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Relations on irredundance and domination number for six regular graph with girth 3

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Abstract
In this paper, we discuss about the irredundant number, upper irredundnat number and domination number denoted by $ir(G(n)), IR(G(n))$ and $\gamma(G(n))$ respectively for 6-regular graphs of n vertices with girth 3. Here, $G(n)$ denotes the 6-regular graphs on n vertices with girth 3. We further establish some relation between $ir(G(n)), IR(G(n))$ and $\gamma(G(n))$.

Keywords
6-regular graph, Girth, Irredundant set, Irredundant number, Dominating set, Domination number.

AMS Subject Classification
05C69.

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Contents

1. Introduction

Let $G=(V,E)$ be a finite, undirected connected graph with neither loops nor multiple edges. For basic definitions and terminologies we refer to West [6]. The open neighbourhood of v is the set $N(v)=\{u \in V | uv \in E\}$ and the closed neighbourhood of v is $N[v]=N(v)\cup\{v\}$. A graph is called K regular if degree of each vertex in the graph is K. The concept of domination was introduced by Ore [5]. A subset D of $V(G)$ is a dominating set of G if every vertex in $V-D$ is adjacent to at least one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number $\gamma(G)$. Domination in graphs is well studied in [3, 4]. A set of vertices $S\subseteq V(G)$ is an *irredundant set* if for every vertex $v\in S$, there is a vertex $u \in V(G)$ such that $u\in N[S]$ but $u \notin N[S-\{v\}]$, v has its own private neighbour with respect to S. An upper irredundant number $IR(G)= max\{|S| : S \text{ is an irredundant set}\}$. The maximum cardinality of S is upper irredundant number. The lower irredundant number $ir(G)=min\{|S| : S \text{ is a maximal irredundant set}\}$. The minimum cardinality of S is an lower irredundant number. Irredundance concepts in graphs was defined by Cockayne in [1].

In [2], C. Jayasekaran and S. Delbin Prema has discussed about some exact results and relations between irredundance, upper irredundance and domination concepts for four regular graphs with girth 3. It stimulates to continue some other results among irredundance, upper irredundance and domination parameters for 6-regular graphs with girth 3. The structure of the 6-regular graphs with girth 3 is defined as follows.

Definition 1.1. If v_1 is adjacent with $v_{n-2},v_{n-1},v_n,v_2,v_3,v_4$; v_2 is adjacent with $v_{n-1},v_n,v_1,v_3,v_4,v_5$; v_i is adjacent with $v_{i-3},v_{i-2},v_{i-1},v_{i+1},v_{i+2},v_{i+3}$, where $i=3$ to $n-2$, v_{n-1} is adjacent with $v_{n-4},v_{n-3},v_{n-2},v_n,v_1,v_2$ and v_n is adjacent with $v_{n-1}v_{n-2},v_{n-1},v_1,v_2,v_3$ such that $v_1v_2\dots v_n$ forms a cycle, then clearly each vertex is of degree 6. Hence, the graph has $3n$ edges. Thus, from the construction, we have a six regular graph of girth 3 with n vertices and $3n$ edges.

2. Exact values for $ir(G(n))$ and $IR(G(n))$

In this section, we investigate some exact values among irredundance and upper irredundance of graph $G(n)$.

Theorem 2.1. *For the 6 - regular graph with girth 3 and 7 vertices $G(7)$, there does not exist any irredundant set.*

Proof. Suppose there exists an irredundant set S for $G(7)$. Then for any vertex $v \in S$, $N[v] \neq N[S-v]$. Since $G(7) = K_7$, each vertex is adjacent to every other vertex in G. Therefore, $N[v] = V(G)$ and $N[S-v] = V(G)$ implies that $N[v] = N[S-v]$ which is a contradiction. Hence, there doesnot exist any irredundant set for $G(7)$. ☐

Observation 2.2. *In $G(n)$ there exists a redundant set only for $n = 7$.*

Proof. Let $S \subseteq V(G)$ be a redundant set in $G(n)$. Then for any vertex $v \in S$, $N[v] = N[S-v]$. If $n \geq 8$, then by the Definition 2.1.1 for any graph $G(n)$, $N[v] \neq N[S-v]$, which is a contradiction. Therefore, S is a redundant set only for $G(7)$. ☐

Theorem 2.3. *In $G(n)$, $S = \{v_i, v_{i+7}, v_{i+14}, \ldots, v_{i+(\lceil \frac{n}{7} \rceil -1)7}\}$ is a minimum irredundant set for $1 \leq i \leq n$ and the suffices modulo n.*

