Malaya Journal of Matematik, Vol. 8, No. 3, 898-902, 2020

https://doi.org/10.26637/MJM0803/0028

# Analytic odd mean labeling of Corona graphs

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**Abstract**

The concept of an analytic odd mean labeling was introduced in [3] and further studied in [4 - 7]. In this work, we show that the graphs $TL_n$, $TL_n \odot K_1$, $T_n \odot K_1$, $Q_n \odot K_1$ and $[A(T_n)]AK_1$ admit an analytic odd mean labeling.

**Keywords**

Analytic odd mean labeling, analytic odd mean graph.

**AMS Subject Classification**

05C78.

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**Article History**: Received **12** January **2020**; Accepted **24** April **2020**



## Contents



## 1. Introduction

The graph represented here are only finite, simple and undirected graph $G=(V,E)$ with $p$ vertices and $q$ edges. For mathematical notations we refer Harary [2]. Over the last six decades, the graph labeling concept gained more popularity in the field of graph theory. During this period, several methods of graph labeling are introduced and studied which are available as a ready reference in [1]. One such labeling is called an analytic mean labeling [8]. A graph $G$ is an analytic mean graph if it admits a bijection $f:V\rightarrow \{0,1,2,\ldots,p-1\}$ such that the induced edge labeling $f^*: E\rightarrow Z$ given by $f^*(uv)=\left\lceil\frac{f(u)^2-f(v)^2}{2}\right\rceil$ with $f(u)>f(v)$ is injective. Motivated by the concept of analytic mean labeling, we extended this concept and introduced a new labeling called analytic odd mean labeling [3]. A graph $G$ is an analytic odd mean if there exist an injective function $f:V\rightarrow \{0,1,3,5\ldots,2q-1\}$ with an induce edge labeling $f^*:E\rightarrow Z$ such that for each edge $uv$ with $f(u)<f(v)$, $f^*(uv)=\begin{cases}\left\lceil\frac{f(v)^2-(f(u)+1)^2}{2}\right\rceil, & if \;\; f(u)\neq 0\\ \left\lceil\frac{f(v)^2}{2}\right\rceil, & if \;\; f(u)=0\end{cases}$ is injective. We say that $f$ is an analytic odd mean labeling of $G$. In [4 - 7], we proved that cycle $C_n$, path $P_n$, $n$-bistar, comb $P_n\odot K_1$, graph $L_n\odot K_1$, wheel graph $W_n$, flower graph $Fl_n$, fan $F_n$, double fan $D(F_n)$, double wheel $D(W_n)$, closed helm $CH_n$, total graph of cycle $T(C_n)$, total graph of path $T(P_n)$, armed crown $C_n\Theta P_m$, generalized Peterson graph $GP(n,2)$, the square graph of $P_n$, $C_n$, $B_{n,n}$, $H$-graph and $H\odot mK_1$, subdivision and super subdivision of cycle $C_n$, star $K_(1,n)$, comb $P_n\odot K_1$, path on the comb and $H$-super subdivision of path and cycle are analytic odd mean graphs.

## 2. Preliminaries

In this section, we recall some definitions which will be used throughout the paper.

**Definition 2.1.** *$TL_n$ graph is obtained from $L_n$ by adding the edges $u_iv_{i+1} 1\leq i\leq n-1$, where $u_i$ and $v_i, 1\leq i\leq n$ are the vertices of $L_n$ such that $u_1,u_2,\ldots,u_n$ and $v_1,v_2,\ldots,v_n$ are two paths of length $n$ in the graph $L_n$.*

**Definition 2.2.** *$T_n$ graph is obtained from a path with vertices $v_1,v_2,\cdots,v_n$ by joining $v_i$ and $v_{i+1}$ to a new vertex $u_i$ for $i=1,2,3,\ldots,n-1$.*

**Definition 2.3.** *$Q_n$ graph is obtained from a path $u_1,u_2,\ldots,u_n$ by joining $u_i$ and $u_{i+1}$ to the new vertices $u_i$ and $w_i, 1\leq i\leq n-1$ respectively and then join $v_i$ and $w_i$.*

**Definition 2.4.** *$A(T_n)$ graph is obtained from a path*

$u_1, u_2, \ldots, u_n$ *by joining* $u_i$ *and* $u_{i+1}$ *(alternately) to a new vertex* $v_i$.

