Malaya Journal of Matematik, Vol. 8, No. 3, 877-881, 2020
https://doi.org/10.26637/MJM0803/0024

Distance matrix from adjacency matrix using Hadamard product

P.A. Asharaf[1*] and Bindhu K. Thomas[2]

Abstract
Distance matrix of a graph has important applications in the field of hierarchical clustering, phylogenetic analysis, bioinformatics, telecommunication etc. There are nice research works on determinant, characteristic polynomial and eigen values of distance matrices. This paper describes a formula for finding the distance matrix of a simple connected undirected graph from the powers of the adjacency matrix using Hadamard product on matrices.

Keywords
Adjacency matrix, Distance matrix, Binary matrix , Diameter, Hadamard product, m-distance matrix.

AMS Subject Classification
05C12, 05C40, 05C50, 05C62, 05B20.

[1,2] *Mathematics Research Centre, Mary Matha Arts and Science College, Affiliated to Kannur University, Mananthavady-670645, Kerala, India.*
*__Corresponding author__: [1] asharafpa@gecwyd.ac.in; [2] bindhukthomas@gmail.com
Article History:Received **21** February **2020**; Accepted **14** May **2020**

Contents

1 Introduction 877
2 Preliminaries 877
3 Main Results 878
4 Illustration 879
5 Conclusion 880
References 880

1. Introduction

The earlier study on distance matrix started by L. Graham, H. O. Pollak and L. Lovasz. Now there are nice research works on determinant, characteristic polynomial [1] and eigen values [2] of distance matrices. But so far, there are not so much significant study on how to find distance matrix from adjacency matrix. Here we define the m-distance matrix and developing a formula for finding this matrix in terms of adjacency matrix. Then distance matrix can be easily calculated from these m-distance matrices.

We are considering a simple, connected, undirected graph $G=(V,E)$ of order n with vertex set V and edge set E throughout this paper unless otherwise specified.

2. Preliminaries

Let A_G be the $n\times n$ adjacency matrix [3] of G. Then ij^{th} entry of A_G(m^{th} power of A_G), represent the number of walks of length m between the vertices v_i and v_j of G.

Definition 2.1. *[4] The distance matrix $D=(d_{ij})$, of G is defined as,*

$$d_{ij}=\begin{cases} d(v_i,v_j), & \text{if } i\neq j\\ 0 & \text{, if } i=j\end{cases}$$

where, $d(v_i,v_j)$ is the distance between the vertices v_i and v_j.

Definition 2.2. *[5] The diameter of a graph G is the maximum distance between any two vertices of G and it is denote by $Diam(G)$.*

Definition 2.3. *(Hadamard product [6]) Consider the vector space $\mathfrak{R}^{m\times n}$ over the field $\mathfrak{R}$. For $C,F\in\mathfrak{R}^{m\times n}$, the Hadamard product $\circ$ is a binary operation on $\mathfrak{R}^{m\times n}$ defined by,*

$$(C\circ F)_{ij}:=(C)_{ij}(F)_{ij},\quad \forall i,j.$$

Properties:

(i) $(\mathfrak{R}^{m\times n},\circ)$ and $(B_{m\times n},\circ)$ are commutative Monoids, where $B_{m\times n}$ denote the set of all $m\times n$ binary matrices of $\mathfrak{R}^{m\times n}$.

(ii) $C,F,E\in\mathfrak{R}^{m\times n}$, $E\circ(C+F)=E\circ C+E\circ F$.

(iii) $C,F\in\mathfrak{R}^{m\times n}$, $k\in\mathfrak{R}$, $(kC)\circ F=C\circ(kF)=k(C\circ F)$

(iv) For $C,F\in\mathfrak{R}^{m\times n}$

$$(C\circ F)_{ij}=\begin{cases} 1, & \text{if } c_{ij} \text{ and } f_{ij}=1\\ 0, & \text{otherwise}\end{cases}$$

Since the first two powers of the adjacency matrix A_G of G, $(A_G^0 = I_n, A_G^1 = A_G)$ are binary matrices, it will be more convenient that its other powers $A_G^2, A_G^3, \dots$ are also be treated as binary matrices.

