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A study on various soft nano continuous functions and soft nano homeomorphism

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Abstract
Some interesting properties of soft nano $g\omega$-continuous functions are discussed and provided with the counter examples. Soft nano $g\omega$-irresolute continuous functions are studied along with their characterization. Specially, we establish some notable results pertaining to soft nano perfectly continuous functions, soft nano strongly continuous functions. Soft nano $g\omega$-homeomorphism is defined and its subclass soft nano $(g\omega)^*$-homeomorphism is studied.

Keywords
Soft nano $g\omega$-continuous, soft nano $g\omega$-irresolute, soft nano perfectly continuous, soft nano strongly continuous, soft nano $g\omega$-homeomorphism, soft nano $(g\omega)^*$-homeomorphism.

AMS Subject Classification
54A05.

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Contents

1. Introduction

Many researchers have worked on the algebraic structure of soft set theory and utilized soft sets in multicriteria decision-making problems, applied the technique of knowledge reduction to the information table induced by the soft set. Also, defined and discussed the several properties of soft images and soft inverse images of soft sets with applications in medical diagnosis. In nano Topology, weaker forms of nano open sets and nano continuous functions, their decomposition and further work was carried by [2–5, 7, 9–11, 13–15, 18, 19, 22]. Sundaram [21], developed generalized homeomorphism concept and in Nano topological spaces, nano homeomorphism is given by Thivagar et.al [23].

The notion of soft nano topology was introduced by [1]. Patil et. al [16] defined soft nano disjoint dense sets whose union forms soft nano resolvable space, soft nano extremally disconnected and soft nano faint homeomorphism. A brief study on soft nano irresolvable spaces, soft nano open hereditarily irresolvable space and comparisons between such spaces, along with levels of soft nano irresolvability has been presented.

Indeed a significant theme in soft nano topology concerns the variously modified forms of soft nano continuity such as soft nano strongly continuous, soft nano perfectly continuous, soft nano irresolute functions. In this paper, analysis of properties of weaker forms of soft nano continuous functions with soft nano $g\omega$-irresolute functions and its compositions are done. This forms the basis for further extension of study in contra soft nano generalized continuous functions. Introducing the concept of soft nano $g\omega$-homeomorphism, its subclass soft nano $(g\omega)^*$-homeomorphism is developed.

2. Preliminaries

Definition 2.1. *[1] Let the set of objects be denoted by U. The soft approximation space is (U, R^1, O_1) where R^1 is a soft equivalence relation. Let $X_1 \subseteq U$:*

1. Then $(L_{R^1}(X_1), O_1) = \cup\{R^1(x_1) : R^1(x_1) \subseteq X_1\}$ is a soft *lower approximation of X_1 concerning to R^1.*

2. *Then $(U_{R^1}(X_1),O_1) = \cup\{R^1(x_1) : R^1(x_1)\cap X_1 \neq \phi\}$ is a soft upper approximation of X_1 concerning to R^1.*

3. *Then $(B_{R^1}(X_1),O_1) = (U_{R^1}(x_1) - L_{R^1}(x_1))$ is a soft boundary region of X_1 concerning to R^1.*

Here are some definitions and results given by various authors, helpful for further study.

Definition 2.2. *[1] Let set of objects be denoted by U, R^1 is a soft equivalence relation and $\tau_{R^1}(X_1) = \{U,\phi,(L_{R^1}(X_1),O_1),(U_{R^1}(X_1),O_1),(B_{R^1}(X_1),O_1)\}$ satisfies the following axioms.*

1. *U and $\phi \in \tau_{R^1}(X_1)$.*
2. *The union of the elements of any finite subcollection $\phi \in \tau_{R^1}(X_1)$ is in $\phi \in \tau_{R^1}(X_1)$.*
3. *The intersection of the elements of any finite subcollection $\phi \in \tau_{R^1}(X_1)$ is in $\phi \in \tau_{R^1}(X_1)$.*

Then $\tau_{R^1}(X_1)$ is soft nano topology on U with respect to X_1, elements of the soft nano topology are known as the soft nano open sets and $(\tau_{R^1}(X_1),U,O_1)$ is called a soft nano topological space.

Definition 2.3. *[1] The soft nano closure of (A^*,O_1) is defined as the intersection of all soft nano closed sets containing (A^*,O_1) and is denoted by sn-cl(A^*,O_1).*

Definition 2.4. *[17] A subset (B_1^*,O_1) of $(\tau_{R^1}(X_1),U,O_1)$ is known as sn-gω-closed if sn-cl$(B_1^*,O_1) \subseteq (V_1^*,O_1)$ whenever $(B_1^*,O_1) \subseteq (V_1^*,O_1)$ and (V_1^*,O_1) is sn-semi-open in $(\tau_{R^1}(X_1),U,$*
$O_1)$. The family of all sn-gω-closed sets over U is denoted by sn-gω-$C(X_1,O_1)$

Definition 2.5. *[17] The sn-gω-closure of subset (A_1^*,O_1) of $(\tau_{R^1}(X_1),U,O_1)$ is defined as sn-cl$_{g\omega}$ $(A_1^*,O_1) = \cap\{(G_1^*,O_1)$: $(G_1^*,O_1) \subseteq (A_1^*,O_1)$, (G_1^*,O_1) is sn-gω-closed$\}$.*

