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Divisor cordial labeling for some cycle and wheel related graphs

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Abstract

Divisor cordial labeling is a variant of cordial labeling. We investigate divisor cordial labeling for Armed Crown, Closed Helm, Web graph and one point union of Cycles.

Keywords

Graph labeling, Cordial labeling, Divisor cordial labeling.

AMS Subject Classification

05C78.

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Contents

1. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$. For all standard terminology and notation we follow Clark and Holton [9]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. *A* graph labeling *is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or edge labeling).*

Labeled graph have applications in many diversified field such as X-Ray crystallography, network design, missile guidance codes etc. A detailed study on verity of applications of graph labeling is reported in Bloom and Golomb [4].

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [8].

In 1987, Cahit [7] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling. Many variants of cordial labeling are also introduced with variation in cordial condition. These labeling are known as equitable labeling.

Definition 1.2. *For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as $f : V(G) \to \{0,1\}$ and induced edge labeling function $f^* : E(G) \to \{0,1\}$ such that for each edge uv, $f^*(uv) = |f(u) - f(v)|$. f is called* cordial labeling *of graph G if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Cordial Labeling is called a* Cordial Graph.

In 2011, R. Varatharajan *et al.* [18] have introduced divisor cordial labeling as follows.

Definition 1.3. *For a graph $G = (V(G), E(G))$, the vertex labeling function is defined as a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ such that an edge uv is assigned the label 1 if one $f(u)$ or $f(v)$ divides the other and 0 otherwise. f is called* Divisor cordial labeling *of graph G if the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Divisor cordial labeling is called a* Divisor cordial graph. *Denote the number of edges labeled with 0 and 1 by $E_f(0)$ and $E_f(1)$ respectively.*

Varatharajan *et al.* [18, 19] have derived many results related to divisor cordial graphs for standard graph families. Vaidya and Shah [20, 21] have investigated divisor cordial labeling for some star related graphs.

Bosamia and Kanani [12, 13] discussed divisor cordial labeling in the context of some graph operations. Raj and Manoharan [14, 15] have discussed divisor cordial labeling for some disconnected graphs while Raj and Valli [16] as well as Maya and Nicholas [17] have obtained divisor cordial labeling for some new graph families. Ghodasara and Adalja [5, 6] have obtained divisor cordial labeling in the context of some graph operations.

Murugan and Devakiruba [3] as well as Rokad and Ghodasara [1] have obtained divisor cordial labeling for some cycle related graphs. Divisor cordial labeling for duplication of graph elements is studied by Thirusangu and Madhu[11]. Devaraj *et. al.*[10] as well as Muthaiyan and Pugalenthi[2] obtained results related to divisor cordial labeling.

Definition 1.4. *A* crown grap*h is cycle with a pendent edge attached at each vertex.*

Definition 1.5. *The* armed crown *is a graph in which path P_2 is attached at each vertex of cycle C_n by an edge. It is denoted by AC_n where n is the number of vertices of cycle C_n.*

Definition 1.6. *The* helm graph H_n *is the graph obtained from a wheel W_n by attaching a pendent edge at each vertex of the cycle.*

Definition 1.7. *A* closed helm *is the graph obtained from a helm by joining each pendent vertex to form a cycle. It is denoted by CH_n.*

Definition 1.8. *A* web graph Wb_n *is the graph obtained by joining the pendent vertices of a helm to form a cycle and then adding a single pendent edge to each vertex of this outer cycle.*

Definition 1.9. *A* One Point Union of Cycles *is consists of t copies of cycle C_n sharing a common vertex. It is denoted by $C_n^{(t)}$.*

In the present paper we have investigated divisor cordial labeling for armed crown, closed helm, web graph and one point union of cycles.

2. Main Results

Theorem 2.1. *The armed crown AC_n is a divisor cordial graph.*

Proof. Consider the graph AC_n with the vertex set $V(AC_n)$ and an edge set $E(AC_n)$ then $|V(AC_n)| = 3n$ and $|E(AC_n)| = 3n$.

