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V_k-Super vertex magic graceful labeling of graphs

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Abstract

Let G be a finite and simple (p,q) graph. An one-one onto function $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,p+q\}$ is called V-super vertex magic graceful labeling if $f(V(G)) = \{1,2,3,\ldots,p\}$ and for any vertex $v \in V(G)$, $\sum_{u \in N(v)} f(uv) - f(v) = M$, where M is a whole number. For an integer $k \geq 1$, let $E_k(v) = \{e \in E(G)$: the distance between e from v is less than or equal to $k\}$. For $v \in V(G)$, we define $w_k(v) = \sum_{e \in E_k(v)} f(e)$. A V_k-super vertex magic graceful labeling (V_k-SVMGL) is a one-one function f from $V(G) \cup E(G)$ onto the set $\{1,2,3,\ldots,p+q\}$ such that $f(V(G)) = \{1,2,3,\ldots,p\}$ and for any element $v \in V(G)$, we have $w_k(v) - f(v) = M$, where M is a whole number. In this paper, we study several properties of V_k-SVMGL and we identify an equivalent condition for the E_k-regular graphs which admits V_k-SVMGL. At last we identify some families of graphs which admit V_2-SVMGL.

Keywords

V-super vertex magic graceful labeling, V_k-super vertex magic graceful labeling, E_k-regular graphs, circulant graphs.

AMS Subject Classification

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Contents

1. Introduction

A graph labeling is a function which has domain as graph elements such as vertices and/or edges with co-domain as a set of numbers. Usually the co-domain has been taken as integers. Many of graph labelings are introduced and discussed by various authors. To know more about graph labeling, refer [1].

In 1967, Rosa [5] introduced a labeling called β-valuation. Golomb [2] called such labeling as graceful. An injection f from the vertices of G to { 0, 1, 2, …, q } is called a graceful labeling of G if when we assign each edge uv the label $|f(u) - f(v)|$, the resulting edge labels are distinct.

In 1966 and 1976, Stewart [12] and Sedlacek [6] introduced magic type labelings. Magic labeling is a one to one map on to the appropriate set of consecutive integers starting from 1, with some kind of 'constant sum' property.

A vertex magic total labeling (VMTL) of G is a one-one function f from $V(G) \cup E(G)$ onto the set $\{1,2,3,\ldots,p+q\}$ such that for any vertex $v \in V(G)$, the sum $f(v) + \sum_{u \in N(v)} f(uv) = M$, where M is a whole number. The whole number M is said to be the magic constant [1].

In 2004, the concept 'super vertex-magic total labeling (SVMTL)' in simple graphs has been defined by MacDougall et al.[3]. They name the VMTL as super if $f(V(G)) = \{1,2,3,\ldots,p\}$. For their labeling, the vertices receive the least integers.

In 2003, Swaminathan and Jeyanthi [10] introduced another labeling called super vertex magic labeling (SVML). They call a VMTL is super if $f(E(G)) = \{1,2,\ldots,q\}$. Here, the smallest labels are assigned to the edges. To avoid confusion, Marimuthu and Balakrishnan [4] called a VMTL is E-super if $f(E(G)) = \{1,2,\ldots,q\}$. A graph G is called E-super vertex magic (E-SVM) if it admits an E-super vertex magic labeling (E-SVML).

An E_k-SVML of G is an one-one function f from $V(G) \cup$

$E(G)$ onto the set $\{1,2,3,\ldots,p+q\}$ such that $f(E(G)) = \{1,2,3,\ldots,q\}$ and for any vertex $v \in V(G)$, the sum $f(v) + w_k(v) = M$, where M is a whole number.

In 2018, Sivagnanam Mutharasu and Duraisamy Kumar [8] introduced V_k-super vertex magic labeling(V_K-SVML) in graphs. A V_k-SVML of G is a one-one function f from $V(G) \cup E(G)$ onto the set $\{1,2,3,\ldots,p+q\}$ such that $f(V(G)) = \{1,2,3,\ldots,p\}$ and for any vertex $v \in V(G)$, the sum $f(v) + w_k(v) = M$, where M is a whole number.