**Definition 2.5.** *$A(Q_n)$ graph is obtained from a path* $u_1, u_2, \ldots, u_n$ *by joining* $u_i$ *and* $u_{i+1}$ *(alternately) to the new vertices* $v_i$ *and* $w_i$.

**Definition 2.6.** *The corona* $G_1 \odot G_2$ *of graphs* $G_1$ *and* $G_2$ *is obtained by taking one copy of* $G_1$, *has* $p_1$*vertices and* $p_1$ *copies of* $G_2$, *and then joining the* $i^{th}$ *vertex of* $G_1$ *by an edge to every vertex in the* $i^{th}$ *copy of* $G_2$.

## 3. Main Results

In this section, we prove that the graphs $TL_n$, $TL_n \odot K_1$, $T_n \odot K_1$, $Q_n \odot K_1$ and $[A(T_n)]AK_1$ admit an analytic odd mean labeling.

**Theorem 3.1.** *Triangular ladder* $TL_n$ *admits an analytic odd mean labeling.*

*Proof.* Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be two paths of length $n$. Join $u_i$ and $v_i$ for $1 \leq i \leq n$. Join $u_i$ and $v_{i+1}$ for $1 \leq i \leq n-1$. The graph obtained $TL_n$ has $2n$ vertices and $4n-3$ edges.
A function $f$ from $V$ to $\{0, 1, 3, 5, \ldots, 8n-7\}$ by
$f(v_i) = 2i-1$ and $f(u_i) = 6n+2i-7$ for $i = 1, 2, \ldots, n$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1, 2, \ldots, n$, $f^*(u_iv_i) = 6n(3n-7) + 2i(6n-7) + 25$.
For $i = 1, 2, \ldots, n-1$,
$f^*(u_iu_{i+1}) = 6n+2i-5$,
$f^*(v_iv_{i+1}) = 2i+1$
and $f^*(u_iv_{i+1}) = 3n(3n-7) + 3i(4n-6) + 23$.
We observe that the edge labels of $u_iu_{i+1}$and $v_iv_{i+1}$ are increased by 2 and the edge labels of $u_iv_i$ are increased by $12n-14$ as $i$ increases from 1 to $n$ and the edge labels of $u_iv_{i+1}$ are increased by $12n-18$ as $i$ increases from 1 to $n-1$. Hence all the edge labels are odd and distinct. Therefore $TL_n$ admits an analytic odd mean labeling. □

**Theorem 3.2.** $TL_n \odot K_1$ *admits an analytic odd mean labeling.*

*Proof.* Let $u_i$ and $v_i, 1 \leq i \leq n$ be the vertices of $TL_n$. Let $x_i$ and $y_i$, $1 \leq i \leq n$ be the vertices are attached with $v_i$ and $u_i$ respectively. The graph obtained $TL_n$ has $4n$ vertices and $6n-3$ edges.
A function $f$ from $V$ to $\{0, 1, 3, 5, \ldots, 12n-7\}$ is defined by
$f(v_i) = 2i-1$ for $i = 1, 2, \ldots, n$,
$f(u_i) = 2n+2i+1$ for $i = 1, 2, \ldots, n$,
$f(x_i) = 4n+2i+5$ for $i = 1, 2, \ldots, n$
and $f(y_i) = 10n+2i-7$ for $i = 1, 2, \ldots, n$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1, 2, \ldots, n$,
$f^*(u_iv_i) = 2n(n+1) + 2i(2n+1) + 1$,
$f^*(v_ix_i) = 4n(2n+5) + 2i(4n+5) + 13$,
$f^*(u_iy_i) = 2n(24n-37) + 2i(8n-9) + 23$.
For $i = 1, 2, \ldots, n-1$,
$f^*(u_iu_{i+1}) = 2n+2i+3$,
$f^*(v_iv_{i+1}) = 2i+1$
and $f^*(u_iv_{i+1}) = 2n(n+1) + 2i(2n-1) - 1$.
We observe that the edge labels of $u_iu_{i+1}$ and $v_iv_{i+1}$ are increased by 2 and the edge labels of $u_iv_i$ are increased by $4n+2$ as $i$ increases from 1 to $n$ and the edge labels of $u_iv_{i+1}$ are increased by $4n-2$ as $i$ increases from 1 to $n-1$. Also the edge labels of $x_iv_i$ are increased by $8n+10$ and the edge labels of $u_iy_i$ are increased by $16n-18$ as $i$ increases from 1 to $n$. Hence all the edge labels are odd and distinct. Therefore $TL_n \odot K_1$ admits an analytic odd mean labeling. □

