Malaya Journal of Matematik, Vol. 8, No. 3, 915-919, 2020

https://doi.org/10.26637/MJM0803/0031

Effects of variable internal heat source and variable gravity field on convection in a porous layer

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Abstract

The present article is to examine the qualitative impact of variable internal heat source and gravity variance field on the onset of convection in a horizontal fluid saturated porous medium is investigated using linear stability analysis. To measure the value of the critical Rayleigh number and the corresponding wave number, the single term Galerkin technique is used. Eight separate sets of gravity variance and heat source functions are chosen, and their effect is addressed on the onset of convection. It is observed that the variable heat source and variable gravity at the start of convection do not affect the shape and size of the convective cell. It seen that the system is to be more unstable for the caseses (ii), (iii), (vi) and (vii) while more stable for the caseses (i), (iv), (v) and (viii).

Keywords

Variable internal heat source, Variable gravity, Heat transfer, Stability.

AMS Subject Classification

35Q30.

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Article History: Received **02** March **2020**; Accepted **19** May **2020**

Contents

1. Introduction

Natural convection (convection powered by bouyancy in which gravitational force plays an important role) in fluid-saturated porous medium has attracted the interest of engineers and scientists for a long time due to its numerous applications in fields such as geothermal energy extraction, oil reservoir modeling, reactor vessel insulation, nuclear waste disposals and building thermal insulation to mention a few. Several authors have thoroughly studied the convective instability of horizontal porous layer that is subjected to a destabilizing temperature gradient. The works of Bejan[1] and Combarnous[2] have also compiled most of the findings relevant to this problem.

There are several studies that have appeared in the literature on how the onset of Rayleigh-Bénard convection is influenced by a periodical boundary temperature. Davis[3] has reviewed several of the results related to those issues. At the other hand, limited attention has been paid to the studies relating to the influence of thermal modulation on the onset of convection in a fluid saturated porous medium. Rudraiah and Malashetty[4] have studied the influence of time dependent wall temperature on the onset of convection in a porous medium. The study of the effect of complex body forces on convection in a fluid- and fluid-saturated porous layer has been of great interest [5–8] The effect of internal heating on convection studied by Joseph and Shir[9] and Joseph[10] exploited nonlinear energy methods to find the critical Rayleigh number for an internal heat source for a fluid saturated porous sheet.

Rionero and Straughan[11] and Straughan[12] exploited the energy method to research the influence of variable gravity and variable internal heat source at the onset of convection in a fluid-saturated porous sheet. Gangadharaiah et al. [13] and Suma et al. [14] used perturbation methods to investigate the combined effect of the transit and variable gravity field on the system stability. Also, they considered only one case relating to the linear height variation of gravity. Nevertheless, in sedimentary basins, orogenic and epeirogenic movements of the crustal structures and Earth's crust, nonlinear variation of gravity field with depth can occur(Cordell[15], Shneiderov[16] and Shi and Zhang[17]). Rao et al. [18] compared the exponential, binomial and parabolic functions and found that the parabolic model fits more closely with most crustal structures. In this paper, therefore, we analyze variable internal heat source and gravity field on system using linear and nonlinear variations for the eight cases:

(i) $H(z) = -z, N(z) = z$

(ii) $H(z) = -z^2, N(z) = z$

(iii) $H(z) = -z^3, N(z) = z$

(iv) $H(z) = -(e^z - 1), N(z) = z$

(v) $H(z) = -z, N(z) = z^5$

(vi) $H(z) = -z^2, N(z) = z^5$

(vii) $H(z) = -z^3, N(z) = z^5$

(viii) $H(z) = -(e^z - 1), N(z) = z^5$

The simulations were performed and evaluated for the parameter of the internal heat source strength and the parameter of the gravity variance are discussed in detail.

2. Conceptual Model

Figure 1 illustrates the physical configuration of the present study. The physical model under consideration is a horizontal isotropic porous bed bounded between planes at $z = 0$ and $z = L$ with changeable gravity $g(z)$. We assume that the gravity vector $\vec{g}$ is

$$\vec{g} = -g_0(1 + \lambda H(z))\vec{k}$$

where λ, the variable gravity coefficient, is assumed to be a constant.