We define the divisor cordial labeling $f : V(AC_n) \to \{1, 2, \dots, 3n\}$ as follows:

$f(v_{3n-2}) = 1$,

$f(v_i) = 2^i$;
for $1 \le i \le p_1$ such that $2^i \le 3n$,

Let $p_1 = 3k_1 + r_1; 0 \le r_1 \le 2$;
$f(v_{i+3(k_1+\lfloor r_1/2 \rfloor)}) = 3 \times 2^{i-1}$;
for $1 \le i \le p_2$ such that $3 \times 2^{i-1} \le 3n$,

Let $p_2 = 3k_2 + r_2; 0 \le r_2 \le 2$;
$f(v_{i+3(k_1+\lfloor r_1/2 \rfloor)+3(k_2+\lfloor r_2/2 \rfloor)}) = 5 \times 2^{i-1}$;
for $1 \le i \le p_3$ such that $5 \times 2^{i-1} \le 3n$,

Let $p_3 = 3k_3 + r_3; 0 \le r_3 \le 2$;
$f(v_{i+3(k_1+\lfloor r_1/2 \rfloor)+3(k_2+\lfloor r_2/2 \rfloor)+3(k_3+\lfloor r_3/2 \rfloor)}) = 7 \times 2^{i-1}$;
for $1 \le i \le p_4$ such that $7 \times 2^{i-1} \le 3n$.

Continuing in this way till we get $\lfloor 3n/2 \rfloor$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In view of above defined labeling pattern we have $E_f(0) = \lceil 3n/2 \rceil$, $E_f(1) = \lfloor 3n/2 \rfloor$. Thus $|E_f(0) - E_f(1)| \le 1$.

Hence the graph armed crown AC_n is a divisor cordial graph. □

Example 2.2. *The armed crown AC_5 and its divisor cordial labeling is shown in Figure 1.*

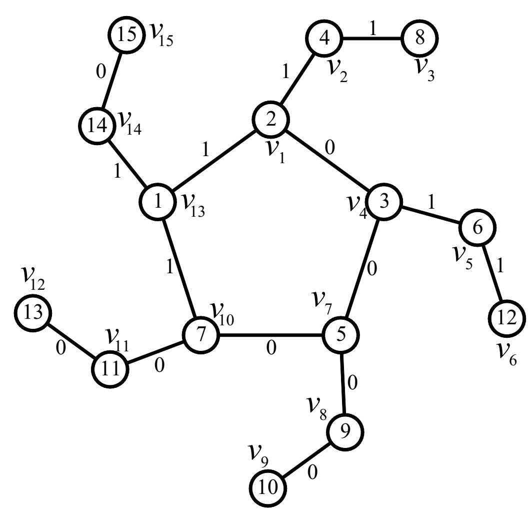

Figure 1: Armed Crown AC_5 and its divisor cordial labeling.

Theorem 2.3. *The Closed Helm CH_n is a divisor cordial graph.*

Proof. Consider the graph CH_n with the vertex set $V(CH_n)$ and an edge set $E(CH_n)$ then $|V(CH_n)| = 2n+1$ and $|E(CH_n)| = 4n$.

We define the divisor cordial labeling $f : V(CH_n) \to \{1, 2, \dots, 2n+1\}$ in following two cases.

Case 1: For $n < 8$.

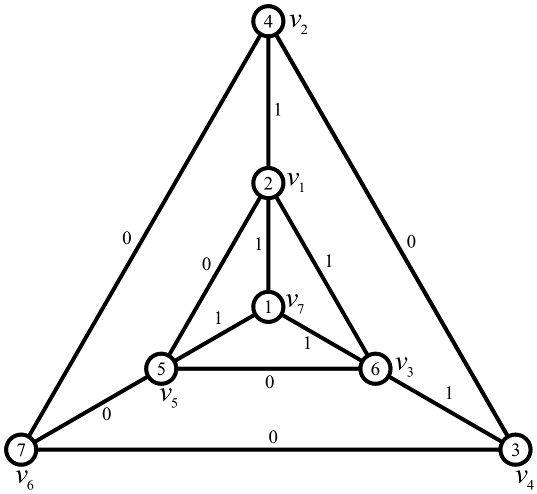

Figure 2: Closed helm CH_3 and its divisor cordial labeling.

