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(T,S)-intuitionistic fuzzy N-subgroup of an N-group

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Abstract

This paper is an attempt to provide the notion of an (T,S)- intuitionistic fuzzy N-subgroup and (T,S)-intuitionistic fuzzy ideal of an N-group in the light of a triangular norm and its corresponding co-norm and also an effort is made to introduce some of their properties.

Keywords

Near-ring, N-group, (T,S)-intuitionistic fuzzy N-subgroup, (T,S)-intuitionistic fuzzy ideal.

AMS Subject Classification

16Y30, 03E72.

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Contents

1. Introduction

Atanassov in [1,2,3,4] provides the notion of an intuitionistic fuzzy set generalizing the concept of fuzzy set introduced by Zadeh in [5]. Thereafter many researcher paying their attention towards the generalization of various algebraic structures. Rosenfield [6] was first to introduce the idea of fuzzy subgroups of a group which was later generalized to intuitionistic fuzzy subgroups by Biswas in [7]. Kim and Jun in [8] gave the concept of N-subgroup of a near ring and the notion of which was later extended to a near ring group by Saikia and Barthakur in [9]. Cho[10], Devi[11], Sharma[12] are some researchers who gave their contribution towards such generalization of N-subgroup and ideal of N-group into intuitionistic fuzzy set.Triangular norms or t-norms play an important role in the study of different algebraic structures. Researcher like Klir and Yuan [13] gave important contribution in this ground. Kim and Lee [14] introduced the concept of intuitionistic fuzzy ideals of rings and Murugadas and Vetrivel in [15] introduced the notion of (T,S)-intuition fuzzy ideals of a near ring.

In this paper, (T,S)-intuitionistic fuzzy N-subgroup and (T,S)-intuitionistic fuzzy ideal of an N-group are defined and their various properties are discussed.

2. Preliminaries

Throughout this section,we recall some notions that are useful for this paper.

Definition 2.1. *A near-ring N is a non empty set together with two binary operation "$+$"and "$.$" if*
$(i)(N,+)$ is a group which is not necessarily abelian)
$(ii)(N,.)$ is a semi group
(iii) for all $a,b,c \in N, a(b+c) = ab+ac$.

Definition 2.2. *A group $(E,+)$ is said to be a near ring group or N-group of a near ring N if there exist a mapping $N \times E \to E, (n,x) \to nx$ such that*
$(i)(n+m)x = nx+mx$
$(ii)(nm)x = n(mx)$
$(iii)1x = x$ for all $n,m \in N, x \in E$
It is clear that N can itself be considered as an N-group which is denoted by N^N.

Definition 2.3. *An N-homomorphism is the mapping $f : E \to F$,where E and F are N-groups, such that*
$(i)f(a+b) = f(a)+f(b)$
$(ii)f(na) = nf(a)$ for all $n \in N$ and $a,b \in E$.

Definition 2.4. *Any subset A of an N-group E is said to be an N-subgroup of E if A is a subgroup of $(E,+)$ and $NA \subseteq A$.*

Definition 2.5. *An ideal A of E is a normal subgroup of E such that $n(a+e)-ne \in A$.*

Definition 2.6. *For a non void set X, a function μ from X to $[0,1]$ is called a fuzzy subset of X and its complement is denoted by $\bar{\mu}$ and is such that $\bar{\mu}(x) = 1-\mu(x)$.*

Definition 2.7. *An intuitionistic fuzzy set (in short IFS) on a non void set X is an object of type $A =< \mu_A, \nu_A > = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$ where μ_A and ν_A are fuzzy subsets of X and the degree of membership and non membership of element $x \in X$ are denoted by $\mu_A(x)$ and $\nu_A(x)$ respectively such that $0 \leq \mu_A(x) + \nu_A \leq 1$.*

Definition 2.8. *For IFSs $A =< \mu_A, \nu_A$ and $B =< \mu_B, \nu_B$ of X, $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$. Obviously every fuzzy set μ_A can also be considered as an IFS as $A = (a, \mu_A(a), \bar{\mu}_A(a)) | a \in X$.Consequently two IFS $\Box$ A and $\Diamond$ A are introduced defined as $\Box A = \{(a, \mu_A(a), \bar{\mu}_A(a)) | a \in X\}$ and $\Diamond A = \{(a, \bar{\nu_A}(a), \nu_A(a)) | a \in X\}$.*

Definition 2.9. *A triangular norm or t-norm is a mapping $T : [0,1] \times [0,1] \to [0,1]$ that satisfies for every $p,q,r \in [0,1]$ the following:*
$(i) T(p,1) = p$(boundary condition);
$(ii) q \leq r \Rightarrow T(p,q) \leq T(p,r)$ (monotonicity);
$(iii) T(p,q) = T(q,p)$ (commutativity);
$(iv) T(p,T(q,r)) = T(T(p,q),r)$ (associativity).