Definition 2.4. *Let us define a function $\delta : \mathfrak{R} \to B = \{0,1\}$ as follows.*

$$\delta(a) = \begin{cases} 0, a \leq 0, a \in \mathfrak{R} \\ 1, \text{ otherwise.} \end{cases}$$

Let $\mathfrak{R}^{m\times n}$ denote the vector space of all $m \times n$ real matrices over the real field $\mathfrak{R}$ and $B_{m\times n}$ denote the set of all $m \times n$ binary matrices of $\mathfrak{R}^{m\times n}$. Define $\delta : \mathfrak{R}^{m\times n} \to B_{m\times n}$ as

$$\delta(D) = (\delta(D))_{ij} := (\delta(d_{ij})), \quad \forall i, j, D \in \mathfrak{R}^{m\times n}.$$

Then $\delta(A_G^m) \in B_{n\times n}$, $\forall m \in \{0,1,2,\dots\}$ and it is the equivalent binary matrix representation of A_G^m. Let it be denoted by $A_G^{(m)}$.

$$(A_G^{(m)})_{ij} = \begin{cases} 1, \text{ if there exist a walk of length } m \text{ form} \\ \qquad v_i \text{ to } v_j \text{ in } G \\ 0, \text{ otherwise.} \end{cases}$$

Also, for $C, F \in B_{m\times n}$, $\delta(C \circ F) = \delta(C) \circ \delta(F)$.

Definition 2.5. *Consider the graph $G = (V,E)$ with n vertices. Then the m-distance matrix D_m of G is an $n \times n$ symmetric binary matrix defined by,*

$$(D_m)_{ij} := \begin{cases} 1, \text{ if } d(v_i, v_j) = m \\ 0, \text{ otherwise.} \end{cases}$$

Properties

(i) $D_m \in B_{n\times n}, \forall m = 0,1,2,\dots$

(ii) $D_0 = I_n$

(iii) $Diam(G) = \max_m\{m : D_m \neq 0\}$

(iv) $D_i \circ D_j = \begin{cases} 0, \text{ if } i \neq j \\ D_i, \text{ if } i = j, \text{ for } 0 \leq i, j \leq d. \end{cases}$

Remark 2.6. *By the property (iv), we get $\{D_0, D_1, \dots, D_d\}$ as an orthogonal subset of $B_{n\times n} \subset \mathfrak{R}^{n\times n}$ with respect to the Hadamard product.*

3. Main Results

Theorem 3.1. *The set $\{D_0, D_1, \dots, D_d\}$ is a Linearly independent subset of the vector space $\mathfrak{R}^{n\times n}$.*

Proof. If $c_0D_0 + c_1D_1 + \dots c_dD_d = 0$ for some $c_0, c_1, \dots, c_d \in \mathfrak{R}$.
Then by taking Hadamard product $\circ$ by $D_j (0 \leq j \leq d)$ on both side,

$$D_j \circ (c_0D_0 + c_1D_1 + \dots c_dD_d) = 0$$
$$c_0D_j \circ D_0 + c_1D_j \circ D_1 + \dots c_jD_j \circ D_j + \dots c_dD_j \circ D_d = 0$$

By property (iv), we have

$$0 + 0 + \cdots + c_jD_j \circ D_j + 0 + \dots + 0 = 0,$$

Since, $D_j \neq 0$, then

$$c_jD_j = 0 \Rightarrow c_j = 0, \quad \forall j.$$

Therefore, $\{D_0, D_1, \dots, D_d\}$ is a linearly independent subset of $\mathfrak{R}^{n\times n}$. □

Theorem 3.2. *Let D be the distance matrix of $G = (V,E)$, then $D = \sum_{t=1}^{d} tD_t$, where $d = Diam(G)$.*

Proof. Let D_t be the t-distance binary matrix of G. Then tD_t is a matrix whose non negative entry t represent the distance between the corresponding vertices of G. Since G is connected, there should be a finite distance varies from 1 to d between any two vertices of G. so,

$$D = \sum_{t=1}^{d} tD_t.$$

□

Remark 3.3. *If we include the zero distance in the above sum, then $D = \sum_{t=0}^{d} tD_t$.*

Remark 3.4. *Let W denote the subspace of all symmetric matrices of $\mathfrak{R}^{n\times n}$ such that all its diagonal entries 0. Consider the $n \times n$ binary matrices $B_{u,t} (u < t), u,t \in N$ in W gives as below*

$$(B_{ut})_{ij} = \begin{cases} 1, \text{ if } (i,j) = (u,t) \text{ or } (i,j) = (t,u) \\ 0, \text{ otherwise.} \end{cases}$$

Then $B_W = \{B_{u,t} : 1 \leq u < t \leq n\}$ will be the standard basis of W and dimension of W is $\frac{n(n-1)}{2}$. Since, $D_m \in W$, it can be represented as a linear combination of elements of B_W.

$$D_m = \sum_{r=1}^{n-1} \sum_{k=r+1}^{n} b_{r,k,m}B_{r,k},$$

where $b_{r,k,m} \in \mathfrak{R}$. Also, the distance matrix $D \in W$, so

$$D = \sum_{u=1}^{n-1} \sum_{t=u+1}^{n} b_{u,t}B_{u,t}. \tag{3.1}$$

By Theorem 3.2, $D = \sum_{m=1}^{d} mD_m$, so

$$D = \sum_{m=1}^{d} m \left(\sum_{r=1}^{n-1} \sum_{k=r+1}^{n} b_{r,k,m}B_{u,k} \right)$$
$$\sum_{r=1}^{n-1} \sum_{k=r+1}^{n} \left(\sum_{m=1}^{d} mb_{r,k,m}B_{r,k} \right) \tag{3.2}$$

Since B_W is basis, (3.1) & (3.2) gives,

$$b_{u,t} = \sum_{m=1}^{d} mb_{r,k,m}.$$

This gives the relationship between scalars $b_{u,t}$ and $b_{r,k,m}$ which are in the linear combination of D and D_m respectively, with respect to the basis B_W.