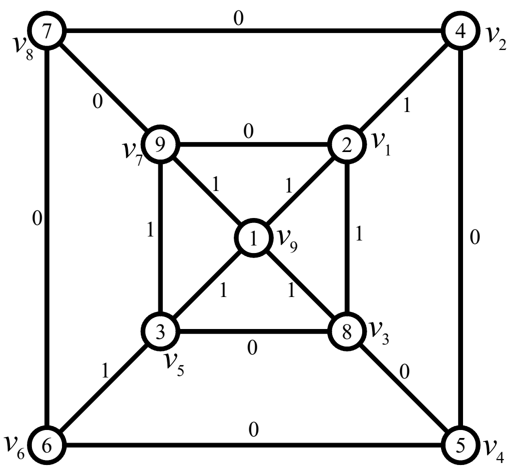

Figure 3: Closed helm CH_4 and its divisor cordial labeling.

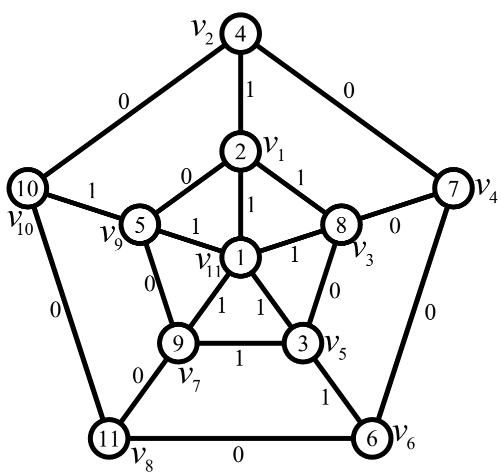

Figure 4: Closed helm CH_5 and its divisor cordial labeling.

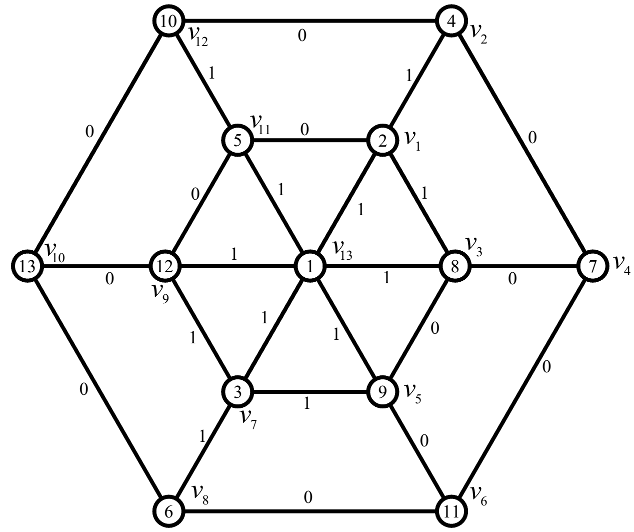

Figure 5: Closed helm CH_6 and its divisor cordial labeling.

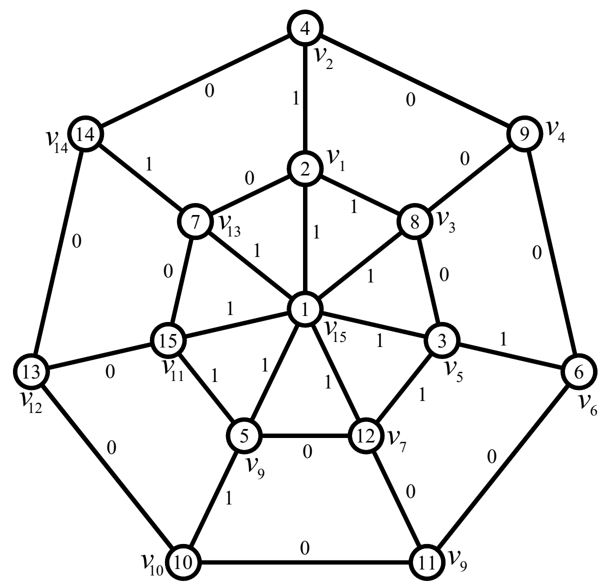

Figure 6: Closed helm CH_7 and its divisor cordial labeling.