Definition 2.10. *A t-co-norm S means a mapping $S : [0,1] \times [0,1] \to [0,1]$, that satisfies following axioms for every $p,q,r \in [0,1]$;*
$(i) S(p,0) = p$(boundary condition);
$(ii) q \leq r \Rightarrow S(p,q) \leq S(p,r)$ (monotonicity);
$(iii) S(p,q) = S(q,p)$ (commutativity);
$(iv) S(p,S(q,r)) = S(S(p,q),r)$ (associativity).

A t-norm T and a t-co-norm S are dual with respect to the standard fuzzy complement if and only if
$(i) 1-T(p,q) = S(1-p,1-q)$ or $1-S(p,q) = T(1-p,1-q)$, where $p,q \in [0,1]$.

Definition 2.11. *Let T be t-norm and S be the t-co-norm. Then for all$1 \leq i \leq n, p_i \in [0,1], n \geq 3$ we have $T_n(p_1,p_2,..,p_n) = T(p_i, T_{n-1}(p_1,p-2,...p_{i-1},p_{i+1},..p_n))$ and $T_2 = T$ and $S_n(p_1,p_2,..,p_n) = S(p_i, S_{n-1}(p_1,p-2,...p_{i-1},p_{i+1},..p_n))$ and $S_2 = T$. Also T_∞ and S_∞ is defined as*

$$T_\infty(p_1,p_2,...) = \lim_{n\to\infty} T_n(p_1,p_2,...,p_n);$$

$$S_\infty(p_1,p_2,...) = \lim_{n\to\infty} S_n(p_1,p_2,...,p_n).$$

Definition 2.12. *Let $\{\mu_1,\mu_2,...\mu_n\}$ be a set of fuzzy subsets of X. Then for all $p \in X$ $(\mu_1 \cap \mu_2 \cap ... \cap \mu_n)(p) = T_n(\mu_1(p),\mu_2(p),..,\mu_n(p))$ defines the t-norm based intersection with respect the t and $(\mu_1 \cup \mu_2 \cup ... \cup \mu_n)(p) = S_n(\mu_1(p),\mu_2(p),..,\mu_n(p))$ defines the union with respect to t-co-norm S.*

Definition 2.13. *Let $\{A_i = (\mu_i,\nu_i) | 1 \leq i \leq n\}$ be any collection of IFSs in a set X. Then their intersection and union are defined as*
$A_1 \cap A_2 \cap ... \cap A_n = \{(p, T_n(\mu_i(p)), S_n(\nu_i(p)) | p \in X\}$ and $A_1 \cup A_2 \cup ... \cup A_n = \{(p, S_n(\mu_i(p)), T_n(\nu_i(p)) | p \in X\}$ where S_T is the dual t-co-norm of T.

Definition 2.14. *Let T be a t-norm (or t-co-norm). A fuzzy set μ in X is said to satisfy idempotent property with respect to T if $Im(\mu) \subseteq \{p \in [0,1] : T(p,p) = p\}$.*

Definition 2.15. *Let μ be a fuzzy subset of a N-group E. Then μ is said to be a fuzzy N-subgroup of E if for all $n \in N$ and $p,q \in E$*
$(i) \mu_(p-q) \geq min\{\mu(p), \mu_(q)\}$
$(ii) \mu(np) \geq \mu(p)$.

Definition 2.16. *Let μ be a fuzzy subset of an N-group E. Then μ is called an intuitonistic fuzzy ideal of E,if for all $n \in N$ and $p,q \in E$*
$(i) \mu_(p-q) \geq min\{\mu(p), \mu_(q)\}$
$(ii) \mu(q+p-q) \geq \mu(p)$
$(iii) \mu(np) \geq \mu(p)$
$(iv) \mu(n(q+p)-nq) \geq \mu(p)$
$(v) \nu_(p-q) \leq max\{\nu(p), \nu_(q)\}$
$(vi) \nu(q+p-q) \leq \nu(p)$
$(vii) \nu(np) \leq \nu(p)$
$(iv) \nu(n(q+p)-nq) \leq \nu(p)$.