**Theorem 3.3.** $T_n \odot K_1$ *admits an analytic odd mean labeling.*

*Proof.* Let $u_1, u_2, \ldots, u_n$ be a path of length $n$. Let $v_i$ be a new vertex joined with $u_i$ and $u_{i+1}$ . The graph obtained is $T_n$. Let $x_i$ be the vertex joined with $u_i$, $1 \leq i \leq n$. Let $y_i$ be the vertex joined with $v_i$, $1 \leq i \leq n-1$. The graph obtained $T_n \odot K_1$ has $4n-2$ vertices and $5n-4$ edges.
A function $f$ from $V$ to $\{0, 1, 3, 5, \ldots, 10n-9\}$ is defined by
$f(u_i) = 2n+2i-3$ for $i = 1, 2, \ldots, n$,
$f(x_i) = 5n+2i-4$ if $n$ is odd for $i = 1, 2, \ldots, n$,
$f(x_i) = 5n+2i-5$ if $n$ is even for $i = 1, 2, \ldots, n$,
$f(v_i) = 2i-1$ and $f(y_i) = 8n+2i-7$ for $i = 1, 2, \ldots, n-1$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1, 2, \ldots, n$,
$f^*(u_ix_i) = \frac{(7n+4i-6)(3n-2)+1}{2}$ if $n$ is odd,
$f^*(u_ix_i) = \frac{(7n+4i-7)(3n-3)+1}{2}$ if $n$ is even.
For $i = 1, 2, \ldots, n-1$,
$f^*(u_iv_i) = 2n(n-3) + 2i(2n-3) + 5$,
$f^*(u_iu_{i+1}) = 2n+2i-1$,
$f^*(v_iu_{i+1}) = 2n(n-1) + 2i(2n-1) + 1$
and $f^*(v_iy_i) = 8n(4n-7) + 2i(8n-7) + 25$.
Hence all the edge labels are odd and distinct. Therefore $T_n \odot K_1$ admits an analytic odd mean labeling. □

**Theorem 3.4.** $Q_n \odot K_1$ *admits an analytic odd mean labeling.*

*Proof.* Let $u_1, u_2, \ldots, u_n$ be a path of length $n$. Let $v_i$ and $w_i$ be two vertices joined with $u_i$ and $u_{i+1}$ and join $v_i$ and $w_i$, $1 \leq i \leq n-1$. The graph obtained is $Q_n$. Let $x_i$ be the vertex joined with $u_i$, $1 \leq i \leq n$. Let $y_i$ be the vertex joined with $v_i$, $1 \leq i \leq n-1$. Let $z_i$ be the vertex joined with $w_i$, $1 \leq i \leq n-1$. The graph obtained $Q_n \odot K_1$ has $4n+2$ vertices and $7n-6$ edges.
A function $f$ from $V$ to $\{0, 1, 3, 5, \ldots, 14n-13\}$ is defined by
$f(u_i) = 2i-1$ for $i = 1, 2, \ldots, n$,
$f(x_i) = 7n+2i-6$ if $n$ is odd for $i = 1, 2, \ldots, n$,
$f(x_i) = 7n+2i-7$ if $n$ is even for $i = 1, 2, \ldots, n$,
$f(v_i) = 2n+2i-1$ for $i = 1, 2, \ldots, n-1$,
$f(w_i) = 4n+2i-3$ for $i = 1, 2, \ldots, n-1$,
$f(y_i) = 10n+2i-9$ for $i = 1, 2, \ldots, n-1$
and $f(z_i) = 12n+2i-11$ for $1 \leq i \leq n-1$.

