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$\mathscr{Z}$- generalized homeomorphism via double fuzzy topological spaces

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Abstract
We introduce and study $df\ \mathscr{Z}$-generalized homeomorphisms and pre $df\ \mathscr{Z}$-generalized homeomorphisms in double fuzzy topological spaces. Additionally, a portion of their principal properties are contemplated.

Keywords
df homeomorphisms, $df\ \mathscr{Z}$-generalized homeomorphisms and pre $df\ \mathscr{Z}$-generalized homeomorphisms.

AMS Subject Classification
54A40, 45D05, 03E72.

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Contents

1. Introduction and Preliminaries

In 1986, Atanassov [1] began "Intuitionistic fuzzy sets" and Coker [2] in 1997, started Intuitionistic fuzzy topological space. The term "double" rather than "intuitionistic" instituted by Garcia and Rodabaugh [5] in 2005. In the past two decades numerous experts achieving more applications on double fuzzy topological spaces.

From 2011, $\mathscr{Z}$-open sets and maps were presented in topological spaces by El-Maghrabi and Mubarki [3].

X denotes a non-empty set, $I_1 = [0,1)$, $I_0 = (0,1]$, $I = [0,1]$, $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $\iota \in I_0$ & $\kappa \in I_1$ and always $1 \geq \iota + \kappa$. I^X is a family of all fss on X. In 2002, double fuzzy topological spaces (briefly, dfts), (X, η, η^*), (ι, κ)-fuzzy open (resp. (ι, κ)-fuzzy closed) (briefly (ι, κ)-fo (resp. (ι, κ)-fc)) set were given by Samanta and Mondal [11].

All other undefined notions are from [11, 15, 16, 18] and cited therein.

2. Double fuzzy $\mathscr{Z}$- generalized homeomorphism

Definition 2.1. *A bijective mapping $g : (X, \zeta_1, \zeta_1^*) \to (Y, \zeta_2, \zeta_2^*)$ is called double fuzzy $\mathscr{Z}$-generalized homeomorphism (resp. double fuzzy homeomorphism) (briefly, $DF\mathscr{Z}gHom$ (resp. $DFHom$)) if g & g^{-1} are $DF\mathscr{Z}gCts$ (resp. $DFCts$).*

Definition 2.2. *A bijective mapping $g : (X, \zeta_1, \zeta_1^*) \to (Y, \zeta_2, \zeta_2^*)$ is called pre double fuzzy $\mathscr{Z}$ generalized homeomorphism (briefly, $pDF\mathscr{Z}gHom$) if g and g^{-1} are $DF\mathscr{Z}gIrr$.*

Theorem 2.3. *Every $DFHom$ is $DF\mathscr{Z}gHom$. But not conversely.*

Proof. Let $g : X \to Y$ be $DFHom$. Then g and g^{-1} are $DFCts$. Let K be any (ι, κ)-fc set in Y. Then $g^{-1}(K)$ is also (ι, κ)-fc in X as f is $DFCts$. As every (ι, κ)-fc set is (ι, κ)-$f\mathscr{Z}gc$, $g^{-1}(K)$ is also (ι, κ)-$f\mathscr{Z}gc$ in X. Thus g is $DF\mathscr{Z}gCts$. Also $g(g^{-1}(K)) = K$ is (ι, κ)-fc in Y implies K is also (ι, κ)-$f\mathscr{Z}gc$ in Y. Hence $g(g^{-1}(K))$ is (ι, κ)-$f\mathscr{Z}gc$ in Y. Thus g^{-1} is also $DF\mathscr{Z}Cts$. Therefore, g is $DF\mathscr{Z}gHom$. □