Proof. Let $v_1, v_2, , \ldots, v_n$ be the vertices of $G(n)$ such that $v_1 v_2 \ldots v_n v_1$ forms a cycle. For an irredundant set S, we have $N[v] \neq N[S-v]$ where $v \in S$. By the construction of $G(n)$,starting with the vertex $v_i$for $1 \leq i \leq n$, we have $N[v_i] = \{v_{i-3}, v_{i-2}, v_{i-1}, v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$ where the suffices modulo n and $|N[v_i]| = 2$. Hence, the next vertex should be chosen as v_{i+7}, where $N[v_{i+7}] = \{v_{i+4}, v_{i+5}, v_{i+6}, v_{i+7}, v_{i+8}, v_{i+9}, v_{i+10}\}$. Clearly $N[v_i] \neq N[v_{i+7}]$. Proceeding like this, we can choose the minimum irredundant set as $\{v_i, v_{i+7}, v_{i+14}, \ldots, v_{i+(\lceil \frac{n}{7} \rceil -1)7}\}$ where the suffices modulo n. Hence the proof.

Example 2.4. Consider the graph $G(13)$ given in figure 2.1. The minimum irredundant set are $\{v_1, v_8\}$, $\{v_2, v_9\}$, $\{v_3, v_{10}\}$, $\{v_4, v_{11}\}$, $\{v_5, v_{12}\}$, $\{v_6, v_{13}\}$, $\{v_7, v_1\}$, $\{v_8, v_2\}$, $\{v_9, v_3\}$, $\{v_{10}, v_4\}$, $\{v_{11}, v_5\}$, $\{v_{12}, v_6\}$ and $\{v_{13}, v_7\}$.

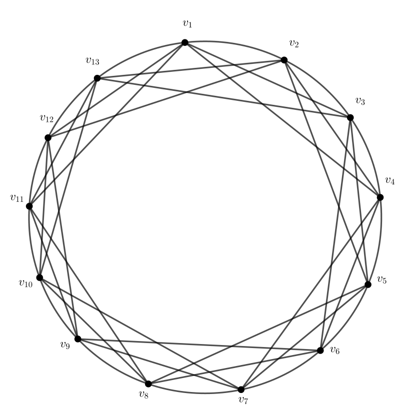

Figure 2.1: $G(13)$

Theorem 2.5. *In $G(n)$, if n is a multiple of 7. Then $G(n)$ contains only 7 minimum irredundant set.*

Proof. Let $G(n)$ be a 6-regular graph with girth 3 and n be a multiple of 2. Then $n = 7m$ where $m \geq 2$. If $m = 1$, then $G(n) = K_7$. Hence by Observation 2.2, $m = 1$ is a contradiction. Hence, $n = 7m$, $m \geq 2$. Now by Theorem 2.3, the minimum irredundant sets for $G(7m)$ are $S_i = \{v_i, v_{i+7}, v_{i+14}, \ldots, v_{i+(\lceil \frac{n}{7} \rceil -1)7}\} = \{v_i, \ v_{i+7}, \ v_{i+14}, \ldots, v_{i+(m-1)7}\} = \{v_i, v_{i+7}, \ldots, v_{i+n-7}\}$ where $1 \leq i \leq n$, $m \geq 2$ and the suffices modulo n.

Now we consider for two cases $m = 2$ and $m > 2$.

Case 1. $m = 2$

Here, $G(n) = G(14)$. Consider the graph $G(14)$ given in figure 2.2. Then the minimum irredundant sets are $\{v_1, v_8\}$, $\{v_2, v_9\}$, $\{v_3, v_{10}\}$, $\{v_4, v_{11}\}$ $\{v_5, v_{12}\}$, $\{v_6, v_{13}\}$ and $\{v_7, v_{14}\}$. Thus, $G(n)$ contains only 7 minimum irredundant sets.