The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1,2,\ldots,n$,
$f^*(u_ix_i) = \frac{(7n+4i-6)(7n-6)+1}{2}$ if $n$ is odd,
$f^*(u_ix_i) = \frac{(7n+4i-7)(7n-7)+1}{2}$ if $n$ is even.
For $i = 1,2,\ldots,n-1$,
$f^*(u_iv_i) = 2n(n-1)+2i(2n-1)+1$,
$f^*(u_iu_{i+1}) = 2i+1$,
$f^*(w_iu_{i+1}) = 4n(2n-3)+2i(4n-5)+3$,
$f^*(y_iv_i) = \frac{(12n+4i-9)(8n-9)}{2} + \frac{1}{2}$,
$f^*(w_iz_i) = \frac{(16n+4i-13)(8n-9)}{2} + \frac{1}{2}$
and $f^*(v_iw_i) = 6n(n-2)+2i(2n-3)+5$.
Hence all the edge labels are odd and distinct. Therefore $Q_n \odot K_1$ admits an analytic odd mean labeling. □

**Theorem 3.5.** $[A(T_n)]AK_1$ *admits an analytic odd mean labeling.*

*Proof.* Let $u_1,u_2,\ldots,u_n$ be a path of length $n$. Let $v_i$ be the vertex joined with $u_i$ and $u_{i+1}$ alternately. The graph obtained is $A(T_n)$. Let $x_i$ and $y_i$ be the vertex joined with $u_i$ and $v_i$ respectively. The graph obtained is $[A(T_n)]AK_1$. Let $G = [A(T_n)]AK_1$. Here we consider two cases.
**Case (1)**: $A(T_n)$ starts from $u_2$. Here we have two subcases.
**Subcase (1)(i)**: If $n$ is odd and $n = 3+4i$ where $i = 0,1,2,\ldots$, the graph has $3n-2$ vertices and $\frac{7n-7}{2}$ edges.
An injective function $f : V(G) \to \{0,1,3,5,\ldots,7n-8\}$ is defined by
$f(v_i) = 2i-1$ for $i = 1,2,\ldots,\frac{n-1}{2}$,
$f(u_{2i-1}) = n+4i-4$ for $i = 1,2,\ldots,\frac{n+1}{2}$,
$f(u_{2i}) = n+4i-2$ for $i = 1,2,\ldots,\frac{n-1}{2}$,
$f(x_{2i-1}) = \frac{7n+8i-11}{2}$ for $i = 1,2,\ldots,\frac{n-1}{2}$,
$f(x_{2i}) = \frac{7n+8i-7}{2}$ for $i = 1,2,\ldots,\frac{n-1}{2}$
and $f(y_i) = 6n+2i-7$ for $i = 1,2,\ldots,\frac{n-1}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1,2,\ldots,\frac{n-1}{2}$,
$f^*(u_{2i-1}u_{2i}) = n+4i-2$,
$f^*(u_{2i}u_{2i+1}) = n+4i$,
$f^*(u_{2i+1}v_i) = \frac{(n+6i)(n+2i)+1}{2}$,
$f^*(u_{2i}v_i) = \frac{(n+6i-2)(n+2i-2)+1}{2}$,
$f^*(u_{2i}x_{2i-1}) = \frac{(9n+16i-13)(5n-9)}{8} + \frac{1}{2}$,
$f^*(u_{2i+1}x_{2i}) = \frac{(9n+16i-5)(5n-9)}{8} + \frac{1}{2}$
and $f^*(y_iv_i) = 6n(3n-7)+2i(6n-7)+25$.
**Subcase (1)(ii)**: If $n$ is odd and $n = 5+4i$ where $i = 0,1,2,\ldots$, the graph has $3n-2$ vertices and $\frac{7n-7}{2}$ edges.
An injective function $f : V(G) \to \{0,1,3,5,\ldots,7n-8\}$ is defined by
$f(v_i), f(u_{2i-1})$, $f(u_{2i})$ and $f(y_i)$ are as in Subcase (1)(i)
$f(x_{2i-1}) = \frac{7n+8i-9}{2}$ for $i = 1,2,\ldots,\frac{n-1}{2}$
and $f(x_{2i}) = \frac{7n+8i-5}{2}$ for $i = 1,2,\ldots,\frac{n-1}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
$f^*(u_{2i-1}u_{2i})$, $f^*(u_{2i}u_{2i+1})$, $f^*(u_{2i+1}v_i)$, $f^*(u_{2i}v_i)$ and $f^*(y_iv_i)$ are as in Subcase (1)(i)