Example 2.4. *Let γ & δ be fuzzy subsets of $X = Y = \{l_1, m_1, n_1\}$ are defined as $\gamma(l_1) = 0.3$, $\gamma(m_1) = 0.4$, $\gamma(n_1) = 0.5$; $\delta(l_1) = 0.4$, $\delta(m_1) = 0.5$, $\delta(n_1) = 0.5$. Consider the $dfts$'s (X, ζ, ζ^*) and (X, η, η^*) with*

$$\zeta(\rho) = \begin{cases} 1, & \text{if } \rho = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \rho = \gamma, \\ 0, & \text{otherwise.} \end{cases} \qquad \zeta^*(\rho) = \begin{cases} 0, & \text{if } \rho = \underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \rho = \gamma, \\ 1, & \text{otherwise.} \end{cases}$$

$$\eta(\rho)=\begin{cases}1, & \text{if } \rho=\underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \rho=\delta, \\ 0, & o.w.\end{cases} \qquad \eta^*(\rho)=\begin{cases}0, & \text{if } \rho=\underline{0} \text{ or } \underline{1}, \\ \frac{1}{2}, & \text{if } \rho=\delta, \\ 1, & o.w.\end{cases}$$

Then the id mapping $g:(X,\zeta,\zeta^*)\to(X,\eta,\eta^*)$ is $df\mathscr{Z}$-Hom. but not DFHom as the fs δ is (ι,κ)-fo set in Y, but $g^{-1}(\delta)=\delta$ is not (ι,κ)-fo set in Y. Hence $g:Y\to X$ is not DFCts.

Theorem 2.5. *Every $pDF\mathscr{Z}gHom$ is $DF\mathscr{Z}gHom$.*

Theorem 2.6. *Let $g:(X,\zeta_1,\zeta_1^*)\to(Y,\zeta_2,\zeta_2^*)$ be a bijective mapping. Then*

(i) g is $DF\mathscr{Z}gHom$.

(ii) g is $DF\mathscr{Z}gCts$ & $DF\mathscr{Z}gO$ map.

(iii) g is $DF\mathscr{Z}gCts$ & $DF\mathscr{Z}gC$ map.

are equivalent.

Proof. $(i)\Rightarrow(ii)$: Let g be $DF\mathscr{Z}gHom$. Then g and g^{-1} are $DF\mathscr{Z}gCts$. To prove that g is $DF\mathscr{Z}gO$ map, let ρ be a (ι,κ)-fo set in X. Since $g^{-1}:Y\to X$ is $DF\mathscr{Z}gCts$, $(g^{-1})^{-1}(\rho)=g(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ in Y. Therefore $g(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ in Y. Hence g is $DF\mathscr{Z}gO$ map.

$(ii)\Rightarrow(iii)$: Let g be $DF\mathscr{Z}gCts$ and $DF\mathscr{Z}gO$ map. To prove that g is $DF\mathscr{Z}gC$ map. Let μ be a (ι,κ)-fc set in X. Then $1-\mu$ is (ι,κ)-fo set in X. Since g is $DF\mathscr{Z}gO$ map, $g(1-\mu)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Now, $g(1-\mu)=1-g(\mu)$. Therefore $g(\mu)$ is (ι,κ)-$f\mathscr{Z}gc$ in Y. Hence, g is a $DF\mathscr{Z}gC$ map.

$(iii)\Rightarrow(i)$: Let g be $DF\mathscr{Z}Cts$ and $DF\mathscr{Z}gC$ map. To prove that g^{-1} is $DF\mathscr{Z}gCts$. Let ρ be a (ι,κ)-fo set in X. Then $1-\rho$ is a fc set in X. Since g is $DF\mathscr{Z}gC$ map, $g(1-\rho)$ is $f\mathscr{Z}gc$ set in Y. Now, $(g^{-1})^{-1}(1-\rho)=g(1-\rho)=1-g(\rho)$. Therefore $g(\rho)$is $f\mathscr{Z}go$ set in Y. Hence $g^{-1}:Y\to X$ is $DF\mathscr{Z}gCts$. Thus g is $DF\mathscr{Z}gHom$. □

Theorem 2.7. *Let $f:(X,\zeta,\zeta^*)\to(Y,\sigma,\sigma^*)$. Then*

(i) f is $DF\mathscr{Z}gIrr$.

(ii) $\forall$ (ι,κ)-$f\mathscr{Z}gc$ set ρ in Y, $f^{-1}(\rho)$ is (ι,κ)-$f\mathscr{Z}gc$ in X.

(iii) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}gnbhd$ ρ of $f(x_p)$, $f^{-1}(\rho)$ is a (ι,κ)-$f\mathscr{Z}gnbhd$ of x_p.