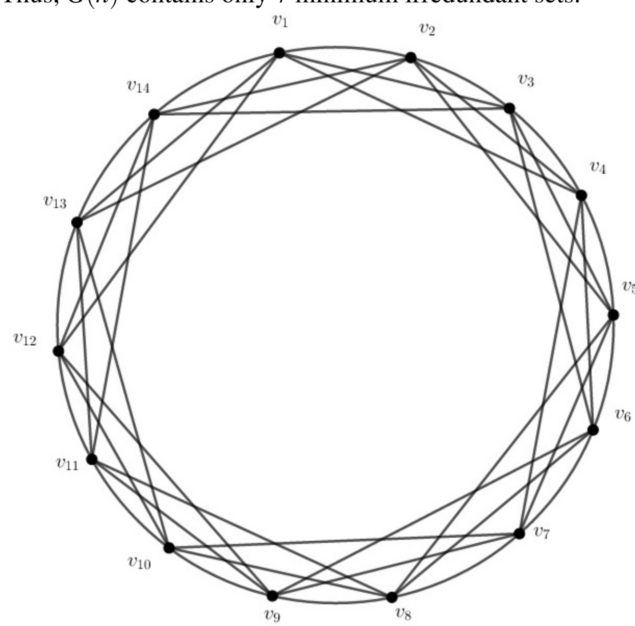

Figure 2.2 : $G(14)$

Case 2. $m > 2$

The minimum irredundant set are $S_i = \{v_i, v_{i+7}, \ldots, v_{i+n-7}\}$ where $1 \leq i \leq n$ and the suffices modulo n. Here, $S_1 = \{v_1, v_8, \ldots, v_{n-6}\}$, $S_2 = \{v_2, v_9, \ldots, v_{n-5}\}$, $S_3 = \{v_3, v_{10}, \ldots, v_{n-4}\}$, $S_4 = \{v_4, v_{11}, \ldots, v_{n-3}\}$, $S_5 = \{v_5, v_{12}, \ldots, v_{n-2}\}$, $S_6 = \{v_6, v_{13}, \ldots, v_{n-1}\}$ and $S_7 = \{v_7, v_{14}, \ldots, v_n\}$. Proceeding like this we get $S_p = \{v_p, v_{p+7}, \ldots, v_{p+n-7}\}$ for $8 \leq p \leq n$. Since, the suffices are modulo n then S_p is either S_1 or S_2 or S_3 or S_4 or S_5 or S_6 or S_7 according as $p \equiv 1\ (mod\ 7)$ or $p \equiv 2\ (mod\ 7)$ or $p \equiv 3\ (mod\ 7)$ or $p \equiv 4\ (mod\ 7)$ or $p \equiv 5\ (mod\ 7)$ or $p \equiv 6\ (mod\ 7)$ or $p \equiv 0\ (mod\ 7)$, respectively. Thus, $G(n)$ contains only 7 minimum irredundant sets. ☐

Example 2.6. For $G(21)$ given in figure 2.3. The minimum irredundant set are $\{v_1, v_8, v_{15}\}$, $\{v_2, v_9, v_{16}\}$,$\{v_3, v_{10}, v_{17}\}$, $\{v_4, v_{11}, v_{18}\}$, $\{v_5, v_{12}, v_{19}\}$, $\{v_6, v_{13}, v_{20}\}$ and $\{v_7, v_{14}, v_{21}\}$. Thus, there are only 7 minimum irredundant set.