For $i = 1,2,\ldots,\frac{n-1}{2}$,
$f^*(u_{2i}x_{2i-1}) = \frac{(9n+16i-11)(5n-7)}{8} + \frac{1}{2}$
and $f^*(u_{2i+1}x_{2i}) = \frac{(9n+16i-3)(5n-7)}{8} + \frac{1}{2}$ .
Clearly all the edge labels are odd and distinct. Therefore the graph $G$ admits an analytic odd mean labeling.
**Subcase (2)(i)**: If $n$ is even and $n = 4+4i$ where $i = 0,1,2,\ldots$, the graph has $3n-4$ vertices and $\frac{7n-12}{2}$ edges.
An injective function $f : V(G) \to \{0,1,3,5,\ldots,7n-13\}$ is defined by
$f(v_i) = 2i-1$ for $i = 1,2,\ldots,\frac{n-2}{2}$,
$f(u_{2i-1}) = n+4i-5$ for $i = 1,2,\ldots,\frac{n}{2}$,
$f(u_{2i}) = n+4i-3$ for $i = 1,2,\ldots,\frac{n}{2}$,
$f(x_{2i-1}) = \frac{7n+8i-14}{2}$ for $i = 1,2,\ldots,\frac{n-2}{2}$,
$f(x_{2i}) = \frac{7n+8i-10}{2}$ for $i = 1,2,\ldots,\frac{n-2}{2}$
and $f(y_i) = 6n+2i-11$ for $i = 1,2,\ldots,\frac{n-2}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i = 1,2,\ldots,\frac{n}{2}$,
$f^*(u_{2i-1}u_{2i}) = n+4i-3$,
$f^*(u_{2i}u_{2i+1}) = n+4i-1$,
For $i = 1,2,\ldots,\frac{n-2}{2}$,
$f^*(u_{2i+1}v_i) = \frac{(n+6i-1)(n+2i-1)+1}{2}$,
$f^*(u_{2i}v_i) = \frac{(n+6i-3)(n+2i-3)+1}{2}$,
$f^*(u_{2i}x_{2i-1}) = \frac{(9n+16i-18)(5n-10)}{8} + \frac{1}{2}$,
$f^*(u_{2i+1}x_{2i}) = \frac{(9n+16i-10)(5n-10)}{8} + \frac{1}{2}$
and $f^*(y_iv_i) = 6n(3n-11)+2i(6n-11)+61$.
**Subcase (2)(ii)**: If $n$ is even and $n = 6+4i$ where $i = 0,1,2,\ldots$, the graph has $3n-4$ vertices and $\frac{7n-12}{2}$ edges.
An injective function $f : V(G) \to \{0,1,3,5,\ldots,7n-13\}$ is defined by
$f(v_i), f(u_{2i-1})$, $f(u_{2i})$ and $f(y_i)$ are as in Subcase (2)(i)
$f(x_{2i-1}) = \frac{7n+8i-12}{2}$ for $i = 1,2,\ldots,\frac{n-2}{2}$
and $f(x_{2i}) = \frac{7n+8i-8}{2}$ for $i = 1,2,\ldots,\frac{n-2}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
$f^*(u_{2i-1}u_{2i})$, $f^*(u_{2i}u_{2i+1})$, $f^*(u_{2i+1}v_i)$, $f^*(u_{2i}v_i)$ and $f^*(y_iv_i)$ are as in Subcase (2)(i)
For $i = 1,2,\ldots,\frac{n-2}{2}$,
$f^*(u_{2i}x_{2i-1}) = \frac{(9n+16i-18)(5n-6)}{8} + \frac{1}{2}$
and $f^*(u_{2i+1}x_{2i}) = \frac{(9n+16i-8)(5n-8)}{8} + \frac{1}{2}$.
Clearly all the edge labels are odd and distinct. Therefore the graph $G$ admits an analytic odd mean labeling.
**Case (2)**: $A(T_n)$ starts from $u_1$. Here also we have two subcases.
**Subcase (1)(i)**: If $n$ is odd and $n = 3+4i$ where $i = 0,1,2,\ldots$, the graph has $3n-2$ vertices and $\frac{7n-7}{2}$ edges.
An injective function $f : V(G) \to \{0,1,3,5,\ldots,7n-8\}$ is defined by
$f(v_i), f(u_{2i-1})$, $f(u_{2i})$, $f(x_{2i-1})$, $f(x_{2i})$ and $f(y_i)$ are as in Subcase (1)(i) in Case (1).
The edge labeling $f^*$ induced by the above function $f$ is as follows:

