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On upper and lower completely contra e-irresolute fuzzy multifunction

K. Kamala[1]* and V. Seenivasan [2]

Abstract
In this paper the concepts of upper and lower fuzzy completely contra e-irresolute fuzzy multifunctions are introduced. Also the concepts of the upper and lower completely weakly e-irresolute fuzzy multifunctions are being discussed. Some characterizations of these classes and some basic interesting properties of such fuzzy multifunctions are obtained and the mutual relationship and with other existing fuzzy multifunctions are also discussed.

Keywords
Upper and lower fuzzy completely contra e-irresolute fuzzy multifunctions, upper and lower completely weakly e-irresolute fuzzy multifunctions.

AMS Subject Classification
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[1,2] *Department of Mathematics, University College of Engineering Panruti, Panruti-607106, TamilNadu, India.*
***Corresponding author**: [1] krsaut@gmail.com; [2] seenujsc@yahoo.co.in
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Contents

1. Introduction

In the year 1985, Papageorgiou[7] introduced the fuzzy multifunction as a function from an ordinary topological space X to a fuzzy topological space Y. Further a group of researchers are engaged themselves and have studied different types of fuzzy multifunctions. The upper and lower inverses of a fuzzy multifunction are discussed and defined Papageorgiou. Mukherjee and Malakar[6] have studied fuzzy multifunctions with q-coincidence with the definition of upper inverse that were discussed and defined in[5] and with the definition of lower inverse which is defined by J. E. Joseph and M. H. Kwack in [4], On the other hand, J. Dontchev[3] introduced and discussed a lot about contracontinuous functions. Joseph and Kwack[4] introduced another form of contra continuous functions. In recent years, several authors have studied some new forms of contra-continuity for functions and multifunctions. Also, Seenivasan.V and Kamala.K, have defined and studied about Fuzzy e-continuity and fuzzy e-open sets[9] and studied about Fuzzy Ce-I(ec,eo) functions and fuzzy completely Ce-I(rc,eo) functions via fuzzy e-open sets[10]. In this paper we introduce fuzzy upper (lower) completely contra e-irresolute multifunctions and fuzzy upper (lower) completely weakly contra e-irresolute multifunctions and give some characterizations and properties of such notions are discussed. Throughout this paper let us use the abbreviations as fuzzy topological space as fts, fuzzy multifunction as fmf, Regular open set as Ro, regular closed as Rc, fuzzy open set as FOS, fuzzy closed set as FCS, fuzzy e-closed set as fe-cs, fuzzy e-open set as fe-os, e-irrsolute multifunction as e-irmf etc.

2. Preliminaries

In this section, we recall some definitions and basic results which will be used. Through this paper, by (X,τ) or simply by X we will mean a topological space in the classical sense, and (Y,σ) or simply Y will stand for a fuzzy topological space as defined by Chang[2]. Fuzzy sets in Y will be denoted by $\lambda,\mu,\rho,\eta,\gamma$ etc., and although subsets of X will be denoted by A,B, M, U, V,W etc. A mapping $F : X \rightarrow Y$ is called a fuzzy multifunction[7] if for each, $x \in X$, F(x) is a fuzzy set in Y. For a fuzzy multifunction $F: X \to Y$, the upper inverse $F^+(\mu)$ and lower inverse $F^-(\mu)$ of a fuzzy set μ in Y are defined as follows: $F^+(\mu) = \{x \in X, F(x) \leq \mu\}$ and $F^-(\mu) = \{x \in X, F(x) q \mu\}$.

Definition 2.1. *[9] A fuzzy set μ of a fts X is called fe-os if $\mu \leq cl(\mathrm{int}_\delta \mu) \vee \mathrm{int}(cl_\delta \mu)$ anf Fe-os if $\mu \geq cl(\mathrm{int}_\delta \mu) \wedge \mathrm{int}(cl_\delta \mu)$. The intersection of all fe-cs's containing μ is called fe-closure of μ and is denoted by fe-cl (μ) and the union of all fe-os's contained in μ is called fuzzy e-interior of μ and is denoted by fe-*$\mathrm{int}(\mu)$.