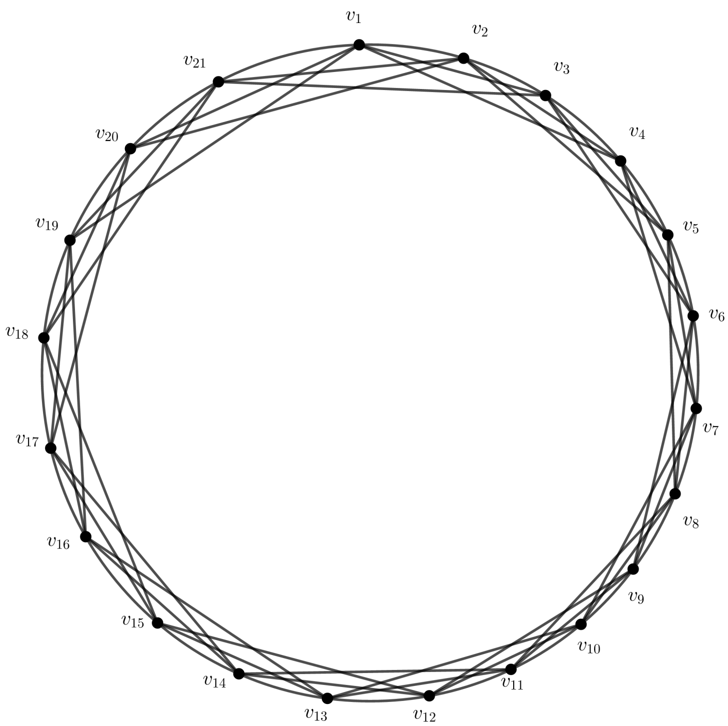

Figure 2.3: $G(21)$

Theorem 2.7. *In* $G(n)$, $S' = \left\{v_i, v_{i+4}, v_{i+8}, \ldots,\ v_{i+\left(\lceil \frac{n-3}{4} \rceil -1\right)4}\right\}$ *is an upper irredundant set where* $1 \le i \le n$ *and the suffices modulo n.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of $G(n)$ such that $v_1 v_2 \ldots v_n v_1$ forms a cycle. Beginning with the vertex v_i for $1 \le i \le n$ where $N[v_i] = \{v_{i-3}, v_{i-2}, v_{i-1}, v_i, v_{i+1}, v_{i+2}, v_{i+3}\}$, the suffices modulo n and $|N[v_i]| = 7$. Since $N[v_i]$ contains the vertex v_{i+3}, the next vertex to be chosen is v_{i+4} where $N[v_{i+4}] = \{v_{i+1}, v_{i+2}, v_{i+3}, v_{i+4}, v_{i+5}, v_{i+6}, v_{i+7}\}$. Clearly, $N[v_i] \neq N[v_{i+4}]$. Now the next vertex to be chosen is v_{i+8} where $N[v_{i+4}] \neq N[v_{i+8}]$. Proceeding like this we choose the upper irredundant set as $\left\{v_i, v_{i+4}, v_{i+8}, \ldots,\ v_{i+\left(\lceil \frac{n-3}{4} \rceil -1\right)4}\right\}$ where the suffices modulo n. Hence the proof. □

Example 2.8. For the graph $G(11)$ given in the figure 2.4, the upper irredundant sets are $\{v_1, v_5\}, \{v_2, v_6\}, \{v_3, v_7\}, \{v_4, v_8\}, \{v_5, v_9\}, \{v_6, v_{10}\}, \{v_7, v_{11}\}, \{v_8, v_1\}, \{v_9, v_2\}, \{v_{10}, v_3\}$ and $\{v_{11}, v_4\}$

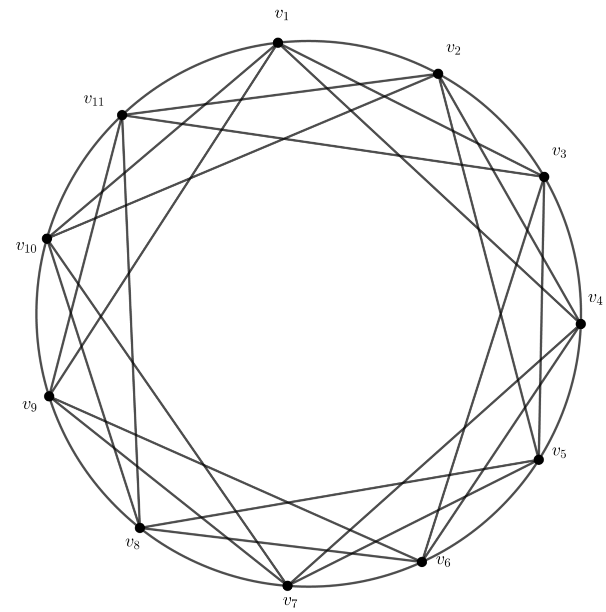

Figure 2.4 : $G(11)$

Theorem 2.9. *In $G(n)$, if n is a multiple of 4. Then $G(n)$ contains only 4 upper irredundant sets.*

Proof. Let $G(n)$ be a 6-regular graph with girth 3 and n be a multiple of 4. Then $n = 4m$ where $m \ge 2$. Now by Theorem 2.7, the upper irredundant set for $G(4m)$ are $S'_i = \left\{v_i, v_{i+4}, v_{i+8}, \ldots,\ v_{i+(m-1)4}\right\} = \{v_i, v_{i+4}, \ldots, v_{i+n-4}\}$ where $1 \le i \le n$, $m \ge 2$ and the suffices modulo n.

Now we consider for two cases $m = 2$ and $m > 2$.

Case 1. $m = 2$

Here, $G(n) = G(8)$. Consider the graph $G(8)$ given in figure 2.5. The upper irredundant sets are $\{v_1, v_5\}$, $\{v_2, v_6\}$, $\{v_3, v_7\}$ and $\{v_4, v_8\}$. Thus $G(n)$ contains only 4 upper irredundant sets.