(iv) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}gnbhd$ ρ of $f(x_p)$, there is a (ι,κ)-$f\mathscr{Z}gnbhd$ μ of $x_p\ni f(\mu)\leq\rho$.

(v) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}go$ ρ of $Y\ni f(x_p)\in A$, $\exists$ a (ι,κ)-$f\mathscr{Z}go$ set $\ni x_p\in\mu$ and $f(\mu)\leq\rho$.

(vi) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}go$ set ρ of $Y\ni f(x_p)q\rho$, $\exists$ a (ι,κ)-$f\mathscr{Z}go$ set μ of $X\ni x_pq\mu$ and $f(\mu)\leq\rho$.

(vii) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}gq$-nbhd ρ of $f(xp)$, $f^{-1}(\rho)$ is a (ι,κ)-$f\mathscr{Z}gq$-nbhd of x_p.

(viii) $\forall$ fp x_p of X and every (ι,κ)-$f\mathscr{Z}gq$-nbhd ρ of $f(xp)$, $\exists$ a (ι,κ)-$f\mathscr{Z}gq$-nbhd μ of $x_p\ni f(\mu)\leq\rho$.

are equivalent.

Proof. (i)⇒(ii), (i)⇒(iii), (v)⇒(i) and (vii)⇒(viii): Obvious.

(ii)⇒(i): Let ρ be a (ι,κ)-$f\mathscr{Z}gc$ set in Y which implies $\underline{1}-\rho$ is (ι,κ)-$f\mathscr{Z}go$-open in Y. $f^{-1}(\underline{1}-\rho)=\underline{1}-f^{-1}(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ in X implies $f^{-1}(\rho)$ is (ι,κ)-$f\mathscr{Z}gc$ in X. Hence f is $DF\mathscr{Z}Irr$.

(iii)⇒(iv): Let x_p be a fp of X \$ ρ be a (ι,κ)-$f\mathscr{Z}$gnbhd of $f(x_p)$. Taking $\mu=f^{-1}(\rho)$ is a (ι,κ)-$f\mathscr{Z}$gnbhd of x_p and $f(\mu)=f(f^{-1}(\rho))\leq\rho$.

(iv)⇒ (v): Let x_p be a fp of X \$ ρ be a (ι,κ)-$f\mathscr{Z}go$ set $\ni f(x_p)\in\rho$. Then ρ is a (ι,κ)-$f\mathscr{Z}$gnbhd of $f(x_p)$. Hence there is (ι,κ)-$f\mathscr{Z}$gnbhd μ of $x_p\in X\ni x_p\in\mu$ and $f(\mu)\leq\rho$. Hence there is (ι,κ)-$f\mathscr{Z}go$ set γ in $X\ni x_p\in\gamma\leq\mu$ and $f(\gamma)\leq f(\mu)\leq\rho$.

(i)⇒(vi): Let x_p be a fp of X \$ ρ be a (ι,κ)-$f\mathscr{Z}go$ set in Y $\ni f(x_p)q\rho$. Let $\mu=f^{-1}(\mu)$. μ is a (ι,κ)-$f\mathscr{Z}go$ set in X, $\ni x_pq\mu$ and $f(\mu)=f(f^{-1}(\rho))\leq\rho$.

(vi)⇒(i): Let ρ be a (ι,κ)-fo set in Y and $x_p\in f^{-1}(\rho)$. Clearly $f(x_p)\in\rho$. $(x_p)^c=\underline{1}-x_p(x)$. Then $f(\underline{1}-x_p)q\rho$. Hence $\exists$ a (ι,κ)-$f\mathscr{Z}go$ set μ of $X\ni(1-x_p)q\mu$ and $f(\mu)\leq A$. Now $(\underline{1}-x_p)q\mu\Rightarrow(\underline{1}-x_p)(x)+\mu(x)=1-p+\mu(x)>1$ $\Rightarrow\mu(x)>p\Rightarrow x_p\in\mu$. Thus $x_p\in\mu\leq f^{-1}(\rho)$. Hence $f^{-1}(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ in X.