3. (T,S)-intuitionistic fuzzy N-subgroup of N-group

This part of the paper constitutes of the definition of (T,S)-intuitionistic fuzzy N-subgroup of an N-group along with some of its properties.

Definition 3.1. *Let E be an N-group of a near-ring N. Let $A =< \mu_A, \nu_A >$ be an IFS of E. Let T be a t-norm and S_T be its dual t-co-norm. Then A is said to be a (T,S)-intuitionistic fuzzy N-subgroup of E if for all $x,y \in E$, $n \in N$*
TIFNS1: $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$
TIFNS2: $\mu_A(nx) \geq \mu_A(x)$
TIFNS3: $\nu_A(x-y) \leq S_T(\nu_A(x), \nu_A(y))$
TIFNS4: $\nu_A(nx) \leq \nu_A(x)$.

Example 3.2. *Consider the Dihedral group $Q = \{0, p, 2p, 3p, q, p+q, 2p+q, 3p+q\}$ over the zero near ring N. Then Q is N-group. Let the IFS $A =< \mu_A, \nu_A >$ is such that*

$$\mu_A(x) = \begin{cases} 1, & x = 0 \\ 0.6, & x \in \{q, 2p+q\} \\ 0.4 & x \in \{p, p+q, 3p+q\} \\ 0.3 & x = 3p; \end{cases} \quad \text{and}$$

$$\nu_A(x) = \begin{cases} 0, & x = 0 \\ 0.2, & x \in \{q, 2p+q\} \\ 0.5 & x \in \{p, p+q, 3p+q\} \\ 0.6 & x = 3p. \end{cases}$$

Then $A = <\mu_A, \nu_A>$ *is an* (T,S)*-intuittionistic fuzzy* N*-subgroup of* E *with respect to the following pair of t-norm and t-co-norms*
$(i)\, \min(x,y), \max(x,y)$
$(ii)(xy, x+y-xy)$
$(iii)\, \max(0, x+y-1), \min(1, x+y)$.

Theorem 3.3. *An IFS* $A = <\mu_A, \nu_A>$ *of* E *is a* (T,S) *-intuitionistic fuzzy* N*-subgroup of* E *if and only if both* $\Box A$ *and* $\Diamond A$ *are* (T,S)*-intutionistic fuzzy* N*-subgroup of* E.

Proof. We have $\Box A = <\mu_A, \bar{\mu}_A>$ and $\Diamond A = <\bar{\nu}_A, \nu_A>$. If $A = <\mu_A, \nu_A>$ be a (T,S)-intuitionistic fuzzy N-subgroup of E then for desired result we only need to show the desired conditions for μ_A^c and ν_A^c. Then for $x,y \in E$,
$\bar{\mu_A}(x-y) = 1-\mu_A(x-y) \leq 1-T(\mu_A(x), \mu_A(y))$
$= S(1-\mu_A(x), 1-\mu_A(y)) = S(\bar{\mu}_A(x), \bar{\mu}_A(y))$ and for
$n \in N, x \in E$, $\bar{\mu}_A(nx) = 1-\mu_A(nx) \leq 1-\mu_A(x) = \bar{\mu}_A(x)$.
Similarly, $\bar{\nu}_A(x-y) = 1-\nu_A(x-y) \geq 1-S(\nu_A(x), \nu_A(y))$
$= T(1-\nu_A(x), 1-\nu_A(y)) = T(\bar{\mu}_A(x), \bar{\nu}_A(y))$ and $\bar{\nu}_A(nx) = 1-\nu_A(nx) \geq 1-\nu_A(x) = \bar{\nu}_A(x)$.Therefore $\Box A$ and $\Diamond A$ are (T,S)-intutitonistic fuzzy subgroup of E.The converse part is obvious. □