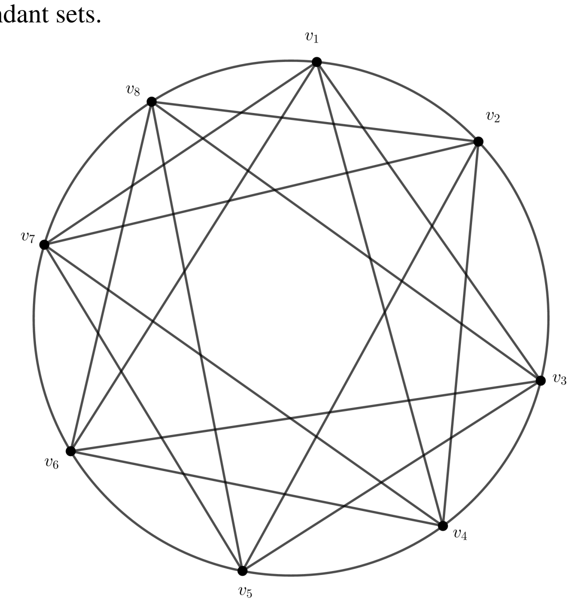

Figure 2.5: $G(8)$

Case 2. $m > 2$

The minimum irredundant set are $S'_i = \{v_i, v_{i+4}, \dots, v_{i+n-4}\}$ where $1 \le i \le n$ and the suffices modulo n. Here, $S'_1 = \{v_1, v_5, \dots, v_{n-3}\}, S'_2 = \{v_2, v_6, \dots, v_{n-2}\}, S'_3 = \{v_3, v_7, \dots, v_{n-1}\}$ and $S'_4 = \{v_4, v_8, \dots, v_n\}$. Proceeding like this we get $S'_p = \{v_p, v_{p+4}, \dots, v_{p+n-4}\}$ where $5 \le p \le n$. Since the suffices are modulo n, S'_p is either $= S'_1$ or S'_2 or S'_3or S'_4 according as $p \equiv 1 \ mod\ 4$ or $p \equiv 2 \ mod\ 4$ or $p \equiv 3 \ mod\ 4$ or $p \equiv 0 \ mod\ 4$, respectively. Thus, $G(n)$ contains only 4 upper irredundant sets. □

Example 2.10. Consider the graph $G(16)$ given in figure 2.6. The upper irredundant sets are $\{v_1, v_5, v_9, v_{13}\}$, $\{v_2, v_6, v_{10}, v_{14}\}$, $\{v_3, v_7, v_{11}, v_{15}\}$ and $\{v_4, v_8, v_{12}, v_{16}\}$. Thus, $G(16)$ contains only 4 upper irredundant set.

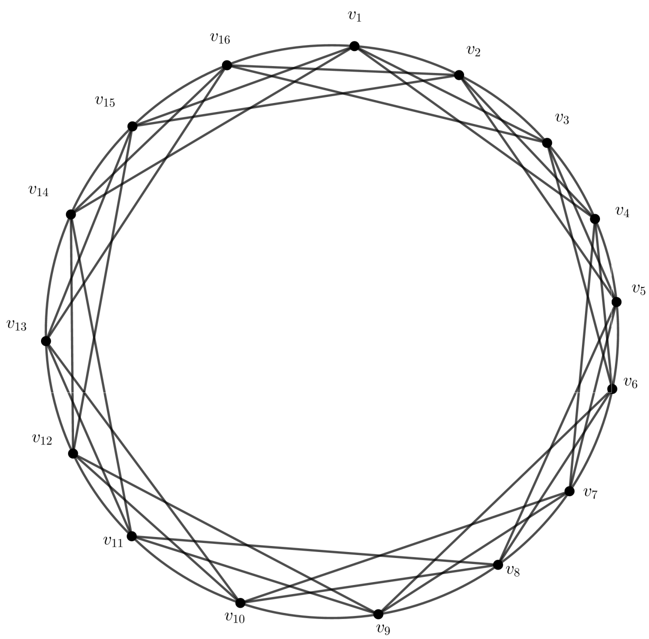

Figure 2.6: $G(16)$

Theorem 2.11. *For the 6-regular graph with girth 3 and n vertices $G(n)$,* $\mathrm{ir(G(n))} = \lceil \frac{n}{7} \rceil$, *for* $n \ge 8$.