Definition 2.2. *[9]A mapping $f: X \to Y$ is said to be a fuzzy e-irresolute (briefly, f e-irresolute) if $f^{-1}(\lambda))$ is fuzzy e-open set in X for every fuzzy e-open set λ in Y.*

Definition 2.3. *[8] A fuzzy set μ is quasi-coincidentwith a fuzzy set λ denoted by $\mu q \lambda$ iff there exist $x \in X$ such that $\mu(x) + \lambda(x) > 1$. If μ and $\lambda$are not quasi-coincident then we write $\mu \bar{q} \lambda$ and $\mu \leq \lambda \Leftrightarrow \mu \bar{q} 1 - \lambda$.*

Definition 2.4. *[8] A fuzzy point x_p is quasi-coincident with a fuzzy set λdenoted by $x_p q \lambda$iff there exist $x \in X$ such that$p + \lambda(x) > 1$.*

Lemma 2.5. *[6]: For a fuzzy multifunction $G: M \to N$, we have $G^-(N - \mu) = M - G^+(\mu)$, for any fuzzy set μ in N.*

3. Fuzzy CC^U e- Irresolute and CC_L e-Irresolute Multifunctions

In this section, we define fuzzy upper and lower completely contra e-irresolute multifunctions and we discuss with some properties.

Definition 3.1. *A fmf $G: P \to Q$ is called fuzzy lower completely contra e-irresolute (briefly, fuzzy CC_L e-irresolute) multifunction if for any fe-cs μ in Q with $x \in G^-(\mu)$ (i.e)$G(x) q \mu$, there exists an Ro set M in P containing x such that $M \subset G^-(\mu)$.*

Definition 3.2. *A fmf $G: P \to Q$ is called fuzzy upper completely contra e-irresolute (briefly, fuzzy CC^U e-irresolute) multifunction if for any fe-cs μ in Q with $x \in G^+(\mu)$, there exists an Ro set M in X containing x such that $M \subset G^+(\mu)$.*

Theorem 3.3. *For a fmf $F: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:*
(i). F is fuzzy CC^U e-irresolute
(ii). For each fe-cs μ and $x \in X$ such that $F(x) \leq \mu$, there exists an Ro set V containing x such that if $y \in V$, then $F(y) \leq \mu$.
(iii). $F^+(\mu)$ is Ro in X for any fe-cs μ in Y.
(iv). $F^-(\rho)$ is Rc in X for any fe-os ρ in Y.
(v). For every FOS μ of Y, F^-(fe-int(μ)) is Rc in X.
(vi). For every FCS η of Y, F^+(fe-cl(η)) is Ro in X.
(vii).$intclF^+(\mu) = F^+$(fe-cl(μ)), for every fuzzy set μ in Y .

Proof : $(i) \Leftrightarrow (ii)$: obvious.
$(i) \Rightarrow (iii)$: Let μ be any fe-cs in Y and $x \in F^+(\mu)$. By (i), there exists a Ro set M containing x such that $M \subset F^+(\mu)$. Thus, $x \in intclF^+(\mu)$ and hence $F^+(\mu)$ is an Ro set in X.

$(iii) \Rightarrow (i)$: Let ρ be any fe-cs in Y and $x \in F^+(\rho)$. By (iii), $F^+(\rho)$ is a Ro set in X. Take $V = F^+(\rho)$. Then $V \subset F^+(\rho)$. Thus, F is fuzzy CC^U e-irresolute.

$(iii) \Rightarrow (iv)$: Let ρ be a fe-os in Y. Then $1_Y - \rho$ is a fe-cs in Y. By (iii), $F^+(1_Y - \rho)$ is a Ro set in X. Since $F^+(1_Y - \rho) = 1_X - F^-(\rho)$, then $F^-(\rho)$ is a Rc set in X.

$(iv) \Rightarrow (iii)$: Let μ be a fe-cs in Y. Then $I_Y - \mu$ is a fe-os in Y. By (iv), $F^-(1_Y - \mu)$ is a Rc set in X. Since $F^-(1_Y - \mu) = 1_X - F^+(\mu)$, then $F^+(\mu)$ is a Ro in X.

$(iv) \Rightarrow (v)$: Let μ be a FOS in Y. Since $fe\text{-}int(\mu)$ is fuzzy e-open, then by (iv), $F^-(fe\text{-}int(\mu))$ is a Rc set in X. Converse is obvious.

