u\|^2 =& \langle(\mathcal{T}-zI)^*u, (\mathcal{T}-zI)^*u\rangle \\ =& \langle(\mathcal{T}-zI)^*(u_1+u_2), (\mathcal{T}-zI)^*(u_1+u_2)\rangle \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-zI)^*(u_1+u_2), \\ &(\mathcal{T}-zI)^*(u_1+u_2), t) < 1\} \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-zI)^*u_1, (\mathcal{T}-zI)^*u_1, t) < 1\} \\ &+ \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}-zI)^*u_2, (\mathcal{T}-zI)^*u_2, t) < 1\} \\ =& \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}_1-zI)^*u_1, (\mathcal{T}_1-zI)^*u_1, t) < 1\} \\ &+ \sup\{t \in \mathrm{R} : \mathcal{F}((\mathcal{T}_2-zI)^*u_2, (\mathcal{T}_2-zI)^*u_2, t) < 1\} \\ =& \langle(\mathcal{T}_1-zI)^*u_1, (\mathcal{T}_1-zI)^*u_1\rangle + \\ &\langle(\mathcal{T}_2-zI)^*u_2, (\mathcal{T}_2-zI)^*u_2\rangle \\ =& \|(\mathcal{T}_1-zI)^*u_1\|^2 + \|(\mathcal{T}_2-zI)^*u_2\|^2 \\ \leq& \mathrm{M}^2|(\mathcal{T}_1-zI)u_1\|^2 + \|(\mathcal{T}_2-zI)u_2\|^2\} \\ \leq& \mathrm{M}^2\{\|(\mathcal{T}-zI)u_1\|^2 + \|(\mathcal{T}-zI)u_2\|^2\} \\ \leq& \mathrm{M}^2\|(\mathcal{T}-zI)(u_1+u_2)\|^2\end{aligned}$$

$$\|(\mathcal{T}-zI)^*u\|^2 \leq \mathrm{M}^2\|(\mathcal{T}-zI)u\|^2$$
$$\Rightarrow \|(\mathcal{T}-zI)^*u\| \leq \mathrm{M}^2\|(\mathcal{T}-zI)u\|$$

Hence $\mathcal{T}$ is M-Fuzzy hyponormal operator □

Theorem 3.13. *If $\mathcal{T}$ is* M*-Fuzzy hyponormal then* $\mathrm{Re}\,\sigma_{ap}(\mathcal{T}) \subset \sigma_{ap}(\mathrm{Re}\,\mathcal{T})$.

Proof. Let $\gamma = \alpha + i\beta \in \mathrm{Re}\,\sigma_{ap}(\mathcal{T})$ then $\exists$ sequence (u_n) of unit vectors of fuzzy Hilbert space $\mathcal{H} \ni \mathcal{T}u_n - \gamma u_n \to 0$ then $\mathcal{T}^*u_n - \bar{\gamma}u_n \to 0$

$$\begin{aligned}(\mathrm{Re}\,\mathcal{T})u_n - \alpha u_n &= \left(\frac{\mathcal{T}+\mathcal{T}^*}{2}\right)u_n - \left(\frac{\gamma+\bar{\gamma}}{2}\right)u_n \\ &= \left(\frac{\mathcal{T}}{2} - \frac{\gamma}{2}\right)u_n + \left(\frac{\mathcal{T}^*}{2} - \frac{\bar{\gamma}}{2}\right)u_n \\ &= \left(\frac{\mathcal{T}-\gamma I}{2}\right)u_n + \left(\frac{\mathcal{T}^*-\bar{\gamma}I}{2}\right)\end{aligned}$$

$$(\mathrm{Re}\,\mathcal{T})u_n - \alpha u_n \to 0$$
$$\Rightarrow (\mathrm{Re}\,\mathcal{T})u_n \to \alpha u_n$$

therefore $\alpha \in \sigma_{ap}(\mathrm{Re}\,\mathcal{T})$.
Hence $\mathrm{Re}\,\sigma_{ap}(\mathcal{T}) \subset \sigma_{ap}(\mathrm{Re}\,\mathcal{T})$. □

Note 3.14. *If a subspace $\mathbb{K}$ reduces on operator $\mathcal{T}$ then $\mathcal{T}$ is fuzzy hyponormal iff $\mathcal{T}|\mathbb{K}$ and $\mathcal{T}|\mathbb{K}^{\perp}$ are Fuzzy hyponormal.*

Theorem 3.15. *If $\mathcal{T}$ is* M- *Fuzzy hyponormal operator then the span of all Eigen vectors of $\mathcal{T}$ reduces $\mathcal{T}$.*