In this paper, by using the natural meaning of both the concepts 'Graceful' and 'V-SVML', a new labeling is introduced in the name 'V_k-super vertex magic graceful labeling'. Let k be a whole number and $1 \le k \le \text{diam}(G)$. For each edge $e \in E(G)$, let $E_k(e) = \{v \in V(G)$: the distance between e from v is less than or equal to $k\}$. If $|E_k(e)| = r$ for some whole number $r \ge 1$ and for each edge $e \in E(G)$, then we call G as E_k-regular. All the connected graphs with at least one edge, are E_1-regular. Consider the following graph $G(V,E)$, with $V(G) = \{a_1,a_2,a_3,a_4,a_5,a_6,a_7\}$ and $E(G) = \{b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8\}$.

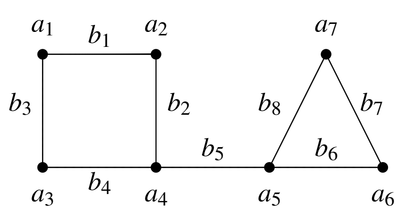

Figure 1. G

The following table give the values of $E_k(a)$ and $E_k(b)$ for $k = 2$.

$E_2(a)$	$E_2(b)$
$E_2(a_1) = \{b_1,b_2,b_3,b_4\}$	$E_2(b_1) = \{a_1,a_2,a_3,a_4\}$
$E_2(a_2) = \{b_1,b_2,b_3,b_4,b_5\}$	$E_2(b_2) = \{a_1,a_2,a_3,a_4,a_5\}$
$E_2(a_3) = \{b_1,b_2,b_3,b_4,b_5\}$	$E_2(b_3) = \{a_1,a_2,a_3,a_4\}$
$E_2(a_4) = \{b_1,b_2,b_3,b_4,b_5,b_6,b_8\}$	$E_2(b_4) = \{a_1,a_2,a_3,a_4,a_5\}$
$E_2(a_5) = \{b_2,b_4,b_5,b_6,b_7,b_8\}$	$E_2(b_5) = \{a_2,a_3,a_4,a_5,a_6,a_7\}$
$E_2(a_6) = \{b_5,b_6,b_7,b_8\}$	$E_2(b_6) = \{a_4,a_5,a_6,a_7\}$
$E_2(a_7) = \{b_5,b_6,b_7,b_8\}$	$E_2(b_7) = \{a_5,a_6,a_7\}$, $E_2(b_8) = \{a_4,a_5,a_6,a_7\}$

A V_k-super vertex magic graceful labeling (V_k-SVMGL) is a one-one function f from $V(G) \cup E(G)$ onto the set $\{1,2,3,\ldots,p+q\}$ such that $f(V(G)) = \{1,2,3,\ldots,p\}$ and for any vertex $v \in V(G)$, the difference $w_k(v) - f(v) = M$ for some whole number M. This whole number M is said to be the magic constant of V_k-SVMGL of G.

If a graph admits a V_k-SVMGL, then we say it is a V_k-super vertex magic graceful(V_k-SVMG) graph. In this paper, we study several properties of V_k-SVMGL and we identify an equivalent condition for the E_k-regular graphs which admits V_k-SVMGL. At last we identify some families of graphs which admit V_2-SVMGL.

2. Main Results

Here, we collect some of the basic properties of V_k-SVMGL.

In a connected graph G with more than one vertex, if $E_k(u) = E_k(v)$ for some vertices $u,v \in V(G)$ and $u \ne v$, then the differences $w_k(u) - f(u)$ and $w_k(v) - f(v)$ are not equal for any V_k-SVMGL f of G(because the function f is one-one). It means that G is not V_k-SVMG and so the next Lemma follows.

Lemma 2.1. *For a connected graph G with more than one vertex, if $E_k(u) = E_k(v)$ for two vertices $u,v \in V(G)$ and $u \ne v$, then G is not V_k-SVMG.*

If G is V_k-SVMG, then the integer k must be lies between 1 and $\text{diam}(G)$ (If not, then for any two vertices $u,v \in V(G)$ and $u \ne v$, we have $E_k(u) = E_k(v)$). Since $\text{diam}(S_n) = 2$ for the star graph S_n, we have the following result.