$(iii) \Rightarrow (vi)$: Let η be a FCS of Y. Since $fe - cl(\eta)$ is fe-cs of Y, then by (iii), $F^+(fe\text{-}cl(\eta))$ is a Ro set in X. Converse is obvious.

$(v) \Rightarrow (vi)$: Let η be a FCS of Y. Then $I_Y - \eta$is a FOS of Y. Since $fe\text{-}int(1_Y - \eta)$ is fe-os of Y. By (v), $F^-(fe\text{-}int(1_Y - \eta)))$ is a Rc set in X. This implies, $F^-(fe\text{-}int(1_Y - \eta)) = F^-(1_Y - fe\text{-}cl\eta) = 1_X - F^+(fe\text{-}cl\eta)$. Then $F^+(fe\text{-}cl(\eta))$ is Ro in X. Converse is obvious.

$(vi) \Rightarrow (vii)$: Let μbe any fuzzy set in Y. Since $fe\text{-}cl(\mu)$ is fuzzy e-closed in Y then by (vii), $F^+(fe\text{-}cl(\mu))$ is Ro in X and $F^+(\mu) = F^+(fe\text{-}cl(\mu))$. Therefore, we obtain $intclF^+(\mu) = intclF^+(fe\text{-}cl\mu)$. Since $F^+(fe\text{-}cl(\mu))$ is Ro in X and hence $intclF^+(\mu) = F^+(fe\text{-}cl\mu))$.

$(vii) \Rightarrow (vi)$: obvious.

Theorem 3.4. *For a fmf $F: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:*
(i). F is fuzzy CC_L- e-irresolute
(ii). For each fe-cs μ and $x \in X$ such that $F(x) \leq \mu$, there exists an Ro set V containing x such that if $y \in V$, then $F(y) \leq \mu$.
(iii). $F^-(\mu)$ is Ro in X for any fe-cs μ in Y.
(iv). $F^+(\rho)$ is Rc in X for any fe-os ρ in Y.
(v). For every FOS μ of Y, $F^+(fe\text{-}int\mu)$ is Rc in X.
(vi). For every FCS η of Y, $F^-(fe\text{-}cl\eta)$ is Ro in X.
(vii).$intclF^-(\mu) = F^-(fe\text{-}cl(\mu))$, for every fuzzy set μ in Y.

Proof : It is similar to that of theorem 3.3

Remark 3.5. *Every fuzzy CC^U e- irresolute (fuzzy CC_Le- irresolute) multifunction is fuzzy upper contra continuous. Converse is not true.*

Example 3.6. *Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $Y = [0, 1]$, $\sigma = \{0, 1, \mu, \lambda, \delta\}$ where $\mu(y) = 0.4$, $\lambda(y) = 0.1$, $\delta(y) = 0.7$, $\nu(y) = 0.3$, $\gamma(y) = 0.5$. Consider the fmfF : $(X, \tau) \to (Y, \sigma)$ is defined as $F(a) = \nu$, $F(b) = \gamma$, $F(c) = \delta$. Then $F^+(1 - \mu) = \{a, b\}$, $F^+(1 - \lambda) = X$, $F^+(1 - \delta) = \{a\}$ and $F^-(1 - \mu) = \{b, c\}$, $F^-(1 - \lambda) = X$, $F^-(1 - \delta) = \phi$ which is open but not Ro in (X, τ). Then F is fuzzy upper contra continuous but not fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute) multifunction.*

Remark 3.7. *Every fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute) multifunction is fuzzy upper(lower) almost continuous. Converse is not true.*

Example 3.8. *Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{b\},\{b,c\}\}$ and $Y = [0,1]$, $\sigma = \{0,1,\mu,\lambda,\eta\}$ where $\mu(y) = 0.4$, $\lambda(y) = 0.1$, $\eta(y) = 0.5$, $\zeta(y) = 0.7$, $\gamma(y) = 0.6$. Consider the fmf $F : (X,\tau) \to (Y,\sigma)$ is defined as $F(a) = \mu$, $F(b) = \zeta$, $F(c) = \gamma$. Then $F^+(1-\mu) = \{a,c\}$, $F^+(1-\lambda) = X$, $F^+(1-\eta) = \{a\}$ which is closed but not Ro in (X,τ). Then F is fuzzy upper almost continuous but not fuzzy CC^U e-irmf.*