Theorem 3.4. *For a non empty subset* A *of an* N*-group* E*,the IFS* $A^{(\alpha,\beta)} = <\mu_A, \nu_A>$*defined as*

$$\mu_A(x) = \begin{cases} 1 x \in A \\ \alpha\, otherwise \end{cases} \quad and\ \nu_A(x) = \begin{cases} 0 x \in A \\ \beta\, otherwise \end{cases}$$

where $\alpha, \beta \in [0,1], \alpha+\beta \leq 1$ *is a* (T,S)*-intuitionistic fuzzy subgroup of* E *if and only if* A *is a* N*-subgroup of* E*.*

Proof. Let A be a N-subgroup of E. Then for $x,y \in E$ if $x,y \in E$ then $x-y \in A$ and so $\mu_A(x-y) = 1 \geq 1 = T(1,1) = T(\mu_A(x), \mu_A(y))$ and $\nu_A(x-y) = 0 \leq 0 = S(0,0)$
$= S(\nu_A(x), \nu_A(y))$.For $x \in A, y \notin A$,we have
$\mu_A(x-y) = \alpha \geq \alpha = T(1,\alpha) = T(\mu_A(x), \mu_A(y))$ and
$\nu_A(x-y) = \beta \leq \beta = S(0,\beta) = S(\nu_A(x), \nu_A(y))$.
For $x \notin A, y \in A$,
$\mu_A(x-y) = \alpha \geq \alpha = T(\alpha,1) = T(\mu_A(x), \mu_A(y))$
and
$\nu_A(x-y) = \beta \leq \beta = S(\beta,0) = S(\nu_A(x), \nu_A(y))$.Again,for $x \in E, n \in N$,if $x \in A$ then $nx \in A$ so that $\mu_A(nx) = 1 \geq 1 = \mu_A(x)$ and $\nu_A(nx) = 0 \leq 0 = \nu_A(x)$. Also if $x \notin A$ then $\mu_A(nx) = 0 \geq 0 = \mu_A(x)$;$\nu_A(nx) = 1 \leq 1 = \nu_A(x)$.Therefore $A^{(\alpha,\beta)}$ is a (T,S)-intuitionistic fuzzy N-subgroup of E.

Conversely, let $A^{(\alpha,\beta)}$ is an (T,S)-intuitionistic fuzzy N-subgroup of E. Since for $x,y \in A, \mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) = T(1,1) = 1 \Rightarrow \mu_A(x-y) = 1 \Rightarrow x-y \in A$. Also for $x \in A, n \in N, \mu_A(nx) \geq \mu_A(x) = 1 \Rightarrow \mu_A(nx) \Rightarrow nx \in A$. Hence A is a N-subgroup of E. □

Theorem 3.5. *If* $A = <\mu_A, \nu_A>$ *is a* (T,S)*-intuitionistic fuzzy* N*-subgroup of* E *then* $A^* = \{x \in E : \mu_A(x) > 0, \nu_A(x) < 1\}$ *is also an* N*-subgroup of* E.

Proof. Let $x,y \in A^*$.Then $\mu_A(x) > 0, \& \nu_A(x) < 1; \mu_A(y) > 0$, $\nu_A(y) < 1$. Therefore $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y)) > T(0,0) = 0$ and $\nu_A(x-y) \leq S(\nu_A(x), \nu_A(y)) = S(1,1) = 1$. Thus $x-y \in A^*$. Again for $x \in A^*$ and $n \in N$, $\mu_A(nx) \geq \mu_A(x) > 0$ and $\nu_A(nx) \leq \nu_A(x) < 1$. Thus $nx \in A^*$. Hence $A^*$is N-subgroup of E. □

Theorem 3.6. *Let* $\{A_i = <\mu_i, \nu_i>: 1 \leq n \leq n\}$ *be a finite collection of* (T,S)*-intuitionistic fuzzy* N*-subgroup of* E*. Then* $A_1 \cap A_2 \cap \ldots \cap A_n$ *is also a* (T,S)*-intuitionistic fuzzy* N*-subgroup of* E.