Corollary 2.2. *For each integer $k \ge 2$, the star graph is not V_k-SVMG.*

Lemma 2.3. *Let $G(p,q)$ be a connected E_k-regular graph. If G admits V_k-SVMGL, then $M = rq + \frac{rq(q+1)}{2p} - \frac{p+1}{2}$, where M is the magic constant and r is the regularity.*

Proof. Since G is V_k-SVMG, there exists a V_k-SVMGL in G, say f. Then we must have $f(V(G))$ is equal to $\{1,2,3,\ldots,p\}$ and $f(E(G))$ is equal to $\{p+1,p+2,p+3,\ldots,p+q\}$. Also, the magic constant M is equal to $w_k(v) - f(v)$ for any element $v \in V(G)$. Hence, $pM = \sum\limits_{v \in V(G)} w_k(v) - \sum\limits_{v \in V(G)} f(v)$
$= \sum\limits_{v \in V(G)} \sum\limits_{e \in V_k(v)} f(e) - \sum\limits_{v \in V(G)} f(v)$
$= r \sum\limits_{e \in E(G)} f(e) - [1+2+\ldots+p]$ (since each edge is counted exactly r times in the sum $\sum\limits_{v \in V(G)} \sum\limits_{e \in V_k(v)} f(e)$)
$= rpq + \frac{rq(q+1)}{2} - \frac{p(p+1)}{2}$ and so $M = rq + \frac{rq(q+1)}{2p} - \frac{p+1}{2}$. □

For $k \ge 1$, Lemma 2.3 gives the magic constant for E_k-regular graphs which are V_k-SVMG. We found the next result which is the particular case of above statement [9].

Lemma 2.4. *[9] If a nontrivial graph G(p, q) is V-SVMG, then the magic number M is given by $M = 2q + \frac{q(q+1)}{p} - \frac{p+1}{2}$.*

For any nontrivial graph G, we have $r = 2$ when $k = 1$. By taking $k = 1$ in Lemma 2.3, we can prove Lemma 2.4.

Theorem 2.5. *For an E_k-regular connected graph G, we have*
(a). $M \ge (p-1)(\frac{3r-1}{2}) - 1$
(b). $M = (p+1)(\frac{3r-1}{2}) - r$ if $q = p$.

Proof. (a) Since G is a connected graph, q is equal to $p-1$. By Lemma 2.3, we have the magice constant $M = rq + \frac{rq(q+1)}{2p} - \frac{p+1}{2} \ge r(p-1) + \frac{r(p-1)(p)}{2p} - \frac{p+1}{2} = (p-1)\frac{3r-1}{2} - 1$.
(b) Since $q = p$, it follows from Lemma 2.3 that $M = rq + \frac{rq(q+1)}{2p} - \frac{p+1}{2} = rp + \frac{rp(p+1)}{2p} - \frac{p+1}{2} = (p+1)\frac{3r-1}{2} - r$. □

Remark 1. In Theorem 2.5(b), we obtained a lower bound for M, where M is the magic constant. In the following example, we prove that the lower bound sharp when $k = 2$. Consider the V_2-SVMGL of the graph C_5 as given below.

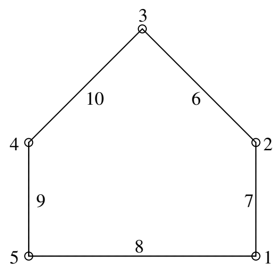

Figure 2: V_2-SVMGL of C_5

The graph C_5 is E_2-regular and the regularity is 4. Also the magic constant M is equal to 29.

Lemma 2.6. *For an integer $k \geq 2$, no tree is E_k-regular and V_k-SVMG.*

Proof. Suppose $\text{diam}(G) = d(\geq 3)$ for a tree G. Let $P = a_0a_1 \ldots a_{d-1}a_d$ be a path of length d. In this case, the edges a_0a_1 and $a_{d-1}a_d$ must be pendent. For $k = d$, we must have $E_k(a_0) = E_k(a_d)$ and so G must not be V_k-SVMG. On the other hand, for $k \leq d-1$, we must have $E_k(a_1a_2)$ is strictly greater than $E_k(a_0a_1)$ and so the tree G is not E_k-regular. Hence, $\text{diam}(G)$ must be less than or equal to 2. If $\text{diam}(G) = 2$, then G is a star. Thus G is not V_k-SVMG(by Corollary 2.2). □