Example 3.9. *Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{b\},\{a,b\}\}$ and $Y = [0,1]$, $\sigma = \{0,1,\mu,\rho,\eta\}$ where $\mu(y) = 0.4$, $\rho(y) = 0.2$, $\eta(y) = 0.5$. Consider the fmf $F : (X,\tau) \to (Y,\sigma)$ is defined as $F(a) = \mu$, $F(b) = \rho$, $F(c) = \eta$. Then $F^-(1-\mu) = \{c\}$, $F^-(1-\rho) = \{a,c\}$, $F^-(1-\eta) = \phi$ which is closed but not Ro in (X,τ). Then F is fuzzy lower almost continuous but not fuzzy CC_L e-irresolute multifunction.*

Theorem 3.10. *Let $\{Y_i : i \in I\}$ be a family of product spaces. If a function $F : X \to \prod Y_i$ is fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute), then $P_i \circ F : X \to Y_i$ is fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute) for each $i \in I$ where P_i is the projection of $\prod_{i\in I} Y_i$ onto Y_i.*

Proof : Let δ_i be any fe-os in Y_i. Since P_i is a fuzzy continuous and fuzzy open set, it is a fe-os. Now$P_i : \prod_{i\in I} Y_i \to Y_i$, $P_i^+(\delta_i)$is a fuzzy e-open in $\prod_{i\in I} Y_i$. Therefore, P_i is a fuzzy e-irresolute function. Now $(P_i \circ F)^+(\delta_i) = F^+(P_i^+(\delta_i)) = F^+\left(\prod_{i\neq j} Y_j \times \delta_i\right)$ since F is fuzzy CC^U e -irresolute. Hence $F^+(P_i^+(\delta_i))$ is a Rc set, since $P_i^+(\delta_i)$ is a fuzzy e-open set. Hence $P_i \circ F$ is fuzzy CC^U e -irresolute.

Theorem 3.11. *If the function $F : \prod_{i\in I} X_i \to \prod_{i\in I} Y_i$, defined by $F(x_i) = \prod_{i\in I} F_i(x_i)$, is fuzzy CC^U e-irresolute (fuzzy CC^L e-irresolute) multifunction, then$F_i : X_i \to Y_i$ is fuzzy CC^U e-irresolute (fuzzy CC^L e-irresolute) multifunction for each $i \in I$.*

Proof : Let υ_i be any fe-cs of Y_i, then $\prod_{i\neq j} Y_j \times \upsilon_i$ is fuzzy e-closed in $\prod_{i\in I} Y_i$. Since F is fuzzy CC^U e - irmf, then

$F^+\left(\prod_{i\neq j} Y_j \times \upsilon_i\right) = \prod_{i\neq j} X_j \times F_i^+(\upsilon_i)$ is Ro in $\prod_{i\in I} X_i$ and hence $F_i^+(\upsilon_i)$ is Ro in X_i. This implies, F_i is fuzzy CC^U e-irmf.

Theorem 3.12. *For a fmf $F : X \to Y$, if $clint(F^-(\eta)) \leq F^-(fe\text{-}K_\eta)$ for every fuzzy set η of Y, then F is fuzzy CC^U e-irresolute.*

Proof : Suppose that $clint(F^-(\eta)) \leq F^-(fe\text{-}K_\eta)$, for every fuzzy set η in Y. By definition, $F^-(fe\text{-}K_\eta) = F^-(\eta)$. This implies that, $clint(F^-(\eta)) = F^-(\eta)$ and $F^-(\eta)$ is Rc in X. Thus, by theorem 3.3, F is fuzzy CC^U e-irresolute.