Proof. Given $\mathcal{T}$ is M- Fuzzy hyponormal operator.
We know that $\mathcal{T}u = zu$ then $\mathcal{T}^*u = \bar{z}u$ for all complex z.
It can be easily seen that the subspace $\mathbb{K} = \{u \in \mathrm{H}\mathcal{T}u = zu, z \in \mathrm{C}\}$ reduces $\mathcal{T}$.
If $z_1 \neq z_2$ then $\{u \in \mathcal{H} : \mathcal{T}u = z_1u, z \in \mathrm{C}\} \perp \{u \in \mathcal{H} : \mathcal{T}v = z_2v, z \in \mathrm{C}\}$.
This follows from the known theorem, $\mathcal{T}u = z_1u$ and $\mathcal{T}v = z_2v, z_1 \neq z_2$ then $\langle u, v\rangle = 0$.
We conclude that the restriction of $\mathcal{T}$ to each of eigen spaces is fuzzy normal.
Hence the span of all eigen vectors of $\mathcal{T}$ reduces $\mathcal{T}$. □

Theorem 3.16. *If $\mathcal{H}$ is finite dimensional and $\mathcal{T}$ is* M-*Fuzzy hyponormal operator then $\mathcal{T}$ is Fuzzy normal.*

Proof. Given $\mathcal{H}$ is finite dimenstional and $\mathcal{T}$ is M-Fuzzy hyponormal operator.
To prove that $\mathcal{T}$ Fuzzy normal
We use mathematical induction to prove this.
Let us assume $\dim \mathcal{H} = n$.
If $n = 1$, then every operator on fuzzy Hilbert space $\mathcal{H}$ is fuzzy normal.
Let us assume that it is true for $\dim \mathcal{H} < n$.
We know that every linear mapping on a finite dimensional space has at least one eigen value.
$\mathcal{T}$ has an eigen value say z.
Let $\mathbb{K} = \mathbb{K}_z$ be the eigen spaces of $\mathcal{T}$. associated with z.
By the above theorem $\mathbb{K}$ reduces $\mathcal{T}$. $\mathcal{T}|\mathbb{K}$ is fuzzy normal. $\mathbb{K}^{\perp}$ also reduces $\mathcal{T}$.
Since $v \in \mathbb{K}^{\perp}$ then for each $u \in \mathbb{K}$, $\langle u, v\rangle = 0$.
Consider,

$$\begin{aligned}
\langle \mathcal{T}v, u\rangle &= \sup\{t \in \mathrm{R} : \mathcal{F}(\mathcal{T}v, u, t) < 1\} \\
&= \sup\{t \in \mathrm{R} : \mathcal{F}(v, \mathcal{T}^*u, t) < 1\} \\
&= \sup\{t \in \mathrm{R} : \mathcal{F}(v, \bar{z}u, t) < 1\} \\
&= z \sup\{t \in \mathrm{R} : \mathcal{F}(v, u, t) < 1\} \\
&= z\langle v, u\rangle \\
\Rightarrow \langle \mathcal{T}v, u\rangle &= 0 \\
\Rightarrow \mathcal{T}v &\in \mathbb{K}^{\perp}
\end{aligned}$$

Again consider

$$\begin{aligned}
\langle \mathcal{T}^*v, u\rangle &= \sup\{t \in \mathrm{R} : \mathcal{F}(\mathcal{T}^*v, u, t) < 1\} \\
&= \sup\{t \in \mathrm{R} : \mathcal{F}(v, \mathcal{T}u, t) < 1\} \\
&= \sup\{t \in \mathrm{R} : \mathcal{F}(v, zu, t) < 1\} \\
&= \bar{z} \sup\{t \in \mathrm{R} : \mathcal{F}(v, u, t) < 1\} \\
&= \bar{z}\langle v, u\rangle \\
\Rightarrow \langle \mathcal{T}^*v, u\rangle &= 0 \\
\Rightarrow \mathcal{T}^*v &\in \mathbb{K}^{\perp}
\end{aligned}$$

It is clear that $\mathcal{T}|\mathbb{K}$ and $\mathcal{T}|\mathbb{K}^{\perp}$ are M-fuzzy hyponormal operator.
By the induction Hypothesis, $\mathcal{T}|\mathbb{K}^{\perp}$ is fuzzy normal.
Hence $\mathcal{T}$ is fuzzy normal. □

4. Conclusion

As the new idea of M-fuzzy hyponormal operator in FH-space is the old form of M- fuzzy hyponormal operator. The conclusion that can be taken from M- fuzzy hyponormal operator in FH- space, example and properties. Including its relationship with Self Adjoint Fuzzy operator, Fuzzy Normal operator and Fuzzy Hyponormal operators. In future we hope it is very helpful to find many types of M-fuzzy hyponormal operator and M-fuzzy paranormal operator.

Acknowledgment

The authors are thankful for the reference to the referees for these beneficial and effective suggestions.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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