Theorem 2.7. *Let $G(p,q)$ be a graph and g be an one-one onto function from the edge set $E(G)$ onto the q successive integers $\{p+1, p+2, \ldots, p+q\}$. Then the function g is extendeble as a V_k-SVMGL of G if and only if the set $\{w_k(u)/u \in V(G)\}$ is a set of p successive integers.*

Proof. Suppose $\{w_k(u)/u \in V(G)\}$ is a set of p successive integers. Take t as the least integer in the set $\{w_k(u)/u \in V(G)\}$. Now define a function f from $V(G) \cup E(G)$ onto $\{1, 2, \ldots, p+q\}$ defined by $f(e) = g(e)$ for $e \in E(G)$ and $f(u) = w_k(u) - t + 1$ for $u \in V(G)$. In this case, the set of edge labelings $f(E(G))$ is equal to $\{p+1, p+2, \ldots, p+q\}$ and the set of vertex labelings $f(V(G))$ is equal to $\{1, 2, \ldots, p\}$ (since $\{w_k(x) - t : x \in V(G)\}$ contain successive integers). Thus f is V_k-SVMGL and the magic constant $M = t - 1$.
On the other hand, suppose g is extendeble as a V_k-SVMGL of G, say f. Let M be the magic constant. Note that $w_k(u) - f(u) = M$ for each vertex $u \in V(G)$. Thus $w_k(u) = M + f(u)$ and so $\{w_k(u)/u \in V(G)\}$ must be equal to $\{M+1, M+2, \ldots, M+p\}$, which is a set of p successive integers. □

3. V_2-SVMGL of cycles and prisms

This section provides some collection of graphs which are V_2-SVMG. For any vertex $u \in V(C_3)$, we have $E_2(u) = E(G)$. Thus by Lemma 2.1, the cycle C_3 is not V_2-SVMG.

Lemma 3.1. *[11] For any integers a and b, we have $gcd(a,b) = gcd(b,a) = gcd(\pm a, \pm b) = gcd(a, b-a) = gcd(a, b+a)$.*

Theorem 3.2. *Let $s(\geq 5)$ be an integer. Then the cycle C_s admits V_2-SVMGL if and only if s is an odd integer.*