Theorem 3.13. *For a fmf $F : X \to Y$, if $clint(F^+(\eta)) \leq F^+(fe\text{-}K_\eta)$ for every fuzzy set η of Y, then F is fuzzy CC_L e-irresolute .*

Proof : It is similar to that of theorem 3.12

Theorem 3.14. *Let $\{V_i : i \in I\}$ be a Ro cover of X and a fmf $F : X \to Y$ is a fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute) iff $F|V_i : V_i \to Y$ is fuzzy CC^U e-irresolute (fuzzy CC_L e-irresolute) for each $i \in I$.*

Proof : Suppose that F is fuzzy CC^U e-irmf. Let $x \in X$and $x \in V_i$ for each $i \in I$. Let λ be a fe-cs of Y containing $F|V_i(x)$. Since F is fuzzy CC^U e-irmf and $F(x) = F|V_i(x)$, there exists an Ro set U containing x such that $U \subset F^+(\lambda)$. Take $W = U \cap V_i$. Then W is a Ro set V_i containing x and $F|V_i(W) = F(W) \leq \lambda$. This implies that $W \subset F^+(\lambda)$. Thus $F|V_i$ is fuzzy CC^U e-irresolute.
Conversely, Let $x \in X$ and λ be fe-cs in Y with $x \in F^+(\lambda)$. Since $\{V_i : i \in I\}$is a Ro cover for X, then $x \in V_i$. Since $F|V_i$ is fuzzy CC^U e-irresolute and $F(x) = F|V_i(x)$, there exists a Ro set W such that $F|V_i(W) \leq \lambda$. Then we have, W is Ro in X and $W \subset F^+(\lambda)$. Therefore, F is fuzzy CC^U e-irresolute.

Theorem 3.15. *For a fmf $F : X \to Y$, if the fuzzy graph multifunction $G_F : X \to X \times Y$ is fuzzy CC^U e-irresolute, then F is fuzzy CC^U e-irresolute.*

Proof : Suppose that fuzzy graph multifunction $G_F : X \to X \times Y$ is fuzzy CC^U e-irresolute and $x \in X$. Let η be fe-cs in Y with $F(x) \leq \eta$. Then $G_F(x) \leq X \times \eta$. Since the graph function G_F is fuzzy CC^U e-irresolute , there exists an Ro set M containing x such that $G_F(M) \leq X \times \eta$. For any $m_0 \in M$ and $y \in Y$, we have $F(m_0)(y) = G_F(m_0)(m_0,y) \leq (X \times \eta)(m_0,y) = \eta(y)$. Then we have $F(m_0)(y) = \eta(y)$ for all $y \in Y$. Thus, $F(m_0) \leq \eta$ for any $m_0 \in M$. Hence, F is fuzzy CC^U e-irresolute.

Theorem 3.16. *For a fmf $F : X \to Y$, if the fuzzy graph multifunction $G_F : X \to X \times Y$ is fuzzy CC_L e-irresolute, then F is fuzzy CC_L e-irresolute.*

Proof : Suppose that fuzzy graph multifunction $G_F : X \to X \times Y$ is fuzzy CC_L e-irresolute and $x \in X$. Let η be fe-cs in Y such that $F(x)q\eta$. Then there exists $y \in Y$ such that $(F(x))(y) + \eta(y) > 1$. Then we have $G_F(x)(x,y) + (X \times \eta)(x,y) > 1$ which implies $G_F(x)q(X \times \eta)$. Since fuzzy graph function G_F is fuzzy CC_L e-irresolute, there exists an Ro set M in X such that $x \in M$ and $G_F(m_0)q(X \times \eta)$ for all $m_0 \in M$. Suppose that there exists a point n_0 in M such that $F(n_0)\bar{q}\eta$. Then for all $y \in Y$, $(F(n_0))(y) + \eta(y) \leq 1$ we have $G_F(n_0)(x,y) \leq F(n_0)(y)$ and $(X \times \eta)(x,y) \leq \eta(y)$. Thus, $G_F(n_0)(x,y) + (X \times \eta)(x,y) \leq 1$. Thus, $G_F(n_0)\bar{q}(X \times \eta)$, for any $n_0 \in M$ which is a contradiction. Hence F is fuzzy CC_L e-irresolute.