Proof. Let $A = A_1 \cap A_2 \cap \ldots \cap A_n$.Through induction on n result can be proved. For $n = 1$,$A = A_1$ so the result is true. Let the result be true now for $n-1$intersections. Now for $x,y \in E$ and $n \in N$,

$$\begin{aligned}
&(\mu_1 \cap \mu_2 \cap .. \cap \mu_n)(x-y)\\
&= T_n(\mu_1(x-y), \mu_2(x-y), ..., \mu_n(x-y)),\\
&\quad T(\mu_1(x-y), T_{n-1}(\mu_2(x-y), ...\mu_n(x-y)))\\
&\geq T(T(\mu_1(x), \mu_1(y)), T(T_{n-1}(\mu_2(x), ..\mu_n(x)), T_{n-1}(\mu_2(y), ...\mu_n(y))))\\
&= T(\mu_1(y), \mu_1(x)), T(T_{n-1}(\mu_2(x), ..\mu_n(x)), T_{n-1}(\mu_2(y), ...\mu_n(y))))\\
&= T(\mu_1(y), T(T(\mu_1(x), T_{n-1}(\mu_2(x), ..\mu_n(x))), T_{n-1}(\mu_2(y), ...\mu_n(y))))\\
&= T(\mu_1(y), T(T_n(\mu_1(x), \mu_2(x), ..\mu_n(x)), T_{n-1}(\mu_2(y), ...\mu_n(y))))\\
&= T(T(\mu_1(y), T_{n-1}(\mu_2(y), ...\mu_n(y))), T_n(\mu_1(x), ...\mu_n(x)))\\
&= T(T_n(\mu_1(x), ...\mu_n(x)), T_n(\mu_1(y), ...\mu_n(y)))\\
&= T((\mu_1 \cap .... \cap \mu_n)(x), (\mu_1 \cap .... \cap \mu_n)(y))
\end{aligned}$$

Also

$$\begin{aligned}
&(\nu_1 \cup \nu_2 \cup .. \cup \nu_n)(x-y)\\
&= S_n(\nu_1(x-y), \nu_2(x-y), ..., \nu_n(x-y)),\\
&\quad S(\nu_1(x-y), S_{n-1}(\nu_2(x-y), ...\nu_n(x-y)))\\
&\leq S(S(\nu_1(x), \nu_1(y)), S(S_{n-1}(\nu_2(x), ..\nu_n(x)), S_{n-1}(\nu_2(y), ...\nu_n(y))))\\
&= S(\nu_1(y), \nu_1(x)), S(S_{n-1}(\nu_2(x), ..\nu_n(x)), S_{n-1}(\nu_2(y), ...\nu_n(y))))\\
&= S(\nu_1(y), S(S(\nu_1(x), S_{n-1}(\nu_2(x), ..\nu_n(x))), S_{n-1}(\nu_2(y), ...\nu_n(y))))\\
&= S(\mu_1(y), S(S_n(\nu_1(x), \nu_2(x), ..\nu_n(x)), S_{n-1}(\nu_2(y), ...\nu_n(y))))\\
&= T(T(\mu_1(y), T_{n-1}(\mu_2(y), ...\mu_n(y))), T_n(\mu_1(x), ...\mu_n(x)))\\
&= S(S_n(\nu_1(x), ...\nu_n(x)), S_n(\nu_1(y), ...\nu_n(y)))\\
&= S((\nu_1 \cap .... \cap \nu_n)(x), (\nu_1 \cap .... \cap \nu_n)(y))
\end{aligned}$$

and,

$$\begin{aligned}
&(\mu_1 \cap \mu_2 \cap .. \cap \mu_n)(nx)\\
&= T_n(\mu_1(nx), \mu_2(nx), ..., \mu_n(nx))\\
&= T(\mu_1(x), T_{n-1}(\mu_2(x), ..., \mu_n(x)))\\
&\geq T(\mu_1(x), T_{n-1}(\mu_2(x), ..., \mu_n(x)))\\
&= T_n(\mu_1(x), \mu_2(x), ..., \mu_n(x)) = (\mu_1 \cap \mu_2 \cap ... \cap \mu_n)(x)
\end{aligned}$$

A*lso*,

$$\begin{aligned}
&(\nu_1 \cup \nu_2 \cup .. \cup \nu_n)(nx)\\
&= S_n(\nu_1(nx), \nu_2(nx), ..., \nu_n(nx))\\
&= S(\nu_1(x), S_{n-1}(\nu_2(x), ..., \nu_n(x)))\\
&\leq S(\nu_1(x), S_{n-1}(\nu_2(x), ..., \nu_n(x)))\\
&= S_n(\nu_1(x), \nu_2(x), ..., \nu_n(x)) = (\nu_1 \cup \nu_2 \cup ... \cup \nu_n)(x).
\end{aligned}$$

Hence $A_1 \cap A_2 \cap \ldots \cap A_n$ is (T,S)- intuitionistic fuzzy N-subgroup of E. □

Theorem 3.7. *Let* $\{A_i = <\mu_i, \nu_i>: i = 1,2,3,...\}$ *be an infinite collection of* (T,S)*-intuitionistic fuzzy* N*-subgroup of an*

N-group E, where T is a continuous t-norm. Then their intersection $\cap_{i=1}^{\infty} A_i$ is also a (T,S)-intuitionistic fuzzy N-subgroup of E.