Proof. Assume that C_s is V_2-SVMG and let f be a V_2-SVMGL of C_s. Note that $|E_2(e)| = r = 4$ for any edge $e \in E(C_s)$. Thus by letting $r = 4$, $p = q = s$ and $k = 2$ in Lemma 2.3, we can have $M = \frac{11s+3}{2}$. Thus s must be odd(since M is an integer).
Conversely, suppose s is odd. Let $V(C_s) = \{v_i/1 \leq i \leq s\}$ and $E(C_s) = \{v_iv_{i\oplus_s 1}/1 \leq i \leq s\}$. Here the operation $\oplus_s$ denotes addition modulo s.
Case A: When $s = 4t+1$ and $t \geq 1$.
Define $f : V(C_s) \cup E(C_s) \to \{1, 2, 3, \ldots, 2s\}$ as given below: $f(v_i) = s + 4 - i$ when $4 \leq i \leq s$ and $f(v_i) = 4 - i$ when $1 \leq i \leq 3$; $f(v_iv_{i\oplus_s 1}) = [(i-1)t \oplus_s 1] + s$, where $(i-1)t \oplus_s 1$ is the positive residue when $(i-1)t + 1$ divides s.
Here we are going to prove that $f(E(C_s)) = \{s+1, s+2, s+3, \ldots, 2s\}$. Take $a = \ell$ and $b = s$ in Lemma 3.1, then we get $\gcd(t,s) = \gcd(t, 4t+1) = \gcd(t, 3t+1) = \gcd(t, 2t+1) = \gcd(t, t+1) = \gcd(t, 1) = 1$. It means that t is a generator of the cyclic group $(Z_s, \oplus_s)$ and so $f(E(C_s)) = \{s+1, s+2, \ldots, 2s\}$.
Claim 1: $w_2(v_i) = 26t + 12 - i$ for integer i with $4 \leq i \leq s$.
Case i: If $i = 4x$ for some integer $1 \leq x \leq t$. Now $w_2(v_i) = f(v_{i-2}v_{i-1}) + f(v_{i-1}v_i) + f(v_iv_{i+1}) + f(v_{i+1}v_{i+2})$.
Since $f(v_{i-2}v_{i-1}) = [(i-3)\frac{s-1}{4} \oplus_s 1] + s = [sx - x - \frac{3s}{4} + \frac{3}{4} \oplus_s 1] + s = [-x - \frac{3s}{4} + \frac{3}{4} \oplus_s 1] + s = [-x - 3t \oplus_s 1] + s$, by the definition of f, we have $w_2(v_i) = [-x - 3t \oplus_s 1] + [-x - 2t \oplus_s 1] + [-x - t \oplus_s 1] + [-x \oplus_s 1] + 4s$.
Since $1 \leq x \leq t$, the first four terms of above equation are not positive. Thus we have $w_2(v_i) = 4s + [s - x - 3t + 1] + [s - x - 2t + 1] + [s - x - t + 1] + [s - x + 1]$. Take $s = 4t+1$. Then we get $w_2(v_i) = 26t + 12 - i$.
Case ii: Suppose $i = 4x+1$ and $1 \leq x \leq t$.
Since $f(v_{i-2}v_{i-1}) = [-x - 2t \oplus_s 1] + s$, we have $w_2(v_i) = [-x - 2t \oplus_s 1] + [-x - t \oplus_s 1] + [-x \oplus_s 1] + [-x + t \oplus_s 1] + 4s$. Since $1 \leq x \leq t$, the first three terms are not positive, we have $w_2(v_i) = [s - x - 2t + 1] + [s - x - t + 1] + [s - x + 1] + [-x + t + 1] + 4s = 26t + 12 - i$. In similar way, we can see that $w_2(v_i) = 26t + 12 - i$ when $i = 4x+2$ and $i = 4x+3$.
Claim 2: $w_2(v_i) = (2t+1)11 - i$ for integers i with $1 \leq i \leq 3$.
Consider v_1. The weight $w_2(v_1) = f(v_{s-1}v_s) + f(v_sv_1) + f(v_1v_2) + f(v_2v_3)$. Since $f(v_{s-1}v_s) = [(s-2)\frac{(s-1)}{4} \oplus_s 1)] + s = [(4t-1)\frac{(s-1)}{4} \oplus_s 1] + s = [-2t \oplus_s 1] + s$, we have $w_2(v_1) = [-2t \oplus_s 1] + [-t \oplus_s 1] + 1 + [t \oplus_s 1] + 4s$. Here, the first two terms are negative or zero. Thus $w_2(v_1) = [s - 2t + 1] + [s - t + 1] + 1 + [t \oplus_s 1] + 4s = (2t+1)11 - 1$. In similar way, we can prove that $w_2(v_2) = (2t+1)11 - 2$ and $w_2(v_3) = (2t+1)11 - 3$.
Note that $t = \frac{s-1}{4}$. Then by Claim 1, $w_2(v_i) - f(v_i) = 26t + 12 - i - (s + 4 - i) = \frac{11s+3}{2} = M$ for $4 \leq i \leq s$. Also from Claim 2, we have $w_2(v_i) - f(v_i) = 11(2t+1) - i - (4 - i) = \frac{11s+3}{2} = M$ for $i = 1, 2, 3$.

Case B: If $s=4t+3$ and $t\geq 1$.
Define $f:V(C_s)\cup E(C_s)\to\{1,2,\dots,2s\}$ as follows:
$f(v_i)=i+1$ when $1\leq i\leq s-1$ and $f(v_s)=1$; $f(v_iv_{i\oplus_s 1})=[(i-1)(t+1)\oplus_s 1]+s$, where $[(i-1)(t+1)\oplus_s 1]+s$ is the positive residue $(i-1)(t+1)+1$ divides s. From Lemma 3.1, $\gcd(t+1,s)=\gcd(t+1,4t+3)=\gcd(t+1,3t+2)=\gcd(t+1,2t+1)=\gcd(t+1,t)=\gcd(t,t+1)=\gcd(t,1)=1$. Hence $t+1$ is a generator for the cyclic group $(Z_s,\oplus_s)$ and hence $f(E(C_s))=\{s+1,s+2,\dots,2s\}$. As proved in Case A, we can prove that the above labeling is a V_2-SVMGL with magic constant $M=\frac{11s+3}{2}$. □