Case 2: For $n \geq 8$.
$f(v_{2n+1}) = 1$,

$f(v_i) = 2^i$;
for $1 \leq i \leq p_1$ such that $2^i \leq 2n+1$,

Let $p_1 = 2k_1 + r_1; 0 \leq r_1 \leq 1$;
$f(v_{i+2(k_1+r_1)}) = 3 \times 2^{i-1}$;
for $1 \leq i \leq p_2$ such that $3 \times 2^{i-1} \leq 2n+1$,

Let $p_2 = 2k_2 + r_2; 0 \leq r_2 \leq 1$;
$f(v_{i+2(k_1+r_1)+2(k_2+r_2)}) = 5 \times 2^{i-1}$;
for $1 \leq i \leq p_3$ such that $5 \times 2^{i-1} \leq 2n+1$,

Let $p_3 = 2k_3 + r_3; 0 \leq r_3 \leq 1$;

$f(v_{i+2(k_1+r_1)+2(k_2+r_2)+2(k_3+r_3)}) = 7 \times 2^{i-1}$;
for $1 \leq i \leq p_4$ such that $7 \times 2^{i-1} \leq 2n+1$.

Continuing in this way till we get $2n$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In the view of above defined labeling pattern we have $E_f(0) = 2n$, $E_f(1) = 2n$. Thus $|E_f(0) - E_f(1)| \leq 1$.

Hence, the graph closed helm CH_n is a divisor cordial graph. □

Example 2.4. *The Closed Helm CH_8 and its divisor cordial labeling is shown in Figure 7.*

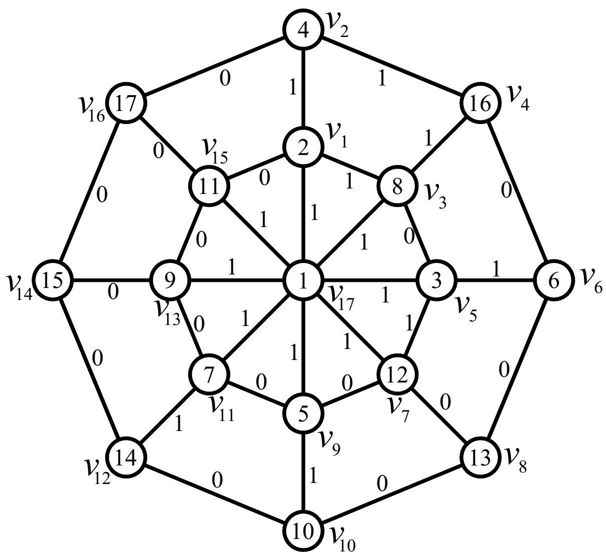

Figure 7: Closed helm CH_8 and its divisor cordial labeling.

Theorem 2.5. *The Web graph Wb_n is a divisor cordial graph.*

Proof. Consider the graph Wb_n with the vertex set $V(Wb_n)$ and an edge set $E(Wb_n)$ then $|V(Wb_n)| = 3n+1$ and $|E(Wb_n)| = 5n$.

We define the divisor cordial labeling $f : V(Wb_n) \to \{1, 2, \ldots, 3n+1\}$ in following two cases.

Case 1: For $n = 4, 6, 8, 10$.

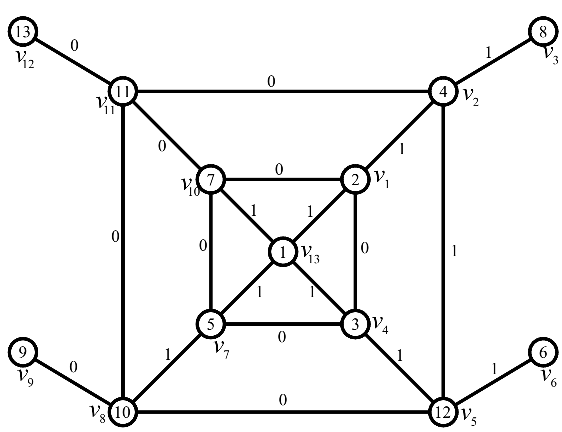

Figure 8: Web graph Wb_4 and its divisor cordial labeling.

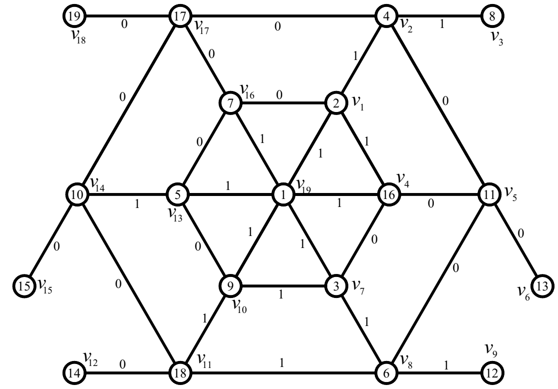

Figure 9: Web graph Wb_6 and its divisor cordial labeling.