Definition 2.6. *[17] The sn-gω-interior of subset (A_1^*,O_1) of $(\tau_{R^1}(X_1),U,O_1)$ is defined as sn-int$_{g\omega}$ $(A_1^*,O_1) = \cup\{(G_1^*,O_1)$: $(G_1^*,O_1) \subseteq (A_1^*,O_1)$, (G_1^*,O_1) is sn-gω-closed$\}$.*

Definition 2.7. *[17] If there exists sn-gω-open set (S_1^*,O_1) such that $x_1 \in (S_1^*,O_1) \subseteq (A_1^*,O_1)$, where (S_1^*,O_1) is a subset of a soft nano topological space $(\tau_{R^1}(X_1),U,O_1)$, then it is said to be sn-gω neighborhood (briefly sn-gω nhd) of a point x_1 of U.*

Definition 2.8. *[17] A function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ is sn-gω continuous if the inverse image of every sn-open in U_2 is sn-gω open in U_1.*

Definition 2.9. *[3] A map $F : (U_1,\tau_R(X_1) \to (U_2,\tau_{R'}(X_2)$ is said to be nano ωg- closed map (resp. nano ωg-open map) if the image of every nano ωg-closed set (resp. nano ωg-open set) in U_2 is nano closed set (resp. nano open set) in U_1.*

Definition 2.10. *[16] A soft nano (Y_1^*,O_1) of soft nano topological space $(\tau_{R'}(X_1),U_1,O_1)$ is sn-dense, if sn- cl(Y_1^*,O_1)=U_1.*

Definition 2.11. *[3] A function $F : U_1 \to U_2$ is called homeomorphism, if*

1. *F is bijective*
2. *F is continuous*
3. *F is open.*

Definition 2.12. *[24] A function $F : (X_1,\tau_1) \to (X_2,\tau_2)$ is regular generalized star b-homeomorphism if F is both rg**b-continuous and rg**b-open.*

3. Soft Nano Continuous Functions

Definition 3.1. *The function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ is said to be;*

1. *soft nano strongly continuous (briefly, sn-δ-continuous), if $F^{-1}(M_1^*,O_1)$ is soft nano clopen in U_1 for each soft nano subset (M_1^*,O_1) in U_2.*
2. *soft nano perfectly continuous, if $F^{-1}(M_1^*,O_1)$ is soft nano clopen in U_1 for each soft nano subset (S_1^*,O_1) in U_2.*

Definition 3.2. *In a soft nano topological space $(\tau_{R'}(X_1),U_1,O_1)$, $B_{sn} = \{U_1,L_{(R'}(X_1),B_{(R'}(X_1)\}$ is soft nano-basis for $\tau_{R'}(X_1)$.*

Theorem 3.3. *A function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ is sn-gω-continuous if and only if the inverse image of every member of B_{sn} is sn-gω-$O(X_1,O_1)$.*

Proof. Let $(B_1^*,O_1) \in B_{sn}$ and $F_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ be sn-gω-continuous on $(\tau_{R'}(X_1),U_1,O_1)$. Then (B_1^*,O_1) is sn-$O(X_1,O_1)$, $F^{-1}(V^*,O_1)$ is sn-gω-$O(X_1,O_1)$, as F is sn-gω-continuous. Therefore the inverse image of every member of B_{sn} is sn-gω-$O(X_1,O_1)$.
Conversely, let inverse image of every member of B_{sn} be sn-gω-$O(X_1,O_1)$. Let $(H_1^*,O_1) = \cap\{(V^*,O_1) : (V^*,O_1) \in (B_1^*,O_1)\}$ where $(B_1^*,O_1) \subset B_{sn}$. Then $F^{-1}(H_1^*,O_1) = F^{-1}(\cup\{(V^*,O_1) : (V^*,O_1) \in (B_1^*,O_1)\} = \cup\{F^{-1}(V^*,O_1) : (V^*,O_1) \in (B_1^*,O_1)\}$ where each $F^{-1}(V^*,O_1)$ is sn-gω-$O(X_1,O_1)$. Also their function $F^{-1}((H_1^*,O_1))$ is sn-gω-$O(X_1,O_1)$. Hence $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ is sn-gω continuous. □

Theorem 3.4. *Let $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_1),U_2,O_2)$ be sn-onto, sn-gω-continuous function. If (G^*,O_1) is sn-gω-dense in $(\tau_{R'}(X_1),U_1,O_1)$, then F(G^*,O_1) is sn-dense in $(\tau_{R''}(X_1),U_2,O_2)$.*

Proof. Given (G^*,O_1) is sn-gω-dense in $(\tau_{R'}(X_1),U_1,O_1)$. Thus sn-cl$_{g\omega}(G^*,O_1) = U_1$. As F is sn-onto, F(sn-cl$_{g\omega}(G^*,O_1)$) = F(U_1) = U_2. Here F(sn-cl$_{g\omega}(G^*,O_1)$) $\subseteq$ sn-cl(F(G^*,O_1)) as

F is sn-gω-continuous. Here sn-cl(F(G^*,O_1))⊆ U_2 and $U_2 \subseteq$ sn-cl(F(G^*,O_1)) implies that sn-cl(F(G^*,O_1)) = U_2. Therefore F(G^*,O_1) is sn-dense in $(\tau_{R'}(X_1),U_1,O_1)$. Hence a sn-continuous function maps sn-gω-dense sets into sn-dense sets whenever it is sn-onto. □

4. Soft Nano-$g\omega$-Irresolute Functions

The stronger form of sn-$g\omega$-continuous functions, sn-$g\omega$-irresolute functions in soft nano topological space is introduced and its characterizations is mentioned.