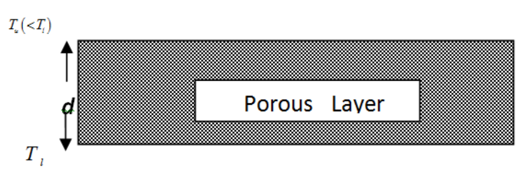

Figure 1. Physical configuration

3. Mathematical Formulation

The porous layer governing equations are

$$\nabla \cdot \vec{V} = 0 \tag{3.1}$$

$$0 = -\nabla p - \frac{\mu}{K}\vec{V} + \rho_0[1 - \beta(T - T_0)]\vec{g}(z) \tag{3.2}$$

$$A\frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla)T = \kappa \nabla^2 T + Q(z) \tag{3.3}$$

In these equations, $\vec{V}$ denotes the velocity vector, p is the pressure, κ is the thermal diffusivity, A is the ratio of heat capacities, ρ_0 is the reference fluid density and T is the temperature. The basic steady state solution is of the form

$$(u, v, w, p, T) = (0, 0, 0, p_b(z), T_b(z))$$

Then equation (3.3) can be written for basic temperature T_b as:

$$\frac{d^2 T_b}{dz^2} - \frac{1}{\kappa}Q(z) = 0$$

Integrating the above equation twice, we get

$$T_b(z) = \frac{-1}{\kappa}\int_0^z \int_0^\xi Q(\lambda)d\lambda d\xi$$

Applying the boundary conditions

$$T_b = T_L \; at \; z = 0 \; \& \; T_b = T_u \; at \; z = d,$$

we obtain

$$T_b(z) = \frac{-1}{\kappa}\int_0^z \int_0^\xi Q(\lambda)d\lambda d\xi - Cz + T_l,$$

where the constant C is given by

$$C = \frac{1}{d}(T_l - T_u) - \frac{1}{\kappa d}\int_0^d \int_0^\xi Q(\lambda)d\lambda d\xi.$$

Basic state is slightly perturbed using the relation given by

$$\vec{V} = \vec{V}', \; p = p_b(z) + p', \; T = T_b(z) + \theta \tag{3.4}$$

Substituting equations (3.4) into equations (3.1)-(3.3). linearizing, by eliminating the term ∇p in the momentum equation and retaining the vertical component, we have:

$$\nabla^2 w = R(1 + \lambda H(z))\nabla^2_{hm} T \tag{3.5}$$

$$\left(A\frac{\partial}{\partial t} - \nabla^2\right)T_m = wN(z). \tag{3.6}$$

We assume the solution are of the form

$$(w, T) = [W(z), \ominus(z)]\, e^{i(lx+my)}. \tag{3.7}$$

Substituting equation (3.7) into equations (3.5)-(3.6), we obtain the following ordinary differential equations

$$(D^2 - a^2)w = -Ra^2(1 + \lambda H(z))\ominus \tag{3.8}$$

$$(D^2 - a^2)\ominus = WL(z) \tag{3.9}$$

where $\ominus$ is the amplitude of perturbed temperature, W is the amplitude of perturbed vertical velocity and $R = \alpha g_0(T_l - T_u)d^3/\nu\kappa$ is the Rayleigh number and $L(z) = 1 + NsN(z)$. The boundary conditions take the form

$$W = \ominus = 0 \; at \; z = 0, 1. \tag{3.10}$$

4. Technique of Solution

Equations (3.8) and (3.9) along with the boundary conditions given by equation (3.10) constitute an eigen value problem with as the eigen value. Accordingly W and $\circleddash$ are written as

$$W = \sum_{i=1}^{n} A_i W_i, \ \circleddash = \sum_{i=1}^{n} B_i \circleddash_i$$
$$W_i = \circleddash_i = sin(iz) \tag{4.1}$$

where A_i & B_i are constants to be determined. Substituting equation (4.1) into equations (3.8)-(3.9) and using trial functions, we obtain a system of linear homogeneous algebraic equations in A_i & B_i. A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation involving the physical parameters R, Ns, λ and a in the form

$$f(R, Ns, \lambda, a) = 0.$$

The critical value of R^c is determined numerically with respect to a for different values of Ns & λ.