Theorem 3.17. *If $F : (X,\tau) \to (Y,\sigma)$ is a fuzzy CC^U e- irresolute (fuzzy CC_L e- irresolute) injective fmf and $F(x)$ be fuzzy e-T_2 space for every $x \in X$, then X is Urysohn space.*

Proof : Let x_1 and x_2 be any two distinct points in X. Since F is injective, $F(x_1) \neq F(x_2)$ in Y. Since Y is fuzzy e-T_2, there exists fe-os η and ρ in Y such that $F(x_1) \in \eta$ and $F(x_2) \in \rho$ and $\eta \wedge \rho = 0$. This implies that fe-$cl(\eta)$ and fe-$cl(\rho)$ are fe-cs in Y. Then, since F is fuzzy CC^U e-irresolute, there exists a Ro sets V and W in X containing x_1 and x_2 respectively, such that $F(V) \leq fe\text{-}cl(\eta)$and $F(W) \leq fe\text{-}cl(\rho)$. This implies that $V \subset F^+(fe\text{-}cl(\eta))$ and $W \subset F^+(fe\text{-}cl(\rho))$, we have $F^+(fe\text{-}cl(\eta))$ and $F^+(fe\text{-}cl(\rho))$ are disjoint and hence $cl(V) \cap cl(W) = \phi$, and by definition, X is Urysohn.

Theorem 3.18. *Let $F : X \to Y$ be a fuzzy CC^U e-irresolute surjective multifunction and F(x) is fuzzy e-closed for each $x \in X$. If X is nearly compact, then Y is fuzzy e-closed.*

Proof : Let $\{\upsilon_\alpha : \alpha \in \Omega\}$ be any cover of F(x) by fe-cs of Y. Since F(x) is fuzzy e-closed for any $x \in X$, there exists a finite subset Δ of Ω such that $F(x) \leq \underset{\alpha\in\Delta}{\vee} fe\text{-}cl(\upsilon_\alpha)$. Take $\lambda = \underset{\alpha\in\Delta}{\vee} fe\text{-}cl(\upsilon_\alpha)$. Since F is fuzzy CC^U e-irresolute, there exists a Ro set A_x of X containing x such that $F(A_x) \leq \lambda$. Then $\{A_x\}, x \in X$ is a Ro cover of X. Since X is nearly compact, there exists $x_i, i = 1,...,n$ in X such that $X = \overset{n}{\underset{i=1}{\cup}} A_{x_i}$ we have $Y = F(X) = F(\overset{n}{\underset{i=1}{\cup}} A_{x_i}) = \overset{n}{\underset{i=1}{\vee}} F(A_{x_i}) \leq \overset{n}{\underset{i=1}{\vee}} \lambda_i = \overset{n}{\underset{i=1}{\vee}} \underset{\alpha\in\Delta_i}{\vee}$ fe-cl (υ_α). Thus, Y is fuzzy e-closed.

Theorem 3.19. *If $F : X \to Y$ is a fuzzy CC^U e-irresolute injection and Y is fuzzy e-normal then X is strongly normal.*

Proof : Let V and W be a disjoint nonempty closed sets of X. Since F is injective, $F(V)$ and $F(W)$ are disjoint FCSs. Since Y is fuzzy e-normal, there exists fe-os μ and λ such that $F(V) \leq \mu$ and $F(W) \leq \lambda$ and $\mu \wedge \lambda = 0$. This implies that $fe\text{-}cl(\mu)$ and $fe\text{-}cl(\lambda)$ are fe-cs in Y. Then, since F is fuzzy CC^U e-irresolute, $F^+(fe\text{-}cl(\mu))$ and $F^+(fe\text{-}cl(\lambda))$ are Ro sets. Then $V \subset F^+(fe\text{-}cl(\mu))$ and $W \subset F^+(fe\text{-}cl(\lambda))$, we have $F^+(fe\text{-}cl(\mu))$ and $F^+(fe\text{-}cl(\lambda))$ are disjoint, and by definition, X is strongly normal.

4. Fuzzy CWC^U e-irresolute and Fuzzy CWC_L e-irresolute Multifunctions

Definition 4.1. *A fmf $F : X \to Y$ is called fuzzy lower completely weakly contra e-irresolute(briefly, fuzzy CWC_L e - irresolute) multifunction if for any fe-cs μ in Y with $x \in F^-(\mu)$ (i.e)$F(x)q\mu$, there exists an open set V in X containing x such that $V \subset F^-(\mu)$.*

Definition 4.2. *A fmf $F : X \to Y$ is called fuzzy upper completely weakly contra e-irresolute(briefly, fuzzy CWC^U e - irresolute) multifunction if for any fe-cs μ in Y with $x \in F^+(\mu)$, there exists an open set V in X containing x such that $V \subset F^+(\mu)$.*