Proof. By Theorem 2.3, the minimum irredundant sets are $S = \{v_i, v_{i+7}, v_{i+14}, \dots, v_{i+(\lceil \frac{n}{7} \rceil - 1)7}\}$ where $1 \le i \le n$ and the suffices modulo n. Hence, $\mathrm{ir}(G(n)) = |S| = \lceil \frac{n}{7} \rceil$, for $n \ge 8$. □

Example 2.12. Consider the graph $G(12)$ given in Figure 2.7. There are 12 minimum irredundant sets for $G(12)$ which are $\{v_1, v_8\}, \{v_2, v_9\}, \{v_3, v_{10}\}, \{v_4, v_{11}\}, \{v_5, v_{12}\}$, $\{v_6, v_1\}, \{v_7, v_2\}, \{v_8, v_3\}, \{v_9, v_4\}$, $\{v_{10}, v_5\}, \{v_{11}, v_6\}$, and $\{v_{12}, v_7\}$. Hence, $ir(G) = \lceil \frac{12}{7} \rceil$. This implies that $ir(G) = 2$.

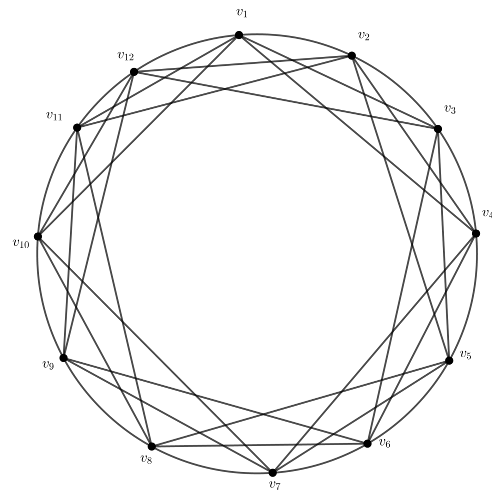

Figure 2.7: $G(12)$

Theorem 2.13. *For the six regular graph with girth 3 and n vertices $G(n)$,* $\mathrm{IR(G(n))} = \lceil \frac{n-3}{4} \rceil$, *for* $n \ge 8$.

Proof. By Theorem 2.7, $S' = \left\{v_i, v_{i+4}, v_{i+8}, \dots, v_{i+(\lceil \frac{n-3}{4} \rceil - 1)4}\right\}$ where $1 \le i \le n$ and $i + \left(\lceil \frac{n-3}{4} \rceil - 1\right)4$ modulo n is an upper irredundant set. Hence, $\mathrm{IR}(G(n)) = |S'| = \lceil \frac{n-3}{4} \rceil$, for $n \ge 8$. □

Example 2.14. Consider the graph $G(8)$ given in Figure 2.5. There are 4 upper irredundant sets for $G(8)$ which are $\{v_1, v_5\}, \{v_2, v_6\}, \{v_3, v_7\}$ and $\{v_4, v_8\}$. Hence, $\mathrm{IR(G)} = \lceil \frac{8-3}{4} \rceil$. This implies that $\mathrm{IR(G)} = 2$.

3. Relation between $ir(G(n))$,$IR(G(n))$ and $\gamma(G(n))$

In this section, we establish the relation among irredundant, upper irredundant number and domination number of six regular graph G(n).

Theorem 3.1. *For the six regular graph $G(n)$ with girth 3,* $\gamma(\mathrm{G(n)}) = \mathrm{ir(G(n))}$, $n \ge 8$.

Proof. Let $G(n)$ be a six regular graph with girth 3. For $n = 7$, the graph $G(7) = K_7$ is complete and so $\gamma(\mathrm{G}) = 1$. But $\mathrm{ir(G)} \ge 2$. Thus, in this case $\gamma(\mathrm{G}) \ne \mathrm{ir(G)}$.

Now let $n \ge 8$. Let S be a minimum dominating set of G. To prove S is an irredundant in G. Suppose there exists v in S such that v is not irredundant in S. Then, $N[v] = N[S - \{v\}]$ implies that S is redundant. Hence by Observation 2.2, $G(n) = K_7$ is complete and hence $n = 7$ which is a contradiction. Therefore, S is an irredundant set in G.

Now to prove S has a minimum cardinality of maximal irredundant set in G. If S is a minimum dominating set in G, then S is a minimal dominating set. Since, every minimal dominating set is a maximal irredundant set[1]. This shows that S is a maximal irredundant set. Since G is a six regular graph of girth 3, S is an irredundant set of minimum cardinality. Hence, $\mathrm{ir(G)} = |\mathrm{S}| = \gamma(\mathrm{G})$. □

Example 3.2. Consider the graph $G(22)$ given in figure 2.8. A minimal dominating set of $G(22)$ is $\{v_1, v_8, v_{15}, v_{22}\}$ and hence $\gamma(\mathrm{G}) = 4$. A minimal irredundant set of $G(22)$ is $\{v_3, v_{10}, v_{17}, v_2\}$ and hence $ir(G) = 4$. Thus $\gamma(\mathrm{G}) = ir(G)$.