5. Results and Discussion

The effect on the onset of convection in a horizontal porous layer with variable gravity and variable internal heat source is analysed. The Galerkin single term approach is used to measure the critical values of the Rayleigh number and the corresponding wave number. To validate the numerical procedure used in the present study, the R^c and the corresponding a_c obtained under the limiting case of $Ns = 0$ (absence of heat source), findings were compared with the results reported in **Table 1** by Rionero and Straughan [11]. **Table 1** shows that the agreement is very good and thus confirms the accuracy of the method used. Eight different cases of linear & non-linear gravity field and internal heat source variation:

(i) $H(z) = -z, N(z) = z$

(ii) $H(z) = -z^2, N(z) = z$

(iii) $H(z) = -z^3, N(z) = z$

(iv) $H(z) = -(e^z - 1), N(z) = z$

(v) $H(z) = -z, N(z) = z^5$

(vi) $H(z) = -z^2, N(z) = z^5$

(vii) $H(z) = -z^3, N(z) = z^5$

(viii) $H(z) = -(e^z - 1), N(z) = z^5$

is investigated. The important values of the critical Rayleigh number and the number of the wave are determined for a broad range of values of the gravity amplitudes and the parameter of the source heat. These values are tabulated in **Table 2-Table**

Table 1. Comparison of R^c and a_c with gravity variation parameter λ in the absence of heat source case($Ns = 0$) for (i) $G(z) = -z$, (ii) $G(z) = -z^2$ and (iv) $G(z) = -(e^z - 1)$

$H(z)$	λ	Present	study	Rionero	[11]
		R^c	a_c^2	R^c	a_c^2
Case: (i)	0	39.478	9.872	39.478	9.870
	1	77.080	10.208	77.020	10.209
	1.5	132.020	12.213	132.020	12.314
	1.8	189.908	17.198	189.908	17.198
	1.9	212.281	19.475	212.280	19.470
Case: (ii)	0	39.478	9.872	39.478	9.870
	0.2	41.832	9.872	41.832	9.874
	0.4	44.455	9.885	44.455	9.887
	0.6	47.389	9.916	47.389	9.915
	0.8	50.682	9.960	50.682	9.961
Case: (iv)	0	39.478	9.872	39.478	9.870
	0.1	42.331	9.872	42.331	9.872
	0.2	45.607	9.885	45.607	9.883
	0.3	49.398	9.904	49.398	9.904
	0.4	53.828	9.941	53.828	9.942
	0.5	59.053	10.005	59.053	10.005

Table 2. $Ns = 0.5$, Case: (i)

λ	a_c	R^c
0	3.1415	31.5827
0.1	3.1415	33.245
0.2	3.1415	35.0919
0.3	3.1415	37.1562
0.4	3.1415	39.4784
0.5	3.1415	42.1103
0.6	3.1415	45.1182
0.7	3.1415	48.5888
0.8	3.1415	52.6379
0.9	3.1415	57.4232
1	3.1415	63.1655

Table 3. $Ns = 0.5$, Case: (ii)

λ	a_c	R^c
0	3.1415	31.5827
0.1	3.1415	32.5015
0.2	3.1415	33.4752
0.3	3.1415	34.5092
0.4	3.1415	35.609
0.5	3.1415	36.7813
0.6	3.1415	38.0333
0.7	3.1415	39.3736
0.8	3.1415	40.8119
0.9	3.1415	42.3591
1	3.1415	44.0283

Table 4. $Ns = 0.5$, Case: (iii)

λ	a_c	R^c
0	3.1415	31.5827
0.1	3.1415	32.142
0.2	3.1415	32.7215
0.3	3.1415	33.3222
0.4	3.1415	33.9455
0.5	3.1415	34.5924
0.6	3.1415	35.2645
0.7	3.1415	35.9633
0.8	3.1415	36.6903
0.9	3.1415	37.4473
1	3.1415	38.2362