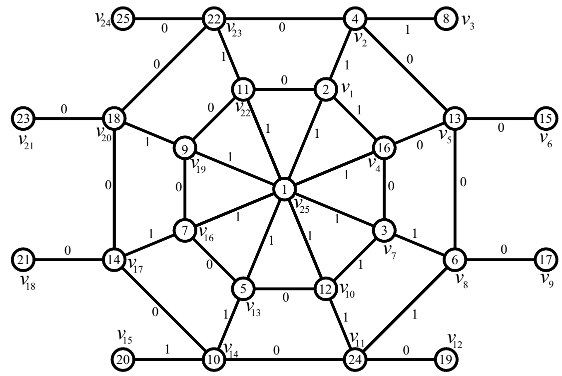

Figure 10: Web graph Wb_8 and its divisor cordial labeling.

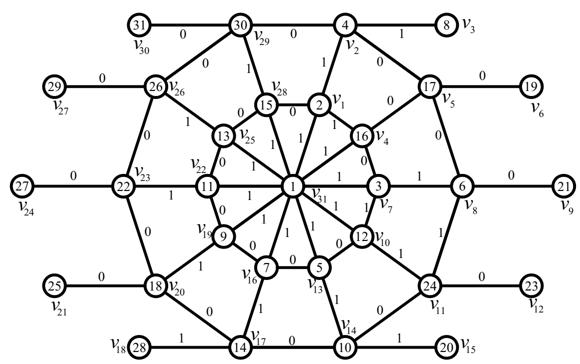

Figure 11: Web graph Wb_{10} and its divisor cordial labeling.

Case 2: For $n \neq 4, 6, 8, 10$.
$f(v_{3n+1}) = 1$,

$f(v_i) = 2^i$;
for $1 \leq i \leq p_1$ such that $2^i \leq 3n+1$,

Let $p_1 = 3k_1 + r_1; 0 \leq r_1 \leq 2$;
$f(v_{i+3(k_1+\lfloor r_1/2 \rfloor)}) = 3 \times 2^{i-1}$;
for $1 \leq i \leq p_2$ such that $3 \times 2^{i-1} \leq 3n+1$,

Let $p_2 = 3k_2 + r_2; 0 \leq r_2 \leq 2$;

$f(v_{i+3(k_1+\lfloor r_1/2\rfloor)+3(k_2+\lfloor r_2/2\rfloor)}) = 5\times 2^{i-1}$;
for $1 \le i \le p_3$ such that $5\times 2^{i-1} \le 3n+1$,

Let $p_3 = 3k_3 + r_3; 0 \le r_3 \le 2$;
$f(v_{i+3(k_1+\lfloor r_1/2\rfloor)+3(k_2+\lfloor r_2/2\rfloor)+3(k_3+\lfloor r_3/2\rfloor)}) = 7\times 2^{i-1}$;
for $1 \le i \le p_4$ such that $7\times 2^{i-1} \le 3n+1$.

Continuing in this way till we get $\lfloor 5n/2\rfloor$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In the view of the above defined labeling pattern we have $E_f(0) = \lceil 5n/2\rceil$, $E_f(1) = \lfloor 5n/2\rfloor$. Thus $|E_f(0) - E_f(1)| \le 1$.

Hence, the web graph Wb_n is a divisor cordial graph. □

Example 2.6. *The Web Graph Wb_5 and its divisor cordial labeling is shown in Figure 12.*

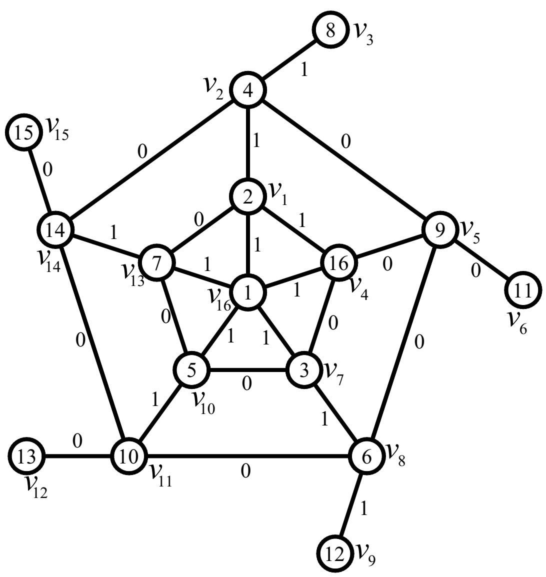

Figure 12: Web graph Wb_5 and its divisor cordial labeling.