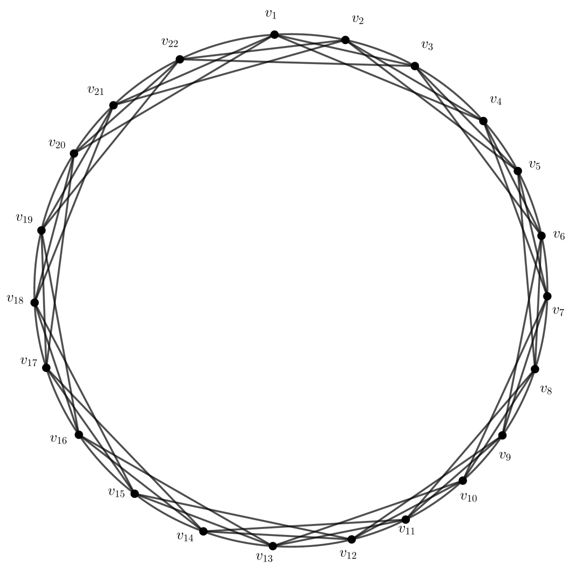

Figure 2.8: $G(22)$

Theorem 3.3. *For the six regular graph $G(n)$ of order n with girth 3, $ir(G(n)) = IR(G(n))$ for $n = 8, 9, 10, 11$ and 15.*

Proof. Let $v_1, v_2 \ldots, v_n$ be the vertices of six regular graph with girth 3 such that $v_1 v_2 \ldots v_n v_1$ forms a cycle. We consider the following two cases.

Case 1. $8 \leq n \leq 11$

Let $S = \{v_1, v_8\}$ and $S' = \{v_1, v_5\}$. Then by Theorem 2.3, S is a minimum irredundant set and by Theorem 2.7, S' is an upper irredundant set. This implies that $ir(G(n)) = 2$ and $IR(G(n)) = 2$ and hence $ir(G(n)) = IR(G(n))$.

Case 2. $n = 15$

Let $S = \{v_1, v_8, v_{15}\}$ and $S' = \{v_1, v_5, v_9\}$. Then by Theorem 2.3, S is a minimum irredundant set and by Theorem 2.7, S' is an upper irredundant set. This implies that $ir(G(n)) = 3$ and $IR(G) = 3$ and hence $ir(G(n)) = IR(G(n))$. The theorem follows from cases 1 and 2. □

Example 3.4. Consider the graph $G(8)$ given in figure 2.5. A minimal irredundant set of $G(8)$ is $\{v_1, v_8\}$ and hence $ir(G) = 2$. An upper irredundant set of $G(8)$ is $\{v_1, v_5\}$ and hence $IR(G) = 2$. Therefore, $ir(G) = IR(G)$.

Theorem 3.5. *For the graph $\mathrm{G}(\mathrm{n})$, $ir(G(n)) < IR(G(n))$, for $n = 12, 13, 14$ and $n \geq 16$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of six regular graph with girth 3 such that $v_1 v_2 \ldots v_n v_1$ form a cycle. We consider the following cases.

Case 1. $n = 12, 13, 14$

By Theorem 2.3, $S = \{v_1, v_8\}$ is a minimum irredundant set and by Theorem 2.7, $S' = \{v_1, v_5, v_9\}$ is an upper irredundant set. This implies that $ir(G(n)) = 2$ and $IR(G(n)) = 3$ and hence $ir(G(n)) < IR(G(n))$.

Case 2. $n \geq 16$

By Theorem 2.3, $S = \{v_i, v_{i+7}, v_{i+14}, \ldots, v_{i+(\lceil \frac{n}{7} \rceil - 1)7}\}$ is a minimum irredundant set and by Theorem 2.7, $S' = \left\{v_i, v_{i+4}, v_{i+8}, \ldots, v_{i+(\lceil \frac{n-3}{4} \rceil - 1)4}\right\}$ is an upper irredundant set. This implies that $|S| < |S'|$ and hence $ir(G(n)) < IR(G(n))$. □

Example 3.6. Consider the graph $G(21)$ given in figure 2.3. A minimal irredundant set of $G(21)$ is $\{v_1, v_8, v_{15}\}$ and hence $ir(G) = 3$. An upper irredundant set of $G(21)$ is $\{v_1, v_5, v_9, v_{13}, v_{17}\}$ and hence $IR(G) = 5$. This implies that $ir(G(n)) < IR(G(n))$.