Proof. Let $x,y \in E$. Then
$\cap_{i=1}^{\infty}\mu_i(x-y) = \lim T_n(\mu_1(x-y),\mu_2(x-y),...,\mu_n(x-y))$
$\geq \lim T(T_n(\mu_1(x),...,\mu_n(x)),T_n(\mu_1(y),...,\mu_n(y)))$
(by Theorem 3.6)
$= T(\lim T_n(\mu_1(x),...,\mu_n(x)),\lim T_n(\mu_1(y),...,\mu_n(y)))$
(since T is continuous)
$= T(\cap_{i=1}^{\infty}\mu_i(x),\cap_{i=1}^{\infty}\mu_i(y))$ and
$\cup_{i=1}^{\infty}\nu_i(x-y) = \lim S_n(\nu_1(x-y),\nu_2(x-y),...,\nu_n(x-y))$
$\leq \lim S(S_n(\nu_1(x),...,\nu_n(x)),S_n(\nu_1(y),...,\nu_n(y)))$
(by Theorem 3.6)
$= S(\lim S_n(\nu_1(x),...,\nu_n(x)),\lim S_n(\nu_1(y),...,\nu_n(y)))$
(since S is continuous)
$= S(\cup_{i=1}^{\infty}\nu_i(x),\cup_{i=1}^{\infty}\nu_i(y))$. Moreover for $x\in E, n\in N$,
$\cap_{i=1}^{\infty}\mu_i(nx) = \lim T_n(\mu_1(nx),\mu_2(nx),...,\mu_n(nx))$
$\geq \lim T_n(\mu_1(x),...,\mu_n(x)) = \cap_{i=1}^{\infty}(x)$;
$\cup\infty_{i=1}(nx) = \lim S_n(\nu_1(nx),..,\nu_n(nx))$
$\leq \lim S_n(\nu_1(x),...,\nu_n(x)) = \cup_{i=1}^{\infty}(x)$, where the limit taken over $n\to\infty$. Thus $\cap_{i=1}^{\infty}A_i$ is a (T,S)-intuitionistic fuzzy N-subgroup of E. □

Theorem 3.8. *For two N-groups E and F, let $f: E\to F$ be an N-epimorphism. If $A = <\mu_A,\nu_A>$ be a (T,S)-intuitionistic fuzzy N-subgroup of E, then $f(A)$ is also a (T,S)-intuitionistic fuzzy N-subgroup of F.*

Proof. Let $y_1,y_2\in F$. Then since f is onto so $z_1,z_2\in E$ such that $y_1 = f(z_1),\&y_2 = f(z_2)$. Now,
$f(\mu_A)(y_1-y_2) = \bigvee_{f(z)=y_1-y_2}\mu_A(z)$
$\geq \bigvee_{\substack{f(z_1)=y_1\\ f(z_2)=y_2}}\mu_A(z_1-z_2)$
$\geq \bigvee_{\substack{f(z_1)=y_1\\ f(z_2)=y_2}} T(\mu_A(z_1),\mu_A(z_2))$
$\geq T(\bigvee_{f(z_1)=y_1}\mu_A(z_1),\bigvee_{f(z_2)=y_2}\mu_A(z_2))$
$= T(f(\mu_A)(y_1),f(\mu_A)(y_2))$
and $f(\nu_A)(y_1-y_2) = \bigwedge_{f(z)=y_1-y_2}\nu_A(z)$
$\leq \bigwedge_{\substack{f(z_1)=y_1\\ f(z_2)=y_2}}\nu_A(z_1-z_2)$
$\leq \bigwedge_{\substack{f(z_1)=y_1\\ f(z_2)=y_2}} T(\nu_A(z_1),\nu_A(z_2))$
$\leq S(\bigwedge_{f(z_1)=y_1}\nu_A(z_1),\bigwedge_{f(z_2)=y_2}\nu_A(z_2))$
$= S(f(\nu_A)(y_1),f(\nu_A)(y_2))$.