$f^*(u_{2i-1}u_{2i})$, $f^*(u_{2i}u_{2i+1})$, $f^*(u_{2i}v_i)$ and $f^*(y_iv_i)$ are as in Subcase (1)(i) in Case (1).
For $i=1,2,\ldots,\frac{n-1}{2}$,
$f^*(u_{2i-1}x_{2i-1}) = \frac{(9n+16i-17)(5n-5)}{8}+\frac{1}{2}$,
$f^*(u_{2i}x_{2i}) = \frac{(9n+16i-9)(5n-5)}{8}+\frac{1}{2}$
and $f^*(u_{2i-1}v_i) = \frac{(n+6i-4)(n+2i-4)+1}{2}$.
**Subcase (1)(ii)**: If $n$ is odd and $n=5+4i$ where $i=0,1,2,\ldots$, the graph has $3n-2$ vertices and $\frac{7n-7}{2}$ edges.
An injective function $f:V(G)\to\{0,1,3,5,\ldots,7n-8\}$ is defined by
$f(v_i), f(u_{2i-1})$, $f(u_{2i})$, $f(x_{2i-1})$, $f(x_{2i})$ and $f(y_i)$ are as in Subcase (1)(i) in Case (1).
The edge labelings $f^*$ induced by the above function $f$ is as follows:
$f^*(u_{2i-1}u_{2i})$, $f^*(u_{2i}u_{2i+1})$, $f^*(u_{2i}v_i)$ and $f^*(y_iv_i)$ are as in Subcase (1)(i) in Case (1).
For $i=1,2,\ldots,\frac{n-1}{2}$,
$f^*(u_{2i-1}x_{2i-1}) = \frac{(9n+16i-15)(5n-3)}{8}+\frac{1}{2}$,
$f^*(u_{2i}x_{2i}) = \frac{(9n+16i-7)(5n-3)}{8}+\frac{1}{2}$
and $f^*(u_{2i-1}v_i) = \frac{(n+6i-4)(n+2i-4)+1}{2}$.
Clearly all the edge labels are odd and distinct. Therefore the graph $G$ admits an analytic odd mean labeling.
The examples of an analytic odd mean labeling of $[A(T_7)]AK_1$ and $[A(T_9)]AK_1$ are given in Figure 9.
**Subcase (2)(i)**: If $n$ is even and $n=4+4i$ where $i=0,1,2,\ldots$, the graph has $3n$ vertices and $\frac{7n-2}{2}$ edges.
An injective function $f:V(G)\to\{0,1,3,5,\ldots,7n-3\}$ by
$f(v_i)=2i-1$ for $i=1,2,\ldots,\frac{n}{2}$,
$f(u_{2i-1})=n+4i-3$ for $i=1,2,\ldots,\frac{n}{2}$,