Definition 4.1. *A function $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_1),U_2,O_2)$ is sn-gω-irresolute, if $F^{-1}(M^*,O_1)$ is sn-gω-open for every sn-gω-open set (M^*,O_1) in U_2.*

Remark 4.2. *The function $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_1),U_2,O_2)$ is sn-gω-irresolute if and only if the inverse image of every sn-gω-closed set U_2 is sn-gω-closed in U_1.*

Theorem 4.3. *Composition of two sn-gω-irresolute functions is again a sn-gω-irresolute function.*

Proof. Let $F_1:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ and $F_2:(\tau_{R''}(X_2),U_2,O_2)\to(\tau_{R'''}(X_3),U_3,O_3)$ are two sn-gω-irresolute functions. Let (M^*,O_3) be a sn-gω-C(X_3,O_3). Since F_2 is sn-gω-irresolute function, $F_2^{-1}(M^*,O_3)$ is sn-gω-C(X_2,O_2). Then $F_1^{-1}(F_2^{-1}(M^*,O_3))$, the inverse image of $F_2^{-1}(M^*,O_3)$ under sn-gω-irresolute function F_1 is sn-gω-C(X_1,O_1). Hence, the composition $F_2\circ F_1$ is sn-gω-irresolute function. □

Theorem 4.4. *Let $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ be a sn-gω-irresolute function, then F is sn-gω-continuous function.*

Proof. Let $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ be a sn-gω-irresolute function and (M^*,O_1) is sn-C(X_1,O_1).Then (M^*,O_1) is sn-gω-closed set [17]. From the definition 4.1, $F^{-1}(M^*,O_1)$ is sn-gω-C(X_1,O_1). Therefore F is sn-gω-continuous function. □

Remark 4.5. *Converse of the above theorem need not be true in general as seen by following example.*

Example 4.6. *Let $U_1=\{\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4\}$, $X_1=\{\varepsilon_1,\varepsilon_3\}$, $U_1/R'=\{\{\varepsilon_1,\varepsilon_4\},\{\varepsilon_2\},\{\varepsilon_3\}\}$, $\tau_{R'}(X_1)=\{U_1,\emptyset,(k_1,\{\varepsilon_3\}),(k_2,\{\varepsilon_3\}),(k_3,\{\varepsilon_3\}),(k_1,\{\varepsilon_1,\varepsilon_4\}),(k_2,\{\varepsilon_1,\varepsilon_4\}),(k_3,\{\varepsilon_1,\varepsilon_4\}),(k_1,\{\varepsilon_1,\varepsilon_3,\varepsilon_4\}),(k_2,\{\varepsilon_1,\varepsilon_3,\varepsilon_4\}),(k_3,\{\varepsilon_1,\varepsilon_3,\varepsilon_4\})\}$. And $\{\varepsilon_1,\varepsilon_2,\varepsilon_4\}$, $\{\varepsilon_2,\varepsilon_3\}$, $\{\varepsilon_2\}\in sn-C(X_1,O_1)$. Let $U_2=\{\varepsilon_1',\varepsilon_2',\varepsilon_3',\varepsilon_4'\}$, $X_2=\{\varepsilon_1',\varepsilon_2',\varepsilon_3'\}$, $U_2/R''=\{\{\varepsilon_1',\varepsilon_3'\},\{\varepsilon_2'\},\{\varepsilon_4'\}\}$ then $\tau_{R''}(X_2)=\{U_2,\emptyset,(k_1',\{\varepsilon_1',\varepsilon_3',\varepsilon_4'\}),(k_2',\{\varepsilon_1',\varepsilon_3',\varepsilon_4'\}),(k_3',\{\varepsilon_1',\varepsilon_3',\varepsilon_4'\}),(k_1',\{\varepsilon_2'\}),(k_2',\{\varepsilon_2'\}),(k_3',\{\varepsilon_2'\}),(k_1',\{\varepsilon_1',\varepsilon_3'k\}),(k_2',\{\varepsilon_1',\varepsilon_3'\}),(k_3',\{\varepsilon_1',\varepsilon_3'\})\}$ and $\{\varepsilon_1',\varepsilon_3',\varepsilon_4'\},\{\varepsilon_2'\},\{\varepsilon_3'\}\in sn-O(X_2,O_2)$. Define a function $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ as $F(\varepsilon_1)=\varepsilon_1',F(\varepsilon_2)=\varepsilon_2',F(\varepsilon_3)=\varepsilon_3'$ and $F(\varepsilon_4)=\varepsilon_4'$. Since $F^{-1}(\{\varepsilon_3'\})=\{\varepsilon_3\}$ is not sn-gω-O(X_1,O_1), but $\{\varepsilon_3'\}$ is sn-gω-O(X_2,O_2). Hence F sn-gω-continuous but not sn-gω-irresolute.*

Theorem 4.7. *If $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ is sn-gω-continuous and $F_2:(\tau_{R''}(X_2),U_2,O_2)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-continuous. Then, $F_2\circ F_1(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-gω-continuous function.*