Again for $n\in N, y\in E$ such that $y = f(x)$ for $x\in E$, we have $f(nx) = nf(x) = ny$. Then
$f(\mu_A)(ny) = \bigvee_{\substack{f(z)=y\\ z\in E}}\mu_A(z) \geq \bigvee_{\substack{f(nx)=ny\\ nx\in E}}\mu_A(nx)$
$\geq \bigvee_{\substack{f(z)=y\\ z\in E}}\mu_A(z) \geq \bigvee_{\substack{f(x)=y\\ x\in E}}\mu_A(x) = f(\mu_A)(y)$ and
$f(\nu_A)(ny) = \bigwedge_{\substack{f(z)=y\\ z\in E}}\nu_A(z) \leq \bigwedge_{\substack{f(nx)=ny\\ nx\in E}}\nu_A(nx)$
$\leq \bigwedge_{\substack{f(z)=y\\ z\in E}}\nu_A(z) \leq \bigwedge_{\substack{f(x)=y\\ x\in E}}\nu_A(x) = f(\nu_A)(y)$. Thus $f(A)$ is a (T,S)-intuitionistic fuzzy N-subgroup of F. □

Theorem 3.9. *Let $f: E\to F$ be a N-homomorphism between N-groups E and F. If $A = <\mu_A,\nu_A>$ be a (T,S)-intuitionistic fuzzy N-subgroup of F then $f^{-1}(A)$ is a (T,S)-intuitionistic fuzzy N-subgroup of E.*

Proof. Let $A = <\mu_A,\nu_A>$ be (T,S)-intuitionistic fuzzy N-subgroup of F. Let $x_1,x_2\in E$. Then
$[f^{-1}(\mu_A)](x_1-x_2) = \mu_A(f(x_1-x_2))$
$\geq T(\mu_A(f(x_1)),\mu_A(f(x_2)))$
$= T(f^{-1}(\mu_A)(x_1),f^{-1}(\mu_A)(x_2))$ and
$[f^{-1}(\nu_A)](x_1-x_2) = \nu_A(f(x_1-x_2)) \leq S(\nu_A(f(x_1)),\nu_A(f(x_2)))$
$= S(f^{-1}(\nu_A)(x_1),f^{-1}(\nu_A)(x_2))$.

Also $f^{-1}(\mu_A)(nx) = \mu_A(f(nx)) = \mu_A(nf(x)) \geq \mu_A(f(x)) = f^{-1}(\mu_A)(x)$.

$f^{-1}(\nu_A)(nx) = \nu_A(f(nx)) = \nu_A(nf(x)) \leq \nu_A(f(x))$
$= f^{-1}(\nu_A)(x)$. Thus $f^{-1}(A)$ is (T,S)-intuitionistic fuzzy N-subgroup of E. □

4. (T,S)-Intuitionistic fuzzy ideal of N-group

Definition 4.1. *An IFS $A = <\mu_A,\nu_A>$ of N-group E is called (T,S)-intuitionistic fuzzy ideal of E if for all $x,y\in E$ and $n\in N$,*

TIFI1: $\mu_A(x-y) \geq T(\mu_A(x),\mu_A(y))$
and $\nu_A(x-y) \leq (\nu_A(x),\nu_A(y))$
TIFI2 : $\mu_A(y+x-y) \geq \mu_A(x)$ and $\nu_A(y+x-y) \leq \nu_A(x)$
TIFI3 : $\mu_A(nx) \geq \mu_A(x)$ and $\nu_A(nx) \leq \nu_A(x)$
TIFI4 : $\mu_A(n(y+x)-ny) \geq \mu_A(x)$
and $\nu_A(n(y+x)-ny) \leq \nu_A(x)$

Example 4.2. *Let us consider a near ring $S = \{0,a,b,c\}$ under the addition and multiplication defined as follows:*