$f(u_{2i})=n+4i-1$ for $i=1,2,\ldots,\frac{n}{2}$,
$f(x_{2i-1})=\frac{7n+8i-6}{2}$ for $i=1,2,\ldots,\frac{n}{2}$,
$f(x_{2i})=\frac{7n+8i-2}{2}$ for $i=1,2,\ldots,\frac{n}{2}$
and $f(y_i)=6n+2i-3$ for $i=1,2,\ldots,\frac{n}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
For $i=1,2,\ldots,\frac{n}{2}$,
$f^*(u_{2i-1}u_{2i})=n+4i-1$,
$f^*(u_{2i}u_{2i+1})=n+4i+1$,
$f^*(u_{2i-1}v_i)=\frac{(n+6i-3)(n+2i-3)+1}{2}$,
$f^*(u_{2i}v_i)=\frac{(n+6i-1)(n+2i-1)+1}{2}$,
$f^*(u_{2i}x_{2i}) = \frac{(9n+16i-2)(5n-2)}{8}+\frac{1}{2}$,
$f^*(u_{2i-1}x_{2i-1}) = \frac{(9n+16i-10)(5n-2)}{8}+\frac{1}{2}$
and $f^*(y_iv_i)=18n(n-1)+6i(2n-1)+5$.
**Subcase (2)(ii)**: If $n$ is even and $n=6+4i$ where $i=0,1,2,\ldots$, the graph has $3n$ vertices and $\frac{7n-2}{2}$ edges.
An injective function $f:V(G)\to\{0,1,3,5,\ldots,7n-3\}$ by
$f(v_i), f(u_{2i-1})$, $f(u_{2i})$ and $f(y_i)$ are as in Subcase (2)(i) in Case (2).
$f(x_{2i-1})=\frac{7n+8i-8}{2}$ for $i=1,2,\ldots,\frac{n}{2}$
and $f(x_{2i})=\frac{7n+8i-4}{2}$ for $i=1,2,\ldots,\frac{n}{2}$.
The edge labeling $f^*$ induced by the above function $f$ is as follows:
$f^*(u_{2i-1}u_{2i})$, $f^*(u_{2i}u_{2i+1})$, $f^*(u_{2i+1}v_i)$, $f^*(u_{2i}v_i)$ and $f^*(y_iv_i)$ are as in Subcase (2)(i) in Case (2).
For $i=1,2,\ldots,\frac{n}{2}$,
$f^*(u_{2i-1}x_{2i-1}) = \frac{(9n+16i-12)(5n-4)}{8}+\frac{1}{2}$
and $f^*(u_{2i}x_{2i}) = \frac{(9n+16i-4)(5n-4)}{8}+\frac{1}{2}$.
Clearly all edge labels are odd and distinct. Therefore the graph $G$ admits an analytic odd mean labeling. □