Theorem 3.3. *Let $s(\geq 5)$ is an integer. The graph $G=\overline{C_s}$, the complement of C_s, is V_2-SVMG and the magic constant is given by $\frac{s^4-2s^3-s^2-22s-8}{8}$.*

Proof. Define $f:V(\overline{C_s})\cup E(\overline{C_s})\to\{1,2,\dots,\frac{s^2-s}{2}\}$ as given below: First the s edges $\{a_1a_3,a_2a_4,\dots,a_na_2\}$ are labeled by $f(a_{i\oplus s-1},a_{i\oplus 1})=s+i$ for $1\leq i\leq s$.
The remaining $\frac{s^2-3s}{2}-s$ edges are randomly labeled onto $\{2s+1,2s+2,\dots,\frac{s^2-s}{2}\}$. Then we label The vertices by $f(a_i)=s-(i-1)$. Remark that for each vertex a_i, the only edge with label $s+i$, is not in $E_2(a_i)$. Thus for each a_i with $1\leq i\leq s$, we have $w_2(a_i)-f(a_i)=\sum\limits_{e\in E(\overline{C_s})} f(e)-(s+i)-f(a_i)=\frac{s^4-2s^3-s^2-6s}{8}-(s+i)-[s-(i-1)]=\frac{s^4-2s^3-s^2-22s-8}{8}$. □

Theorem 3.4. *Let $s(\geq 3)$ be an integer. Then the prism graph D_s admits V_2-SVMGL if and only if s is an even integer.*

Proof. Assume that D_s admits V_2-SVMGL, say f. Let M be the corresponding magic constant. Note that $|E_2(e)|=r=6$ for any edge $e\in E(D_s)$. By letting $k=2$, $r=6$, $q=3s$ and $p=2s$ in Lemma 2.3, we can have $M=\frac{61s+8}{2}$. Then s must be an even integer(since M is an integer).
Conversely suppose s is an even integer. Let $V(D_s)=\{a_i,b_i: 1\leq i\leq s\}$ and $E(D_s)=\{a_ib_i,a_ia_{i\oplus_s 1},b_ib_{i\oplus_s 1}/1\leq i\leq s\}$.
Define $f:V(D_s)\cup E(D_s)\to\{1,2,\dots,5s\}$ as follows:
$f(a_i)=\frac{i}{2}-1$ if $i\geq 4$ and i is even; The range is given by $\{1,2,\dots,\frac{s}{2}-1\}$,
$f(a_2)=\frac{s}{2}$; $\{\frac{s}{2}\}$,
$f(a_i)=\frac{s}{2}+\frac{i+1}{2}$ if i is odd; $\{\frac{s}{2}+1,\frac{s}{2}+2,\dots,s\}$,
$f(b_2)=s+1$; $\{s+1\}$,
$f(b_i)=s+\frac{s}{2}+2-\frac{i}{2}$ if $i\geq 4$ and i is even; $\{s+2,\dots,s+\frac{s}{2}\}$,
$f(b_i)=2s-\frac{i-1}{2}$ if i is odd; $\{s+\frac{s}{2}+1,s+\frac{s}{2}+2,\dots,2s\}$,
$f(a_ib_i)=2s+\frac{i+1}{2}$ if i is odd; $\{2s+1,2s+2,\dots,2s+\frac{s}{2}\}$,
$f(a_ib_i)=2s+\frac{s}{2}+\frac{i}{2}$ if i is even; $\{2s+\frac{s}{2}+1,2s+\frac{s}{2}+2,\dots,3s\}$,
$f(a_ia_{i\oplus_s 1})=3s+\frac{s}{2}-\frac{i-1}{2}$ if i is odd; $\{3s+1,3s+2,\dots,3s+\frac{s}{2}\}$,
$f(b_ib_{i\oplus_s 1})=4s-(\frac{i}{2}-1)$ if i is even; $\{3s+\frac{s}{2}+1,3s+\frac{s}{2}+2,\dots,4s\}$,
$f(a_ia_{i\oplus_s 1})=4s+\frac{i}{2}$ if i is even; $\{4s+1,4s+2,\dots,4s+\frac{s}{2}\}$,
$f(b_ib_{i\oplus_s 1})=5s-\frac{i-1}{2}$ if i is odd; $\{4s+\frac{s}{2}+1,4s+\frac{s}{2}+2,\dots,5s\}$.
It is easily seen that f is a V_2-SVMGL with the magic constant $M=\frac{61s+8}{2}$. □