+	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

.	0	p	q	r
0	0	0	0	0
p	0	p	q	r
q	0	0	0	0
r	0	0	p	q

Now we define an IFS $A = <\mu_A,\nu_A>$ on N-group S^S such that $\mu_A(0) = 1,\mu_A(p) = 0.3,\mu_A(q) = 0.5,\mu_A(r) = 0.4$ and $\nu_A(0) = 0,\nu_A(p) = 0.6,\nu_A(q) = 0.3,\nu_A(r) = 0.5$. Then A is a (T,S)-intuitionistic fuzzy ideal of S^S with respect to the t-norm and t-co-norm $(ab,a+b-ab)$. But this is not intuitionistic fuzzy ideal of S^Sas $\mu_A(q-r) = \mu_A(p) = 0.3 \ngeq \min\{\mu_A(q),\mu_A(r)\}$. Moreover it is clear that every (T,S)-intuitonistic fuzzy ideal is a (T,S)-intuitionistic fuzzy N-subgroup.

Definition 4.3. *An IFS $A = <\mu_A,\nu_A>$ is said to be (T,S)-idempotent if μ_A is idempotent with respect to t-norm T and ν_A is idempotent with respect to t-co-norm S.*

Lemma 4.4. *An idempotent IFS $A = <\mu_A,\nu_A>$ of N-group is (T,S)-intuitionistic fuzzy N-subgroup of E if and only if $A_{(s,t)} = x\in E : \mu_A(x)\geq s,\nu_A(x)\leq t$ for all $s\in Im(\mu_A), t\in Im(\nu_A)$ is N-subgroup of E.*

Theorem 4.5. *An idempotent IFS $A = <\mu_A,\nu_A>$ of N-group E is (T,S)-intuitionistic fuzzy ideal of E if and only if $A_{(s,t)} = \{x\in E : \mu_A(x)\geq s,\nu_A(x)\leq t\}$ for all $s\in Im(\mu_A), t\in Im(\nu_A)$ is an ideal of E.*

Proof. Let $A = <\mu_A, \nu_A>$ be an idempotent (T,S)-intuitionistic fuzzy ideal of E. Then clearly $A_{(s,t)}$ is a subgroup of E for all $s \in Im(\mu_A), t \in Im(\nu_A)$. Now for $x \in A_{(s,t)}, y \in E$, $\mu_A(y+x) = \mu(y+x) \Rightarrow \mu_A(y+x-y) = \mu_A(-y+y+x) = \mu_A(x) \geq s$ and $\nu_A(y+x) = \nu(y+x) \Rightarrow \nu_A(y+x-y) = \nu_A(-y+y+x) = \nu_A(x) \leq t$ gives $x+y-x \in A_{(s,t)}$. Again for $n \in N, x \in A_{(s,t)}, y \in E$ we have $\mu_A(n(y+x)-ny) \geq \mu_A(x) \geq s$ and $\nu_A(n(y+x)-ny) \leq \nu_A(x) \leq t$ gives $n(y+x)-ny \in A_{(s,t)}$. Hence $A_{(s,t)}$ is an ideal of E.

Conversely, let $A_{(s,t)}$ is an ideal of E. Then clearly $A = <\mu_A, \nu_A>$ is (T,S)-intuitionistic fuzzy N-subgroup of E. Now for $x, y \in E$ let $\mu_A(x) = s, \nu_A(x) = t$. Thus $x \in A_{(s,t)}$ and as $A_{(s,t)}$ is an ideal of E so $y+x-y \in A_{(s,t)}$ and for $n \in N$, $n(y+x)-ny \in A_{(s,t)}$ which implies that $\mu_A(y+x-y) \geq s \geq \mu_A(x)$&;$\nu_A(y+x-y) \leq t \leq \nu_A(x)$ and $\mu_A(n(y+x)-ny) \geq s \geq \mu_A(x)$&;$\nu_A(n(y+x)-ny) \leq t \leq \nu(x)$. Hence $A_{(s,t)}$ is a (T,S)-intuitionistic fuzzy ideal of E.

□

Theorem 4.6. *A non empty subset P of a N-group E is an ideal of E if and only if the characteristic function $<\chi_P, \bar{\chi}_P>$ is a (T,S)-intuitionistic fuzzy ideal of E.*

5. Conclusion

The notion of (T,S)-intuitionistic fuzzy N-subgroup and (T,S)-intuitionistic fuzzy ideal of a N-group finds a way to study other substructures like prime and semi prime N-subgroups, bi-ideals, quasi-ideals etc. The researcher is currently studying on some characterization of (T,S)-intuitionistic fuzzy N-subgroups with (α,β)-cut sets.

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