(vi)⇒(vii): Let x_p be a fp of X \$ ρ be (ι,κ)-$f\mathscr{Z}$gq-nbhd of $f(x_p)$. Then there is (ι,κ)-$f\mathscr{Z}go$ set γ in Y $\ni x_pq\gamma$. By hypothesis there is a (ι,κ)-$f\mathscr{Z}go$ set μ of $X\ni x_pq\mu$ and $f(\mu)\leq\gamma$. Thus $x_pq\mu\leq f^{-1}(\gamma)\leq f^{-1}(\rho)$. Hence $f^{-1}(\rho)$ is a (ι,κ)-$f\mathscr{Z}$gq-nbhd of x_p.

(viii)⇒(vi) Let x_p be a fp of X \$ ρ be (ι,κ)-$f\mathscr{Z}go$ in Y $\ni f(x_p)q\rho$. Then ρ is (ι,κ)-$f\mathscr{Z}$gq-nbhd of $f(x_p)$. So there is a (ι,κ)-$f\mathscr{Z}$gq-nbhd γ of $x_p\ni f(\gamma)\leq\rho$. Since γ is a (ι,κ)-$f\mathscr{Z}$gq-nbhd of x_p $\exists$ a (ι,κ)-$f\mathscr{Z}go$ set μ of $X\ni x_pq\mu\leq\gamma$. Hence $x_pq\mu$ and $f(\mu)\leq\rho$. □

Theorem 2.8. *Let $g:(X,\zeta_1,\zeta_1^*)\to(Y,\zeta_2,\zeta_2^*)$ be a bijective function. Then*

(i) g is $pDF\mathscr{Z}gHom$.

(ii) g is $DF\mathscr{Z}gIrr$ and $pDF\mathscr{Z}gO$ map.

(iii) g is $DF\mathscr{Z}gIrr$ and $pDF\mathscr{Z}gC$ map.

are equivalent.

Proof. (i)⇒(ii): Let g be $pDF\mathscr{Z}gHom$. Then g and g^{-1} are $DF\mathscr{Z}gIrr$. It is enough to prove that g is $pDF\mathscr{Z}gO$ map, let ρ be (ι,κ)-$f\mathscr{Z}go$ set in X. Since $g^{-1}:Y\to X$ is $DF\mathscr{Z}Irr$, $(g^{-1})^{-1}(\rho)=g(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Therefore $g(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Hence g is $pDF\mathscr{Z}gO$ map.

(ii)⇒(iii): Let g be $DF\mathscr{Z}gIrr$ and $pDF\mathscr{Z}gO$ map. To prove that g is $pDF\mathscr{Z}gC$ map, let μ be a (ι,κ)-$f\mathscr{Z}gc$ set in X. Then $1-\mu$ is (ι,κ)-$f\mathscr{Z}go$ set in X. Since g is $pDF\mathscr{Z}gO$ map, $g(1-\mu)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Now, $g(1-\mu)=$

$1-g(\mu)$ implies $1-g(\mu)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Therefore $g(\mu)$ is (ι,κ)-$f\mathscr{Z}gc$ in Y. Hence, g is a $pDF\mathscr{Z}gC$ map.

(iii)⇒(i): Let g be $DF\mathscr{Z}gIrr$ and $pDF\mathscr{Z}gC$ map. To prove that g is $pDF\mathscr{Z}gHom$, let ρ be (ι,κ)-$f\mathscr{Z}go$ set in X. Then $1-\rho$ is (ι,κ)-$f\mathscr{Z}gc$ set in X. Since g is $pDF\mathscr{Z}gC$ map, $g(1-\rho)$ is also (ι,κ)-$f\mathscr{Z}gc$ set in Y. Now, $(g^{-1})^{-1}(\underline{1}-\rho)=g(\underline{1}-\rho)=\underline{1}-g(\rho)$ is (ι,κ)-$f\mathscr{Z}gc$ set in Y. Thus $g(\rho)$ is (ι,κ)-$f\mathscr{Z}go$ set in Y. Hence $g^{-1}:Y\rightarrow X$ is $DF\mathscr{Z}Cts$. Since g is $DF\mathscr{Z}Irr$ and by Theorem 2.7, every $DF\mathscr{Z}Irr$ map is $DF\mathscr{Z}Cts$. Thus g is $DF\mathscr{Z}Cts$. Hence g is $DF\mathscr{Z}Hom$. □

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