Table 5. $Ns = 0.5$, Case: (iv)

λ	a_c	R^c
0	3.1415	31.5827
0.1	3.1415	33.8719
0.2	3.1415	36.5189
0.3	3.1415	39.6146
0.4	3.1415	43.2838
0.5	3.1415	47.702
0.6	3.1415	53.1248
0.7	3.1415	59.9387
0.8	3.1415	68.7577
0.9	3.1415	80.6195
1	3.1415	97.4272

Table 6. $Ns = 0.5$, Case: (v)

λ	a_c	R^c
0	3.1415	37.9872
0.1	3.1415	39.9865
0.2	3.1415	42.208
0.3	3.1415	44.6908
0.4	3.1415	47.484
0.5	3.1415	50.6496
0.6	3.1415	54.2674
0.7	3.1415	58.4418
0.8	3.1415	63.312
0.9	3.1415	69.0676
1	3.1415	75.9744

Table 7. $Ns = 0.5$, Case: (vi)

λ	a_c	R^c
0	3.1415	37.9872
0.1	3.1415	39.0922
0.2	3.1415	40.2635
0.3	3.1415	41.5071
0.4	3.1415	42.8299
0.5	3.1415	44.2399
0.6	3.1415	45.7458
0.7	3.1415	47.3579
0.8	3.1415	49.0878
0.9	3.1415	50.9488
1	3.1415	52.9566

Table 8. $Ns = 0.5$, Case:(vii)

λ	a_c	R^c
0	3.1415	37.9872
0.1	3.1415	38.6599
0.2	3.1415	39.3569
0.3	3.1415	40.0794
0.4	3.1415	40.829
0.5	3.1415	41.6072
0.6	3.1415	42.4156
0.7	3.1415	43.256
0.8	3.1415	44.1305
0.9	3.1415	45.041
1	3.1415	45.9898

Table 9. $Ns = 0.5$, Case:(viii)

λ	a_c	R^c
0	3.1415	37.9872
0.1	3.1415	40.7406
0.2	3.1415	43.9243
0.3	3.1415	47.6478
0.4	3.1415	52.061
0.5	3.1415	57.3752
0.6	3.1415	63.8976
0.7	3.1415	72.0933
0.8	3.1415	82.7006
0.9	3.1415	96.9677
1	3.1415	117.184

9. The following conclusions are drawn.

For case (i) and case (v) i.e., for linear gravity variation with linear and the polynomial functional variation of internal heat source and for case (iv) and case (viii) i.e., for exponential gravity variation with linear and the polynomial functional variation of internal heat source. For these four cases, he critical value of the Rayleigh number inecreases with an increase in the value of λ, the amplitude of the gravitational disturbance with internal heat source value $Ns = 0.5$. Thus such a variable gravity more stabilizes the system (**Tables 2,5,6 and 9**). Furthermore, it is noticed that the system more stable for these four caseses. The effect of variable heat source is found to be stabilizing in all cases. This is because variable internal heat source produce more heat and this intern helps for the early onset of convection.

For case (ii) case (iii) case (vi) and case (vii) i.e., for polynomial gravity variation with linear and the polynomial functional variation of internal heat source For these four caseses, he critical value of the Rayleigh number increases with an increase in the value of λ, the amplitude of the gravitational disturbance with internal heat source value $Ns = 0.5$ Thus such a variable gravity less stabilizes the system (**Tables 3,4,7 and 8**). Furthermore, it is noticed that the system less stable for these four caseses and the effect of variable heat source is found to be stabilizing in all cases.

6. Conclusion

The linear theory predicts that, the variable gravity and the variable internal heat source will not change the shape and size of the convection cell. It is noted that for case (i), case (iv) case (v) and case (viii) the system is more stable, while for caseses case (ii), case (iii) case (vi) and case (vii) the system is less stable. The onset of convection can be advanced or delayed by choosing proper values internal heat source parameter Ns and gravity parameter λ. This will helps in heat transfer applications and as well as material processing applications.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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