Theorem 3.7. *If* $\mathrm{G}(\mathrm{n})$ *is a six regular graph with girth 3, then* $\gamma(\mathrm{G}) \leq \mathrm{IR}(\mathrm{G})$ for $\mathrm{n} \geq 8$.

Proof. Let S be a minimum dominating set of $\mathrm{G}(\mathrm{n})$. Then for every vertex v in $V - S$ which are adjacent to atleast one vertex in S. Since S is a minimal dominating set, it is an irredundant set. Also, since $G(n)$ is a six regular graph with girth 3, by Theorem 3.1, $\gamma(\mathrm{G}) = \mathrm{ir}(\mathrm{G})$. Therefore, $\mathrm{ir}(\mathrm{G}) = |\mathrm{S}|$. Also by Theorem 3.3 and by Theorem 3.7, $\mathrm{ir}(\mathrm{G}) \leq \mathrm{IR}(\mathrm{G})$ implies that the cardinality of an irredundant set is always less than or equal to the cardinality of the upper irredundant set. So, we conclude that the domination number is less than or equal to the upper irredundant number. Hence, $\gamma(\mathrm{G}) \leq \mathrm{IR}(\mathrm{G})$, $\mathrm{n} \geq 8$. □

Example 3.8. Consider the graph $G(23)$ given in figure 2.9. A minimal domination set of $G(23)$ is $\{v_1, v_8, v_{15}, v_{22}\}$ and hence $\gamma(\mathrm{G}) = 4$. An upper irredundant set of $G(23)$ is $\{v_1, v_5, v_9, v_{13}, v_{17}, v_{21}\}$ and hence $IR(G) = 5$. This implies that $\gamma(\mathrm{G(n)}) \leq \mathrm{IR}(\mathrm{G(n)})$.

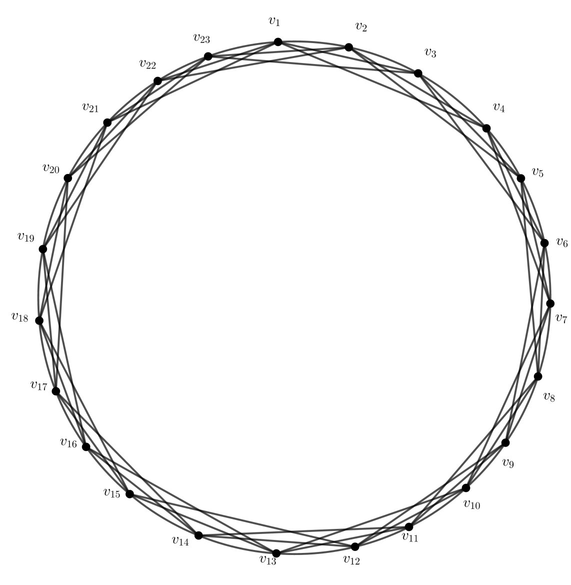

Figure 2.9: $G(23)$

4. Conclusion

In this paper we have discussed the exact values for $ir(G(n))$ and $IR(G(n))$. Also we have proved $\gamma(G(n)) = ir(G(n)), n \geq 8; ir(G(n)) = IR(G(n))$ for $n = 8, 9, 10, 11$ and $15; \gamma(G) \leq IR(G(n))$ for $n \geq 8$.

References

[1] E.J. Cockayne and C.M. Mynhardt, The sequence of upper and lower domination, independence and irredundance numbers of a graph, *Discrete Mathematics*, 122(1993), 89–102.

[2] S. Delbin Prema and C. Jayasekaran, Relations on Irredundance and domination number in Quartic graphs with girth 3, *Journal of Advanced Research in Dynamical and Control Systems*, 10(14)(2018), 1719–1729.

[3] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker Inc. New York., Vol. 208 (1998).

[4] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Domination in Graphs: Advanced Topics*, Marcel Dekker, New York , (1998).

[5] O. Ore, Theory of Graphs, *American Mathematical Society. Colloq. Publ. (American Mathematical Soc. Providence. RI)*, 38(1962).

[6] D.B. West, *Introduction to Graph Theory,* Second Ed., Prentice-Hall, Upper Saddle River, NJ, (2001).

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