Let Γ be a group and e be the identity element of Γ. Let X be a generating set of Γ such that $e\notin X$ and $X=X^{-1}=\{x^{-1}/x\in X\}$. A Cayley graph $G=Cay(\Gamma,X)$ is a graph with $V(G)=\Gamma$ and $E(G)=\{(x,xa)/x\in V(G),a\in X\}$. Since X is a generating set of Γ, $Cay(\Gamma,X)$ is a connected regular graph of degree $|X|$. When $\Gamma=Z_n$, the corresponding Cayley graph is a circulant graph, denoted by $Cir(n,A)$.

Lemma 2.3 give the magic constant for E_k-regular graphs which are V_k-SVM. For $A=\{1,2,s-1,s-2\}$, the corresponding circulant graph $Cir(s,A)$ is not E_2-regular. In the following result, we obtain the magic constant for this family of graphs.

Theorem 3.5. *For an integer $s\geq 7$. $G=Cir(s,\{1,2,s-1,s-2\})$ is V_2-SVMG with the magic constant $M=26s+6$.*

Proof. Let $V(G)=\{a_1,a_2,\dots,a_s\}$ and $E(G)=\{a_ia_{i\oplus_s 1},a_ia_{i\oplus_s 2}:1\leq i\leq s\}$. Define $f:V(G)\cup E(G)\to\{1,2,\dots,3s\}$ as follows:
$f(a_i)=s+5-i$ for $5\leq i\leq s$; $f(a_i)=5-i$ for $1\leq i\leq 4$;
$f(a_ia_{i\oplus_s 1})=s+i$ for $1\leq i\leq s$ and $f(a_ia_{i\oplus_s 2})=3s+1-i$ for $1\leq i\leq s$.
Let $v\in V(G)$. If $v=a_i$ for i with $5\leq i\leq s$.
Then $w_2(a_i)-f(a_i)=f(a_{i\oplus_s s-3}a_{i\oplus_s s-2})+f(a_{i\oplus_s s-2}a_{i\oplus_s s-1})+f(a_{i\oplus_s s-1}a_i)+f(a_ia_{i\oplus_s 1})+f(a_{i\oplus_s 1}a_{i\oplus_s 2})+f(a_{i\oplus_s 2}a_{i\oplus_s 3})+f(a_{i\oplus_s s-4}a_{i\oplus_s s-2})+f(a_{i\oplus_s s-3}a_{i\oplus_s s-1})+f(a_{i\oplus_s s-2}a_i)+f(a_{i\oplus_s s-1}a_{i\oplus_s 1})+f(a_ia_{i\oplus_s 2})+f(a_{i\oplus_s 1}a_{i\oplus_s 3})+f(a_{i\oplus_s 2}a_{i\oplus_s 4})-f(a_i)$
$=[s+i\oplus_s s-3]+[s+i\oplus_s s-2]+[s+i\oplus_s s-1]+[s+i]+[s+i+1]+[s+i+2]+[3s+1-(i\oplus_s s-4)]+[3s+1-(i\oplus_s s-3)]+[3s+1-(i\oplus_s s-2)]+[3s+1-(i\oplus_s s-1)]+[3s+1-i]+[3s+1-(i+1)]+[3s+1-(i+2)]-[s+5-i]=[s+i-3]+[s+i-2]+[s+i-1]+[s+i]+[s+i+1]+[s+i+2]+[3s+1-(i-4)]+[3s+1-(i-3)]+[3s+1-(i-2)]+[3s+1-(i-1)]+[3s+1-i]+[3s+1-(i+1)]+[3s+1-(i+2)]-[s+5-i]=26s+6=M$.
Similarly, we can prove that $f(a_i)+w_2(a_i)=26s+6$ for $i=1,2,3,4$. □

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