Proof. Let $(P^*,O_3)\in$ sn-O(X_3,O_3). Then $F_2^{-1}(P^*,O_3)$ is sn-O(X_2,O_2) as F_2 is sn-continuous. Thus $F_2^{-1}(P^*,O_3)$ is sn-gω- O(X_2,O_2) by [17]. Here, $F_1^{-1}(F_2^{-1}(P^*,O_3))=(F_2\circ F_1)^{-1}(P^*,O_3)$ is sn-gω-O(X_1,O_1) and $F_2\circ F_1$ is sn-gω-continu ous. □

Theorem 4.8. *If $F_1:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ is sn-gω-irresolute and $F_2:(\tau_{R''}(X_2),U_2,O_2)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-g-continuous. Then, $F_2\circ F_1(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-gω-continuous function.*

Proof. Let $(H^*,O_3)\in$ sn-O(X_3,O_3). Then $F_2^{-1}(H^*,O_3)$ is sn-g-O(X_2,O_2) as F_2 is sn-g-continuous. Thus $F_2^{-1}(H^*,O_3)$ is sn-gω- O(X_2,O_2). Then $F_1^{-1}(F_2^{-1}(H^*,O_3))=(F_2\circ F_1)^{-1}(H^*,O_3)$ is sn-gω-O(X_1,O_1) and $F_2\circ F_1$ is sn-gω-continuous. □

Theorem 4.9. *If $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ is sn-gω irresolute and $F_2:(\tau_{R''}(X_2),U_2,O_2)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-gω-continuous. Then, $F_2\circ F_1(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R'''}(X_3),U_3,O_3)$ is sn-gω-continuous function.*

Proof. Let (M^*,O_3) be a member of sn-O(X_3,O_3). Then $F_2^{-1}(M^*,O_3)$ is sn-gω-O(X_2,O_2) as F_1 is sn-gω-irresolute, then $F_1^{-1}(F_2^{-1}(M^*,O_3))=(F_2\circ F_1)^{-1}(H^*,O_3)$ is sn-gω-O(X_1,O_1) and therefore $F_2\circ F_1$ is sn-gω-continuous. □

5. Soft Nano-$g\omega$-homeomorphisms

In this section soft nano-$g\omega$-homeomorphism is introduced and its several properties are discussed.

Definition 5.1. *The function $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ is a called sn-gω homeomorphism, if*

1. *F is bijective.*
2. *F is sn-gω-continuous.*
3. *F is sn-gω-open.*

Theorem 5.2. *A bijective function $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ is sn-gω-homeomorphism if and only if F is sn-gω-closed and sn-gω-continuous.*

Proof. Let $F:(\tau_{R'}(X_1),U_1,O_1)\to(\tau_{R''}(X_2),U_2,O_2)$ be a sn-$g\omega$-homeomorphism and by definition 5.1, F is sn-$g\omega$-continuous. Let $(P^*,O_1)\in$ sn-C(X_1,O_1), then $U_1-(P^*,O_1)$ is sn-C(X_1,O_1) and $F(U_1-(P^*,O_1))$ is sn-$g\omega$-O(X_2,O_2) as F is sn-$g\omega$-open. That is $U_2-F(P^*,O_1)$ is sn-$g\omega$-O(X_2,O_2). Thus $F(P^*,O_1)$ is sn-$g\omega$-C(X_2,O_2) for every sn-closed set (P^*,O_1) in $(\tau_{R'}(X_1),U_1,O_1)$. Therefore the function F is sn-$g\omega$-closed. Conversely, let F be sn-gω-continuous function and sn-$g\omega$-closed. Let $(S^*,O_1)\in$ sn-O(X_1,O_1). As F is sn-$g\omega$-closed, $F(U_1-(S^*,O_1))$ is sn-$g\omega$-C(X_2,O_2). Here $F(U_1-(S^*,O_1))$

$= U_2 - F(S^*, O_1)$ is sn-$g\omega$-C(X_2, O_2). Thus $F(S^*, O_1)$ is sn-$g\omega$-O(X_2, O_2) for every sn-open set (P^*, O_1) and $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is sn-$g\omega$-homeomorphism. □

Theorem 5.3. *If a function $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is soft nano homeomorphism, then it is soft nano-$g\omega$-homemorphism but the converse is not true.*

Proof. A soft nano homeomorphism function F is soft nano continuous, bijective and soft nano open. Then F is sn-$g\omega$-continuous by [17] and thus inverse image of every sn-$g\omega$-O(X_2, O_2) is sn-O(X_1, O_1). □