## 4. Example

An example of an analytic odd mean labeling of $TL_6$ is given in Figure 1.

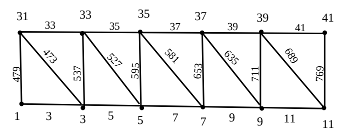

Figure-1

An example of an analytic odd mean labeling of $TL_6 \odot K_1$ is given in Figure 2.

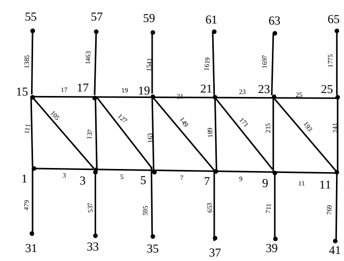

Figure-2

The examples of an analytic odd mean labeling of $T_6 \odot K_1$ and $T_7 \odot K_1$ are given in Figure 3 and 4 respectively.

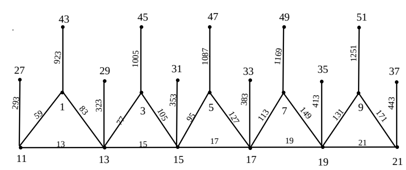

Figure-3

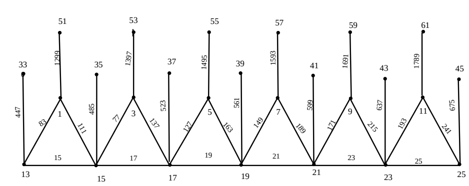

Figure-4

The examples of an analytic odd mean labeling of $Q_3 \odot K_1$ and $Q_4 \odot K_1$ are given in Figure 5 and 6 respectively.

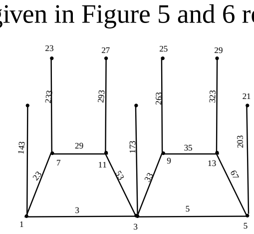

Figure-5

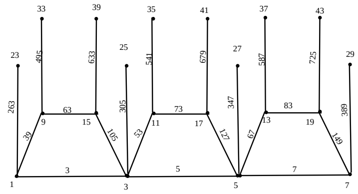

Figure-6

The examples of an analytic odd mean labeling of $[A(T_7)]AK_1$ and $[A(T_9)]AK_1$ are given in Figure 7.

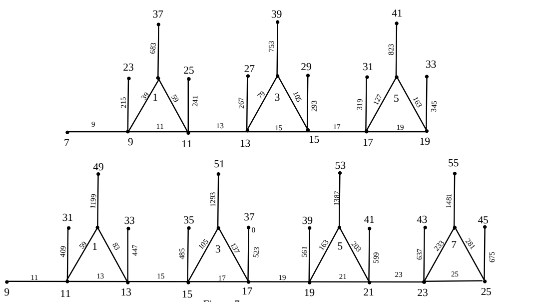

Figure-7

The examples of an analytic odd mean labeling of $[A(T_6)]AK_1$ and $[A(T_8)]AK_1$ are given in Figure 8.

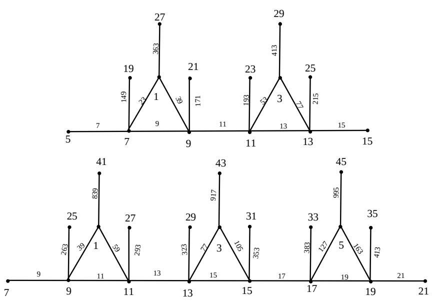

Figure-8

The examples of an analytic odd mean labeling of $[A(T_7)]AK_1$ and $[A(T_9)]AK_1$ are given in Figure 9.

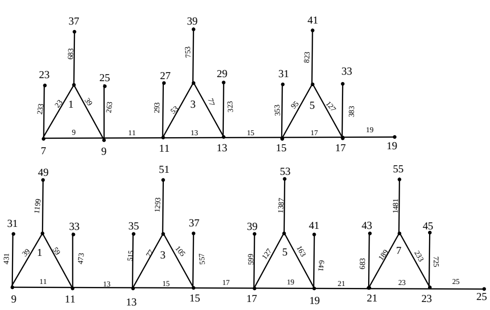

Figure-9

The examples of an analytic odd mean labeling of $[A(T_6)]AK_1$ and $[A(T_8)]AK_1$ are given in Figure 10.

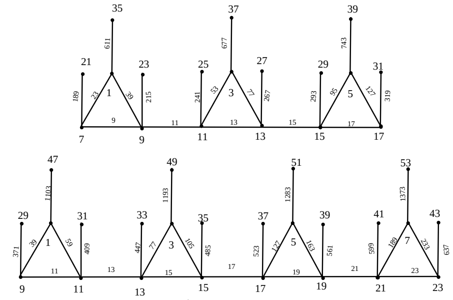

Figure-10

## 5. Conclusion

In this paper, we proved that the chain of graphs such as triangular ladder $TL_n$, the corona product of triangular ladder with $K_1$. triangular snake with $K_1$, quadrilateral snake with $K_1$ and alternate triangular snake with $K_1$ are analytic odd mean graphs.

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ISSN(O):2321 – 5666
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