Theorem 2.7. *The graph $C_3^{(t)}$ is a divisor cordial graph.*

Proof. Consider the graph $C_3^{(t)}$ with the vertex set $V(C_3^{(t)})$ and an edge set $E(C_3^{(t)})$ then $|V(C_3^{(t)})| = 2t+1$ and $|E(C_3^{(t)})| = 3t$.

We define the divisor cordial labeling $f : V(C_3^{(t)}) \to \{1, 2, \ldots, 2t+1\}$ as follows:

$f(v_1) = 1$,
$f(v_{2t+1}) = 2$,
$f(v_{2i}) = 2i$; for $2 \le i \le t$,
$f(v_{4i+1}) = 2i+1$; for $i \ge 1$ such that $2i+1 \le 2t+1$.

Continuing in this way till we get $\lfloor 3t/2\rfloor$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In the view of above defined labeling pattern we have $E_f(0) = \lceil 3t/2\rceil$, $E_f(1) = \lfloor 3t/2\rfloor$. Thus $|E_f(0) - E_f(1)| \le 1$.

Hence, The graph $C_3^{(t)}$ is a divisor cordial graph. □

Example 2.8. *The graph $C_3^{(5)}$ and its divisor cordial labeling is shown in Figure 13.*

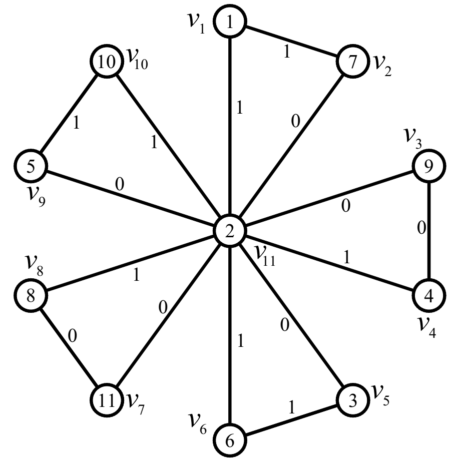

Figure 13: The graph $C_3^{(5)}$ and its divisor cordial labeling.

Theorem 2.9. *The graph $C_4^{(t)}$ is a divisor cordial graph.*

Proof. Consider the graph $C_4^{(t)}$ with the vertex set $V(C_4^{(t)})$ and an edge set $E(C_4^{(t)})$ then $|V(C_4^{(t)})| = 3t+1$ and $|E(C_4^{(t)})| = 4t$. We define the divisor cordial labeling $f : V(C_4^{(t)}) \to \{1, 2, \ldots, 3t+1\}$ as follows:

$f(v_{3t+1}) = 1$

In this way we get $2t$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In view of above define labeling pattern we have $E_f(0) = 2t$, $E_f(1) = 2t$. Thus $|E_f(0) - E_f(1)| \le 1$.

Hence, The graph $C_4^{(t)}$ is a divisor cordial graph. □

Example 2.10. *The graph $C_4^{(4)}$ and its divisor cordial labeling is shown in Figure 14.*

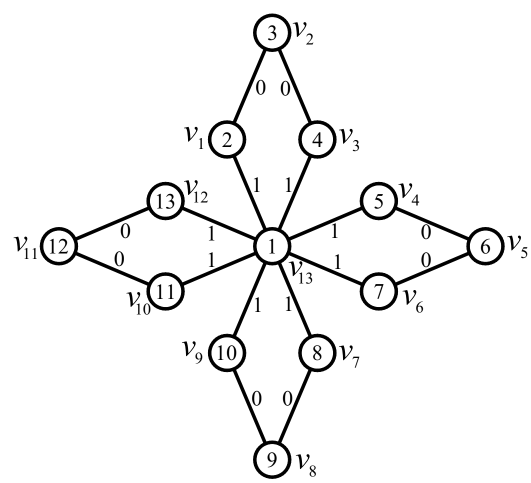

Figure 14:The graph $C_4^{(4)}$ and its divisor cordial labeling.