Remark 5.4. *Converse of the above theorem 5.3 is not true in general.*

Example 5.5. *Let $U_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $X_1 = \{\varepsilon_1, \varepsilon_2, \varepsilon_4\} \subseteq U_1, O_1 = \{k_1, k_2, k_3\}$. $U_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3', \varepsilon_4'\}$, $O_2 = \{k_1', k_2', k_3'\}$, $X_2 = \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\} \subseteq U_2$. $\tau_{R'}(X_1) = \{U_1, \emptyset, (k_1, \{\varepsilon_1\}), (k_2, \{\varepsilon_1\}), (k_3, \{\varepsilon_1\})\ \{(k_1, \{\varepsilon_2, \varepsilon_4\}), (k_2, \{\varepsilon_2, \varepsilon_4\}), (k_3, \{\varepsilon_2, \varepsilon_4\}), (k_1, \{\varepsilon_1, \varepsilon_2, \varepsilon_4\}), (k_2, \{\varepsilon_1, \varepsilon_2, \varepsilon_4\}), (k_3, \{\varepsilon_1, \varepsilon_2, \varepsilon_4\})\}$. $U_2/R'' = \{\{\varepsilon_1', \varepsilon_3'\}, \{\varepsilon_2'\}, \{\varepsilon_4'\}\}$ then $\tau_{R''}(X_2) = \{U_2, \emptyset, (k_1', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}), (k_2', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}), (k_3', \{\varepsilon_1', \varepsilon_2', \varepsilon_3'\}), (k_1', \{\varepsilon_2'\}), (k_2', \{\varepsilon_2'\}), (k_3', \{\varepsilon_2'\}), (k_1', \{\varepsilon_1', \varepsilon_3'\}), (k_2', \{\varepsilon_1', \varepsilon_3'\}), (k_3', \{\varepsilon_1', \varepsilon_3'\})$ Define a function $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ as $F(\varepsilon_1) = \varepsilon_1'$, $F(\varepsilon_2) = \varepsilon_2'$, $F(\varepsilon_3) = \varepsilon_4'$, $F(\varepsilon_4) = \varepsilon_3'$. Here sn-g$\omega$ closed sets in U_1 are $U_1 = \{\varepsilon_3\}, \{\varepsilon_1, \varepsilon_3\}, \{\varepsilon_2, \varepsilon_3\}, \{\varepsilon_3, \varepsilon_4\}, \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}, \{\varepsilon_1, \varepsilon_3, \varepsilon_4\}, \{\varepsilon_2, \varepsilon_3, \varepsilon_4\}$ and sn-gω closed sets in U_2 are, $U_2 = \{\varepsilon_4'\}, \{\varepsilon_1', \varepsilon_4'\}, \{\varepsilon_2', \varepsilon_4'\}, \{\varepsilon_3', \varepsilon_4'\}, \{\varepsilon_1', \varepsilon_2', \varepsilon_4'\}, \{\varepsilon_1', \varepsilon_3', \varepsilon_4'\}, \{\varepsilon_2', \varepsilon_3', \varepsilon_4'\}$. Here F is bijective and inverse image of every soft nano closed set in U_2 is sn-gω closed in U_1. Thus F is sn-gω-continuous. The image of every soft nano open set in U_1 is sn-gω open in U_2. Thus F is sn-gω open.Therefore F is sn-gω-homeomorphism.But F is not sn-homeomorphism, as $F^{-1}(\varepsilon_3', \varepsilon_4') = \{\varepsilon_2, \varepsilon_3\}$is not sn-closed in U_1.*

Theorem 5.6. *A one to one mapping $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is sn-gω-homeomorphism if and only if $F(sn - cl_{g\omega}(M^*, O_1)) = sn\text{-}cl(F(M^*, O_1))$ for every sn-subset (M^*, O_1) of $(\tau_{R'}(X_1), U_1, O_1)$.*

Proof. Let $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ be an sn-$g\omega$-homeomorphism. Then F is sn-$g\omega$-closed and sn-$g\omega$-continuous. As F is sn-$g\omega$-continuous, for $(M^*, O_1) \subseteq U_1$, we have $F(sn - cl_{g\omega}(M^*, O_1)) \subseteq$ sn-cl$(F(M^*, O_1))$. Since $sn - cl_{g\omega}(M^*, O_1)$ is sn-cl(X_1, O_1) and F is sn-$g\omega$-closed function, $F(sn\text{-}cl_{g\omega}(M^*, O_1))$ is sn-$g\omega$-$cl(X_2, O_2)$. Also, $sn\text{-}Cl_{g\omega}(F(sn\text{-}Cl_{g\omega}(M^*, O_1))) = F(sn\text{-}Cl_{g\omega}(F(M^*, O_1)))$. Since $(M^*, O_1) \subseteq sn\text{-}cl_{g\omega}(M^*, O_1)$, $F(M^*, O_1) \subseteq F(sn\text{-}cl_{g\omega}(M^*, O_1))$ and thus it follows that sn-cl$(F(M^*, O_1)) \subseteq$ sn-cl$(F(sn - Cl_{g\omega}(M^*, O_1)))$ $= F(sn - cl_{g\omega}(M^*, O_1))$. Therefore sn-cl$(F(M^*, O_1)) \subseteq F(sn\text{-}cl_{g\omega}(M^*, O_1))$. Hence $F(sn - cl_{g\omega}(M^*, O_1))$= sn-cl$(F(M^*, O_1))$ if F is sn-$g\omega$-homeomorphism. □

Theorem 5.7. *For the sn-gω-continuous function $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$, the following are equivalent.*