Lemma 3.5. *Let A_G be the adjacency matrix of the graph $G = (V, E)$. Let $d = Diam(G)$. Then for $0 \leq k, m \leq d$,*

$$(A_G^{(k)} \circ A_G^{(m)})_{ij} = \begin{cases} 1, & \text{if there exist a walk of length } m \text{ and} \\ & \text{a walk of length } k \text{ between } v_i \text{ and } v_j \\ 0, & \text{otherwise.} \end{cases}$$

Proof. Since, $(A_G^{(k)} \circ A_G^{(m)})$ is a binary matrix , its entries are either 1 or 0.
If $(A_G^{(k)} \circ A_G^{(m)})_{ij} = (A_G^{(k)})_{ij} \circ (A_G^{(m)})_{ij} = 1$, then $(A_G^{(k)})_{ij} = 1$ and $(A_G^{(m)})_{ij} = 1$
$(A_G^{(k)})_{ij} = 1 \Rightarrow$ a walk of length k between v_i and v_j,
$(A_G^{(m)})_{ij} = 1 \Rightarrow$ a walk of length m between v_i and v_j.
$\therefore (A_G^{(k)} \circ A_G^{(m)})_{ij} = 1 \Rightarrow$ a walk of length k and a walk of length m between v_i and v_j. Similarly, $(A_G^{(k)} \circ A_G^{(m)})_{ij} = 0 \Rightarrow$ either $(A_G^{(k)})_{ij} = 0$ or $(A_G^{(m)})_{ij} = 0$, which means a walk of length k or a walk of length m between v_i and v_j does not exist. Therefore

$$(A_G^{(k)} \circ A_G^{(m)})_{ij} = 0,$$

when there does not exist a walk of length k or a walk of length m between v_i and v_j.
By Theorem 3.2, to express the distance matrix D in terms of adjacency matrix A_G, it is enough to express the m-distance matrix D_m in terms of the adjacency matrix A_G. The following result gives relation between D_m and A_G. □

Theorem 3.6. *Let A_G be the adjacency matrix of $G = (V, E)$. Let $d = Diam(G)$, Then for $1 \leq m \leq d$,*

$$D_m = A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right),$$

where D_m is the m-distance matrix of G.

Proof. By Lemma 3.5,

$$(A_G^{(m)} \circ A_G^{(s)})_{ij} = \begin{cases} 1, & \text{if there exist a walk of length } m \text{ and} \\ & \text{a walk of length } s \text{ between } v_i \text{ and } v_j \\ 0, & \text{otherwise.} \end{cases}$$

Then, $\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}$ need not be a binary matrix . But $\delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)$ will be the equivalent binary matrix of it.

$$\left(\delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)\right)_{ij} = \delta\left(\sum_{s=0}^{m-1} (A_G^{(m)})_{ij} \circ (A_G^{(s)})_{ij}\right)$$
$$= \begin{cases} 1, & \text{if there exist a walk of length } m \text{ between } v_i \text{ and } v_j \\ & \text{that can also be joined by a walk of length } < m \\ 0, & \text{otherwise.} \end{cases}$$

Then the subtraction of $\delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)$ from $A_G^{(m)}$ removes all the 1's in $A_G^{(m)}$ that corresponds to the m-length walks joining any two vertices of G which can also be joined by a walk of length fewer than m. The remaining 1's in $A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)$ corresponds to all the m-length paths joining vertices of G. These paths will be the shortest m-length paths joining the vertices because those vertices cannot be joined by a path of length fewer than m. Thus the 1's of $A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)$ corresponds to all the m-distance paths joining the vertices of G. In other words,

$$D_m = A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right).$$

□

Theorem 3.7. *Let A_G be the adjacency matrix of $G = (V, E)$ with diameter d. Then,*

$$D = \sum_{m=1}^{d} m\left(A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)\right).$$

Proof. By Theorem 3.2, $D = \sum_{m=1}^{d} mD_m$. Also, by Theorem 3.6,

$$D_m = A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right).$$

Combining above two, we have

$$D = \sum_{m=1}^{d} m\left(A_G^{(m)} - \delta\left(\sum_{s=0}^{m-1} A_G^{(m)} \circ A_G^{(s)}\right)\right).$$

□

This is the formula for finding the distance matrix D of a graph G from the powers of its adjacency matrix using Hadamard product.