Theorem 2.11. *The graph $C_n^{(t)}$ is a divisor cordial graph for $n \ge 5$.*

Proof. Consider the graph $C_n^{(t)}$ with the vertex set $V(C_n^{(t)})$ and an edge set $E(C_n^{(t)})$ then $|V(C_n^{(t)})| = (n-1)t+1$ and $|E(C_n^{(t)})| = nt$.

We define the divisor cordial labeling $f : V(C_n^{(t)}) \to \{1,2,\dots,(n-1)t+1\}$ in following two cases:

Case 1: For $5 \le n \le 9$

$f(v_{(n-1)t+1}) = 1$,

$f(v_i) = 2^i$;

for $1 \le i \le p_1$ such that $2^i \le (n-1)t+1$,

Let $p_1 = (n-1)k_1 + r_1; 0 \le r_1 \le (n-2)$;

$f(v_{i+(n-1)(k_1+\lfloor r_1/(n-2)\rfloor)}) = 3\times 2^{i-1}$;

for $1 \le i \le p_2$ such that $3\times 2^{i-1} \le (n-1)t+1$,

Let $p_2 = (n-1)k_2 + r_2; 0 \le r_2 \le (n-2)$;

$f(v_{i+(n-1)(k_1+\lfloor r_1/(n-2)\rfloor)+(n-1)(k_2+\lfloor r_2/(n-2)\rfloor)}) = 5\times 2^{i-1}$;

for $1 \le i \le p_3$ such that $5\times 2^{i-1} \le (n-1)t+1$,

Continuing in this way till we get $\lfloor nt/2\rfloor$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

Case 2: For $n \ge 10$

$f(v_i) = 2^{i-1}$;

for $1 \le i \le p_1$ such that $2^{i-1} \le (n-1)t+1$,

$f(v_{i+p_1}) = 3\times 2^{i-1}$;

for $1 \le i \le p_2$ such that $3\times 2^{i-1} \le (n-1)t+1$,

$f(v_{i+p_1+p_2}) = 5\times 2^{i-1}$;

for $1 \le i \le p_3$ such that $5\times 2^{i-1} \le (n-1)t+1$,

$f(v_{i+p_1+p_2+p_3}) = 7\times 2^{i-1}$;

for $1 \le i \le p_4$ such that $7\times 2^{i-1} \le (n-1)t+1$.

Continuing in this way till we get $\lfloor nt/2\rfloor$ edges with label 1. Now for remaining vertices label them in such a way that it does not divide the label of adjacent vertices.

In view of above defined labeling pattern we have $E_f(0) = \lceil nt/2\rceil$, $E_f(1) = \lfloor nt/2\rfloor$. Thus $|E_f(0) - E_f(1)| \le 1$.

Hence, The graph $C_n^{(t)}$ is a divisor cordial graph. □

Example 2.12. *The graph $C_7^{(3)}$ and its divisor cordial labeling is shown in Figure 15.*

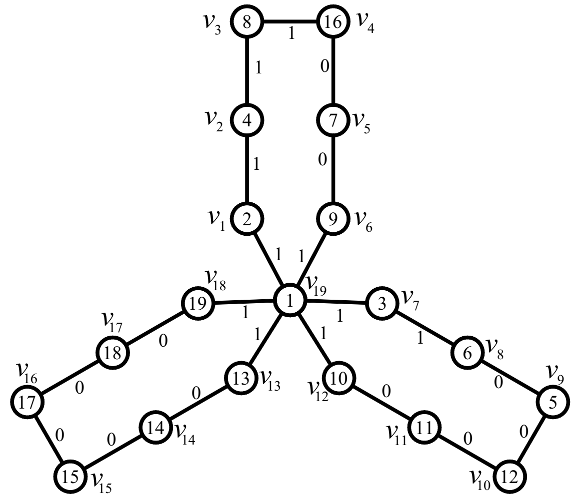

Figure 15: The graph $C_7^{(3)}$ and its divisor cordial labeling.

3. Conclusion

In this paper we have investigated divisor cordial labeling of Armed Crown, Closed Helm, Web Graph and One Point Union of Cycle. To investigate analogous results for different graphs as well as in the context of various graph labeling problems is an open area of research.

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