1. *F is sn-gω-open*
2. *F is sn-gω-homeomorphism*
3. *F is sn-gω-closed.*

Proof. (i) ⇒ (ii), By hypothesis, $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is bijective, sn-$g\omega$-continuous, and sn-$g\omega$-open. Thus F is sn-$g\omega$-homeomorphism.
(ii) ⇒ (iii), Let (M^*, O_1) be sn-O(X_1, O_1), then $(M^*, O_1)^c$ is sn-O(X_1, O_1). By the hypothesis, F is sn-$g\omega$-homeomorphism and thus sn-$g\omega$-open. By assumption, $F((M^*, O_1)^c)$ is sn-$g\omega$-O(X_2, O_2). Thus $F((M^*, O_1)^c) = (F(M^*, O_1))^c$ is sn-$g\omega$-O(X_2, O_2). Therefore $F(M^*, O_1)$ is sn-$g\omega$-C(X_2, O_2) for every sn-closed set (M^*, O_1) in $(\tau_{R'}(X_1), U_1, O_1)$. Hence F is sn-$g\omega$-closed function.
(iii) ⇒ (i), Let (V^*, O_1) be sn-O(X_1, O_1), then $(V^*, O_1)^c$ is sn-C(X_1, O_1). By the hypothesis, $F((V^*, O_1)^c)$ is sn-$g\omega$-C(X_2, O_2) is sn-$g\omega$-closed in $(\tau_{R''}(X_2), U_2, O_2)$. Here $F((V^*, O_1)^c) = (F(V^*, O_1))^c$ is sn-$g\omega$-C(X_2, O_2). That is $F(V^*, O_1)$ is sn-$g\omega$-O(X_2, O_2) for every sn-open set (V^*, O_1) in $(\tau_{R'}(X_1), U_1, O_1)$. Therefore F is sn-$g\omega$-open function. □

6. Soft Nano-$g\omega^*$-homeomorphisms

Here a new class of mapping known as sn-$(g\omega)^*$-homeomorphisms are introduced. These are the subclasses of sn-$g\omega$-homeomorphisms and which include the class of sn-homeomorphisms.

Definition 6.1. *A bijective map $F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is said to be sn-$(g\omega)^*$-homeomorphism if both F and F^{-1} are sn-gω-irresolute. The spaces $(\tau_{R'}(X_1), U_1, O_1)$ and $(\tau_{R''}(X_2), U_2, O_2)$ are sn-$(g\omega)^*$-homeomorphism if there exists a sn-$(g\omega)^*$-homeomorphism from $(\tau_{R'}(X_1), U_1, O_1)$ onto $(\tau_{R''}(X_2), U_2, O_2)$.*

The family of all sn-$g\omega$-homeomorphism of $(\tau_{R'}(X_1), U_1, O_1)$ onto itself is denoted by sn-$g\omega$-H$(\tau_{R'}(X_1), U_1, O_1)$ and family of all sn-$(g\omega)^*$-homeomorphism of $(\tau_{R''}(X_2), U_2, O_2)$ onto itself is denoted by sn-$(g\omega)^*$-H$(\tau_{R''}(X_2), U_2, O_2)$.
To denote the algebraic structure of the set of all sn-$(g\omega)^*$-homeomorphisms, we have the following.
sn-$(g\omega)^*$-H$(\tau_{R'}(X_1), U_1, O_1) = \{F /\ F : (\tau_{R'}(X_1), U_1, O_1) \rightarrow (\tau_{R''}(X_2), U_2, O_2)$ is $sn - (g\omega)^* - homeomorphism\}$.

Theorem 6.2. *For the space$(\tau_{R'}(X_1), U_1, O_1)$, sn-$(g\omega)^*$-H$(\tau_{R''}(X_2), U_2, O_2) \subseteq$ sn-gω-H$(\tau_{R'}(X_1), U_1, O_1)$.*

Proof. The proof follows by the fact that every sn-$g\omega$-irresolute function is sn-$g\omega$-continuous and every sn-$(g\omega)^*$-open map is sn-$g\omega$-open. □

Theorem 6.3. *The set sn-$(g\omega)^*$-H$(\tau_{R'}(X_1), U_1, O_1)$ is a group under composition of functions.*

Proof. Let $* : sn - (g\omega)^*$-H$(\tau_{R'}(X_1), U_1, O_1) \rightarrow$ sn-$(g\omega)^*$-H$(\tau_{R'}(X_1), U_1, O_1)$ be a binary operation defined as $F_1 * F_2 = F_2 \circ F_1$. For all $F_1, F_1 \in sn - (g\omega)^*$-H$(\tau_{R'}(X_1), U_1, O_1)$ and $\circ$

is the usual operation under composition of functions. As $F_2 \circ F_1 \in sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$. The associative law is satisfied by the composition of functions. The identity function $I_F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is the identity element and belongs to $sn-(g\omega)^*$-H$(\tau_{R'}(X_1),U_1,O_1)$. As $F \in sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$ then $F^{-1} \in sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$ such that $F \circ F^{-1} = F^{-1} \circ F = I_F$ and thus the inverse exists for each element of $sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$. Therefore $(sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1),\circ)$ is group under the composition of functions. □

Theorem 6.4. *Let $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ be a $(sn-(g\omega)^*$-homeomorphism, then F induces an isomorphism from the group $sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$ onto the group $sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$.*

Proof. For the function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$, a mapping is defined as $\psi_F^* : sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1) \to sn-(g\omega)^*-H(\tau_{R''}(X_2),U_2,O_2)$ by $\psi_F^*(H) = F \circ H \circ F^{-1} = \psi_F^*(H)$ for every $H \in sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$. By the hypothesis, ψ_F^* is a bijection. Therefore for all $H_1,H_2 \in sn-(g\omega)^*-H(\tau_{R'}(X_1),U_1,O_1)$, $F \circ (H_1 \circ H_2) \circ F^{-1} = (F \circ H_1 \circ F^{-1}) \circ (F \circ H_2 \circ F^{-1}) = \psi_F^*(H_1) \circ \psi_F^*(H_2)$. Hence ψ_F^* is sn-homemorphism and thus it is sn-isomorphism induced by F. □

It is clear from the following example, that the converse of the Theorem 6.4 need not be true in general. This shows that there exists a function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ which is not sn-$(g\omega)^*$-homeomorphism but induced an isomorphism ψ_F^* : *sn-$(g\omega)^*$-H$(\tau_{R'}(X_1),U_1,O_1) \to$ sn-$(g\omega)^*$-H$(\tau_{R''}(X_2),U_2)$*.