Remark 4.3. *Every fuzzy CWC^U e-irresolute (fuzzy CWC_L e-irresolute) multifunction is fuzzy CWC^U e-irresolute (fuzzy CWC_L e-irresolute) multifunction.*

Example 4.4. *Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{b\},\{b,c\}\}$ and $Y = [0,1]$, $\sigma = \{0,1,\mu,\lambda,\eta\}$ where $\mu(y) = 0.4$, $\lambda(y) = 0.2$, $\eta(y) = 0.8$, $\zeta(y) = 0.5$, $\gamma(y) = 0.6$. Consider the fmf $F : (X,\tau) \to (Y,\sigma)$ is defined as $F(a) = \zeta$, $F(b) = \eta$, $F(c) = \gamma$. Then $F^+(1-\mu) = \{a,c\}$, $F^+(1-\lambda) = X$, $F^+(1-\eta) = \{a\}$ which is open but not Ro in (X,τ). Then, F is fuzzy CWC^U e-irresolute but not fuzzy CC^U e-irmf.*

Example 4.5. *Let $X = \{a,b,c\}$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $Y = [0,1]$, $\sigma = \{0,1,\zeta,\eta,\rho\}$ where $\zeta(y) = 0.1$, $\rho(y) = 0.6$, $\eta(y) = 0.4$. Consider the fmf$F : (X,\tau) \to (Y,\sigma)$ is defined as $F(a) = \eta$, $F(b) = \rho$, $F(c) = \zeta$. Then $F^-(1-\zeta) = \{a,b\}$, $F^-(1-\eta) = \{b\}$, $F^-(1-\rho) = \phi$ which is open but not Ro in (X,τ). Then F is fuzzy CWC_L e-irresolute but not fuzzy CC_L e-irmf.*

Theorem 4.6. *For a fmf $F : (X,\tau) \to (Y,\sigma)$ the following statements are equivalent:*
(i). F is fuzzy CWC^U e-irresolute
(ii). For each fe-cs μ and $x \in X$ such that $F(x) \leq \mu$, there exists an open set V containing x such that if $y \in V$, then $F(y) \leq \mu$.
(iii). $F^+(\mu)$ is open in X for any fe-cs μ in Y.
(iv). $F^-(\rho)$ is closed in X for any fe-os ρ in Y.
(v). For every FOS μ of Y, $F^-(fe\text{-}int\mu)$ is closed in X.
(vi). For every FCS η of Y, $F^+(fe\text{-}cl\eta)$ is open in X.

Proof :$(i) \Leftrightarrow (ii)$: obvious.

$i \Rightarrow (iii)$: Let μ be any fe-cs in Y and $x \in F^+(\mu)$. By (i), there exists an open set V containing x such that $V \subset F^+(\mu)$. Thus, $x \in intF^+(\mu)$ and hence $F^+(\mu)$ is an open set in X.

$(iii) \Rightarrow (i)$: Let ρ be any fe-cs in Y and $x \in F^+(\rho)$. By (iii), $F^+(\rho)$ is an open set in X. Take $V = F^+(\rho)$. Then $V \subset F^+(\rho)$. Thus, F is fuzzy CWC^U e-irresolute.

$(iii) \Rightarrow (iv)$: Let ρ be a fe-os in Y. Then $I_Y - \rho$ is a fe-cs in Y. By (iii), $F^+(1_Y - \rho)$ is an open set in X. Since $F^+(1_Y - \rho) = 1_X - F^-(\rho)$, then $F^-(\rho)$ is a closed set in X.

$(iv) \Rightarrow (iii)$: Let μ be a fe-cs in Y. Then $1_Y - \mu$ is a fe-os in Y. By (iv), $F^-(1_Y - \mu)$ is a closed set in X. Since $F^-(1_Y - \mu) = 1_X - F^+(\mu)$, then $F^+(\mu)$ is a fuzzy open in X.

$(iv) \Rightarrow (v)$: Let μ be a FOS in Y. Since $fe\text{-}int(\mu)$ is fuzzy e-open, then by (iv), $F^-(fe\text{-}int(\mu))$ is a closed set in X. Converse is obvious.