4. Illustration

Consider the following graph G. Then the adjacency ma-

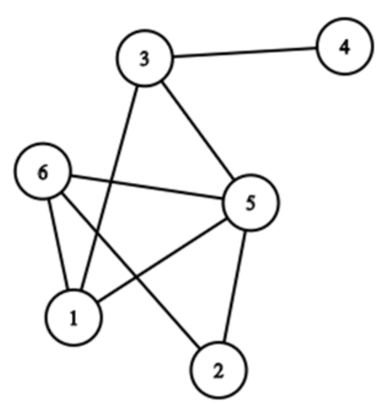

Figure 1

trix A_G of G is

$$A_G = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Here $d = Diam(G) = 3, A_G^{(0)} = I, A_G^{(1)} = \delta(A_G) = A_G$

$$A_G^2 = \begin{bmatrix} 3 & 2 & 1 & 1 & 2 & 1 \\ 2 & 2 & 1 & 0 & 1 & 1 \\ 1 & 1 & 3 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 0 & 2 & 3 \end{bmatrix}, \delta(A_G^2) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$A_G^{(2)} \circ A_G^{(1)} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_G^3 = \begin{bmatrix} 4 & 3 & 6 & 1 & 7 & 7 \\ 3 & 2 & 3 & 1 & 6 & 5 \\ 6 & 3 & 2 & 3 & 7 & 3 \\ 1 & 1 & 3 & 0 & 1 & 2 \\ 7 & 6 & 7 & 1 & 6 & 7 \\ 7 & 5 & 3 & 2 & 7 & 4 \end{bmatrix}, \delta(A_G^3) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_G^{(3)} \circ A_G^{(1)} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A_G^{(3)} \circ A_G^{(2)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$A_G^{(1)} \circ I = 0, D_0 = A_G^{(0)} = I$
$D_1 = A_G^{(1)} - \delta(A_G^{(1)} \circ A_G^{(0)}) = A_G^{(1)} - 0 = A_G^{(1)}$

$$\left(A_G^{(2)} \circ A_G^{(0)} + A_G^{(2)} \circ A_G^{(1)}\right) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$D_2 = A_G^{(2)} - \delta\left(A_G^{(2)} \circ A_G^{(0)} + A_G^{(2)} \circ A_G^{(1)}\right) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\left(A_G^{(3)} \circ I + A_G^{(3)} \circ A_G^{(1)} + A_G^{(3)} \circ A_G^{(2)}\right) = \begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 2 \\ 1 & 2 & 1 & 0 & 2 & 2 \\ 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 0 & 2 & 2 \end{bmatrix}$$

$$D_3 = A_G^{(3)} - \delta\left(A_G^{(3)} \circ I + A_G^{(3)} \circ A_G^{(1)} + A_G^{(3)} \circ A_G^{(2)}\right)$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Now

$$D = \sum_{m=1}^{3} mD_m = 1.D_1 + 2.D_2 + 3.D_3$$
$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$
$$+3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 3 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 2 \\ 2 & 3 & 1 & 0 & 2 & 3 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 3 & 1 & 0 \end{bmatrix}$$

5. Conclusion

Generally it is difficult to find the distance matrix of a large order simple undirected graph. Here we provide a formula for finding the distance matrix from the adjacency matrix of a simple, connected , undirected graph of any finite order . A computer program can be easily written in any of the programming languages for computing distance matrix of the graph using this formula that shall give instant result. So far we considered only simple connected undirected graphs. We can extend this formula for weighted undirected graphs as well as digraphs.

References

[1] L. Graham and L. Lovasz, Distance matrix polynomials of trees, *Advances in Mathematics*, 29(1978), 60–88.

[2] Bapat, *Graphs and Matrices,* Universitext, Springer, 2010.

[3] Narsingh Deo, *Graph Theory with Applications to Engineering and Computer Science*, Courier Dover Publications, 2016.

[4] F. Harary, *Graph Theory*, Addison-Wesley, 1994.

[5] R. J. Wilson, *Introduction to Graph Theory*, 3rd ed. New York, Longman, 1985.

[6] Horn, A. Roger, Johnson, R. Charles, *Matrix Analysis*, Cambridge University Press, 2012.

[7] M. Edelberg, M. R. Garey, and R. L. Graham, On the distance matrix of a tree, *Discrete Mathematics*, 14(1976), 23–39.

[8] R. L. Graham and H. O. Pollak, On the addressing problem for loop switching, *The Bell System Technical Journal*, 50(1971), 2495–2519.

⋆⋆⋆⋆⋆⋆⋆⋆⋆

ISSN(P):2319 − 3786

Malaya Journal of Matematik

ISSN(O):2321 − 5666

⋆⋆⋆⋆⋆⋆⋆⋆⋆