Example 6.5. *Let $U_1 = \{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$, $X_1 = \{\varepsilon_1\} \subseteq U_1, O_1 = \{k_1,k_2,k_3\}$. $U_2 = \{\varepsilon_1^{'},\varepsilon_2^{'},\varepsilon_3^{'}\}$, $O_2 = \{k_1^{'},k_2^{'},k_3^{'}\}$,$X_2 = \{\varepsilon_1^{'},\varepsilon_3^{'}\} \subseteq U_2$, $\tau_{R'}(X_1)$= $\{U_1,\emptyset,(k_1,\{\varepsilon_1\}),(k_2,\{\varepsilon_1\}),(k_3,\{\varepsilon_1\})$ and $U_2/R^{''} = \{\{\varepsilon_1^{'},\varepsilon_3^{'}\}$ then $\tau_{R''}(X_2) = \{U_2,\emptyset,(k_1^{'},\{\varepsilon_1^{'},\varepsilon_3^{'}\}),(k_2^{'},\{\varepsilon_1^{'},\varepsilon_3^{'}\}),(k_3^{'},\{\varepsilon_1^{'},\varepsilon_3^{'}\})\}\}$ Define the function $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ as $F(\varepsilon_1) = \varepsilon_2^{'}$, $F(\varepsilon_2) = \varepsilon_3^{'}$,$F(\varepsilon_3) = \varepsilon_1^{'}$. Here sn-$g\omega$ closed sets in $U_1 = \{\varepsilon_2\},\{\varepsilon_3$ and sn-$g\omega$ closed sets in $U_2 = \{\{\varepsilon_2^{'}\},\{\varepsilon_1^{'},\varepsilon_2^{'}\},\{\varepsilon_2^{'},\varepsilon_3^{'}\},\{\varepsilon_3^{'},\varepsilon_4^{'}\}\}$. Here F and F^{-1} are not sn-$g\omega$ irresolute and so F is not sn-$(g\omega)^*$-homeomorphism. Define the functions $H_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R'}(X_1),U_1,O_1)$ as $H_1(\varepsilon_1) = \varepsilon_1^{'},H_1(\varepsilon_2) = \varepsilon_3^{'},H_1(\varepsilon_3) = \varepsilon_2^{'}$ and $H_2 : (\tau_{R''}(X_2),U_2,O_2) \to (\tau_{R''}(X_2),U_2,O_2)$ defined as $H_2(\varepsilon_1) = \varepsilon_3^{'},H_2(\varepsilon_2) = \varepsilon_2^{'},H_2(\varepsilon_3) = \varepsilon_1^{'}$. Here H_1 and H_2 are sn-$(g\omega)^*$-homeomorphism and it follows that sn-$(g\omega)^*$-H$(\tau_{R'}(X_1),U_1,O_1)$= $\{H_1,I_{U_1}\}$ and sn-$(g\omega)^*$-H$(\tau_{R''}(X_2),U_2,O_2)$ = $\{H_2,I_{U_2}\}$ where $I_{U_1} : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R'}(X_1),U_1,O_1)$ and $I_{U_2} : (\tau_{R''}(X_2),U_2,O_2) \to (\tau_{R''}(X_2),U_2,O_2)$ are identity functions. Now $\psi_F^*(H_1)$= $F \circ H_1 \circ F^{-1}$ = H_2 with $\psi_F^*(I_{U_1})$ = I_{U_2} and hence the induced homeomorphism ψ_F^* : sn-$(g\omega)^*$-H$(\tau_{R'}(X_1),U_1,O_1) \to$ sn-$(g\omega)^*$-H$(\tau_{R''}(X_2),U_2,O_2)$ is an isomorphism.*

Theorem 6.6. *If the function: $F_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ and $F_2 : (\tau_{R''}(X_2),U_2,O_2) \to (\tau_{R'''}(X_3),U_3,O_3)$ are $sn-(g\omega)^*$-homeomorphisms, then the composition $F_2 \circ F_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is also $sn-(g\omega)^*$-homeomorphism.*

Proof. Let $(P^*,O_1) \in$ sn-$g\omega$-O(X_3,O_3). Here $(F_2 \circ F_1)^{-1}(P^*,O_1) = F_1^{-1}(F_2^{-1}(P^*,O_1)) = F_1^{-1}(M^*,O_1)$ where $(M^*,O_1) = F_2^{-1}(P^*,O_1)$ as F_2 is sn-$(g\omega)^*$-homeomorphism and thus (M^*,O_1) is sn-$g\omega$-O(X_2,O_2), by the hypothesis. Also, $F_1^{-1}(M^*,O_1)$ is sn-$g\omega$-O(X_1,O_1). Therefore $(F_2 \circ F_1)^{-1}(P^*,O_1)$= $F_1^{-1}(M^*,O_1)$ is sn-$g\omega$-O(X_1,O_1) for every sn-$g\omega$-open set (P^*,O_1) in $(\tau_{R'''}(X_3),U_3,O_3)$. Thus the composition $F_2 \circ F_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is sn-$(g\omega)^*$-irresolute. □