$(iii) \Rightarrow (vi)$: Let η be a FCS of Y. Since $fe\text{-}cl(\eta)$ is fe-cs of Y, then by (3), $F^+(fe\text{-}cl(\eta))$ is a open set in X. Converse is obvious.

$(v) \Rightarrow (vi)$: Let η be a FCS of Y. Then $1_Y - \eta$ is a FOS of Y. Since $fe\text{-}int(1_Y - \eta)$ is fe-os of Y. By (5), $F^-(fe\text{-}int(1_Y - \eta))$ is a closed set in X. This implies, $F^-(fe\text{-}int(1_Y - \eta)) = F^-(1_Y - fe\text{-}cl(\eta)) = 1_X - F^+(fe\text{-}cl(\eta))$.
Then $F^+(fe\text{-}cl(\eta))$ is open in X. The converse is obvious.

Theorem 4.7. *If $F : X \to Y$ is an upper almost continuous multifunction where X and Y are topological spaces and $G : Y \to Z$ is a fuzzy CC^U e-irresolute multifunction where Z is a fuzzy topological space, then $G \circ F : X \to Z$ is fuzzy CWC^U e-irresolute .*

Proof : Let $x \in X$ and ρ be a fuzzy e- closed set in Z we have $(G \circ F)^+(\rho) = F^+(G^+)(\rho)$. Since G is fuzzy CC^U e-irresolute, $G^+(\rho)$ is Ro in Y. Since F is upper almost continuous, $F^+(G^+)(\rho) = (G \circ F)^+(\rho)$ is open in X. Thus, $G \circ F$ is fuzzy CWC^U e-irresolute.

Theorem 4.8. *If $F_i : X \to Y$ for $i = 1,2,...,n$, are fuzzy CWC^U e-irmf, then $\vee_{n=1}^n F_i$ is a fuzzy CWC^U e-irmf.*

Proof : Let η be a fe-cs of Y and $F_i : X \to Y$ for $i = 1,2,...,n$, are fuzzy CWC^U e-irmf. Let $x \in (\vee_{n=1}^n F_i)^+(\eta)$. Then, $\vee_{n=1}^n$ $F_i(x) \leq \eta$. Since $F_i, i = 1,2,...,n$ are fuzzy CWC^U e-irmf's, then there exists an open set V_x containing x such that $F_i(x_0) \leq \eta$ for every $x_0 \in V_x$. Let $V = \bigcup_{i=1}^n V_{x_i}$. Then $V \subset (\vee_{n=1}^n)F_i^+(\eta)$. Thus $(\vee_{n=1}^n)F_i^+(\eta)$ is open in X and hence $(\vee_{n=1}^n)F_i$ is a fuzzy CWC^U e-irmf.

Theorem 4.9. *If $F_i : X \to Y$ for $i = 1,2,...,n$, is fuzzy CWC_L e-irmf, then $\vee_{n=1}^n F_i$ is a fuzzy CWC_L e-irmf.*

Proof : Let η be a fe-cs of Y and $F_i : X \to Y$ for $i = 1,2,...,n$, are fuzzy CWC_L e-irresolute. Let $x \in (\vee_{n=1}^n F_i)^-(\eta)$. Then, $\vee_{n=1}^n F_i(x) q \eta$. Then, $\vee_{n=1}^n F_i(x) q \eta$. Since $F_i, i = 1,2,...,n$ are fuzzy CWC_L e-irmf's, then there exists an open set V_x containing x such that $F_i(x_0) q \eta$ for every $x_0 \in V_x$. Let $V = \bigcup_{i=1}^n V_{x_i}$. Then $V \subset (\vee_{n=1}^n)F_i^-(\eta)$. Thus $(\vee_{n=1}^n)F_i^-(\eta)$ is open in X and hence $(\vee_{n=1}^n)F_i$ is a fuzzy CWC_L e-irmf.

5. Conclusion

Thus in this paper the concepts of upper and lower fuzzy completely contra e-irresolute fuzzy multifunctions were introduced. Also the concepts of the upper and lower completely weakly e-irresolute fuzzy multifunctions were being discussed. Some characterizations of these classes and some basic interesting properties of such fuzzy multifunctions were obtained and the mutual relationship with other existing fuzzy multifunctions were also discussed.

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