Theorem 6.7. *A function $F_1 : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is sn-$(g\omega)^*$-homeomorphism, then sn-$g\omega$-cl$(F_1^{-1}(M^*,O_1)) = F_1^{-1}(sn-g\omega-cl(M^*,O_1))$ for all $(M^*,O_1) \subseteq (\tau_{R''}(X_2),U_2,O_2)$.*

Proof. The function F is sn-$(g\omega)^*$-homeomorphism, it implies that F is sn-$g\omega$-irresolute. As sn-$g\omega$-cl(M^*,O_1) is sn-$g\omega$-C(X_2,O_2),$F_1^{-1}(sn-g\omega-cl(M^*,O_1))$ is an sn-$g\omega$-C(X_1,O_1). Now $F_1^{-1}(M^*,O_1) \in (sn-g\omega-cl(M^*,O_1))$ and so sn-$g\omega$-cl $(F_1^{-1}(M^*,O_1)) \in F_1^{-1}(sn-g\omega-cl(M^*,O_1))$. Aslo, as F is sn-$(g\omega)^*$-homeomorphism, F_1^{-1} is sn-$g\omega$-irresolute and as sn-$g\omega$-cl$(F_1^{-1}(M^*,O_1))$ is sn-$g\omega$-cl(X_1,O_1), $(F_1^{-1})^{-1}(sn-g\omega-cl(F_1^{-1}(M^*,O_1))) = F(sn-g\omega-cl(F_1^{-1}(M^*,O_1)))$ is sn-$g\omega$-cl(X_2,O_2). Now $(M^*,O_1) \subseteq (F_1^{-1})^{-1}(F_1^{-1}(M^*,O_1) \subseteq (F_1^{-1})^{-1}(sn-g\omega-cl(F_1^{-1}(M^*,O_1))) = F(sn-g\omega-cl(F_1^{-1}(M^*,O_1)))$ and so, sn-$g\omega$-cl$(M^*,O_1) \subseteq F(sn-g\omega-cl(F_1^{-1}(M^*,O_1))$. It is clear that $F_1^{-1}(sn-g\omega-cl(M^*,O_1)) \subseteq F_1^{-1}(F(sn-g\omega-cl(F_1^{-1}(M^*,O_1)))) \subseteq sn-g\omega-cl(F_1^{-1}(M^*,O_1))$ and therefore the equality holds. □

Corollary 6.8. *Let $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ be an sn-$(g\omega)^*$-homeomorphism, then sn-$g\omega$-cl$(F(M^*,O_1))$ = $F(sn-g\omega-cl(M^*,O_1))$ for all $(M^*,O_1) \subseteq (\tau_{R'}(X_1),U_1,O_1)$.*

Proof. As $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is sn-$(g\omega)^*$-homeomorphism, it follows that $F^{-1} : (\tau_{R''}(X_2),U_2,O_2) \to (\tau_{R'}(X_1),U_1,O_1)$ is also sn-$(g\omega)^*$-homeomorphism. Therefore, sn-$g\omega$-cl$(F_1^{-1})^{-1}(M^*,O_1) = (F_1^{-1})^{-1}(sn-g\omega-cl(M^*,O_1))$ for all $(M^*,O_1) \subseteq (\tau_{R'}(X_1),U_1,O_1)$. That is sn-$g\omega$-cl$(F(M^*,O_1)) = F(sn-g\omega-cl(M^*,O_1))$. □

Corollary 6.9. *If $F : (\tau_{R'}(X_1),U_1,O_1) \to (\tau_{R''}(X_2),U_2,O_2)$ is sn-$(g\omega)^*$-homeomorphism, then $F(sn-g\omega-int(M^*,O_1)) = sn-g\omega-int(F(M^*,O_1))$ for all $(M^*,O_1) \subseteq (\tau_{R'}(X_1),U_1,O_1)$.*

Proof. For the set $(M^*,O_1) \subseteq (\tau_{R'}(X_1),U_1,O_1)$, it follows that $sn-g\omega-int(M^*,O_1) = sn-g\omega-cl((M^*,O_1)^c)^c$. Therefore $F(sn-g\omega-int(M^*,O_1)) = F(sn-g\omega-cl((M^*,O_1)^c)^c)$ $= (F(sn\text{-}g\omega\text{-}cl((M^*,O_1)^c))^c = (sn\text{-}g\omega\text{-}cl(F(M^*,O_1))^c)^c$ $= sn\text{-}g\omega\text{-}int(F(M^*,O_1))$. □

Corollary 6.10. *If the function* $F : (\tau_{R'}(X_1), U_1, O_1) \to (\tau_{R''}(X_2), U_2, O_2)$ *is sn-*$(g\omega)^*$*-homeomorphism, then* $F^{-1}(sn - g\omega - int(M^*, O_1) = sn - g\omega - int(F^{-1}(M^*, O_1))$ *for all* $(M^*, O_1) \subseteq (\tau_{R''}(X_2), U_2, O_2)$.

Proof. Proof follows from the corollary 6.9. □

7. Conclusion

Extensive research work has been carried out by researchers in the field of soft nano topology. The present paper depicts the importance of soft nano gω-continuous functions, soft nano gω-irresolute, soft nano homeomorphism, soft nano gω-homeomorphism and soft nano $(g\omega)^*$ homeomorphism. This work is helpful in the development of different forms of soft nano generalized homeomorphism and their interrelationship.

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