Malaya Journal of Matematik, Vol. 8, No. 3, 977-981, 2020

https://doi.org/10.26637/MJM0803/0043

On a class of b-γ-open sets in a topological space

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Abstract

In this paper, we analyze the properties of b-γ-open sets in a topological space. Further, the concept of b-γ-boundary, b-γ-exterior, b-γ-limit point, b-γ-neighborhood, locally b-γ-closed and b-γ-generalized closed sets are introduced and investigated.

Keywords

b-γ-open sets, b-γ-boundary, b-γ-exterior, b-γ-limit point, b-γ-neighborhood, b-γ-generalized closed.

AMS Subject Classification

54A05, 54A10.

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Article History: Received **09** March **2020**; Accepted **21** June **2020**

Contents

1. Introduction

Kasahara [2] introduced the notion of an operation γ in 1979. The notion of γ-open sets were introduced and investigated by Ogata [4] in 1991. Ibrahim [3] introduced the concept of b-γ-open set by using the operation γ. Further, he continued studying the weak forms of γ-open sets in his work. Andrijevic [1] introduced the notion of b-open sets in 1996. In [5], Sivashanmugaraja and Vadivel introduced the notion of fuzzy b-γ-open sets. The aim of this paper is to analyze some properties of b-γ-open sets in a topological space. Further the concepts of b-γ-boundary, b-γ-exterior, b-γ-limit point, b-γ-neighborhood, b-γ-generalized closed set and locally b-γ-closed spaces are introduced. Also, the relationship among these sets are discussed.

2. Preliminaries

Throughout this paper, $(X,\ \tau)$ or X always mean topological space.

Definition 2.1. *[4] Let (X,τ) be a space and γ be an operation on τ. $A\subseteq X$ is called γ-open if $\forall\ x\in A$, $\exists$ an open set U such that $x\in U$ and $\gamma(U)\subseteq A$. Then the collection of all γ-open sets in X are denoted by τ_γ. Evidently $\tau_\gamma\subseteq\tau$. A subset A of X is called γ-closed $\Leftrightarrow$ its complement is γ-open.*

Definition 2.2. *[4] Let (X,τ) be a space and γ be an operation on τ. Then X is said to be γ-regular, if $\forall\ x\in X$ and $\forall$ open neighborhood V of x, $\exists$ an open neighborhood U of x, such that $\gamma(U)\subseteq V$. A space X is γ-regular space $\Leftrightarrow \tau=\tau_\gamma$.*

Definition 2.3. *[3] Let (X,τ) be a space. $A\subseteq X$ is said to be b-γ-open if $A\subseteq \tau_\gamma\text{-}int(cl(A))\cup cl(\tau_\gamma\text{-}int(A))$.*

Definition 2.4. *[1] Let (X,τ) be a space. $A\subseteq X$ is said to be b-open if $A\subseteq int(cl(A))\cup cl(int(A))$.*

Definition 2.5. *[3] Let (X,τ) be a space with an operation γ on the topology τ. Then the intersection of two b-γ-open sets may not be b-γ-open.*

Definition 2.6. *[3] Let (X,τ) be a space with an operation γ on the topology τ. Then if $\{A_i : i\in\Delta\}$ is a collection of b-γ-open sets of a space (X,τ), then $\cup_{i\in\Delta}A_i$ is a b-γ-open set.*

3. b-γ-open and b-γ-closed sets

Remark 3.1. *Let (X,τ) be a space and B is a subset of X. Then B is said to be b-γ-closed $\Leftrightarrow B^c$ is b-γ-open.*

Further, the set of all b-γ-open sets and b-γ-closed sets of (X,τ) are denoted by b-$\gamma O(X)$ and b-$\gamma C(X)$ respectively.

Definition 3.2. *Let $(X,\ \tau)$ be a space and $A \subseteq X$. Then the b-γ-closure of A (briefly, $bcl_\gamma(A)$) is given by $bcl_\gamma(A) = \bigcap\{B : A \subseteq B$ and $B \in b\text{-}\gamma C(X)\}$.*

Definition 3.3. *Let $(X,\ \tau)$ be a space and $A \subseteq X$. Then the b-γ-interior of A (briefly, $bint_\gamma(A)$) is given by $bint_\gamma(A) = \bigcup\{B : A \supseteq B$ and $B \in b\text{-}\gamma O(X)\}$.*

Theorem 3.4. *Let (X,τ) be a space with an operation γ on the topology τ. Then the below statements hold:*

(i) Each γ-open set of (X,τ) is b-γ-open set in (X,τ);

(ii) Each b-γ-open set of (X,τ) is b-open set in (X,τ).

Proof. (i) Let B is a γ-open set. Then $B = \tau_\gamma\text{-}int(B)$. Since, $B \subseteq cl(B), B \subseteq cl(\tau_\gamma\text{-}int(B)) \subseteq cl(\tau_\gamma\text{-}int(B)) \cup \tau_\gamma\text{-}int(cl(B))$. Therefore, B is b-γ-open.

(ii) Evident. □

Remark 3.5. *The converse of the above Theorem 3.4 may not be true as shown in the below examples.*

Example 3.6. *Let $X = \{a,b,c\}$ and $\tau_X = \{X, \phi, \{a,c\}\}$. Define an operation γ on τ_X by $\gamma(B) = B$. Here, the set $\{b,c\}$ is not γ-open but it is b-γ-open.*

Example 3.7. *Let $X = \{a,b,c\}$ and $\sigma = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$. Define an operation γ on σ by*

$$\gamma(B) = \begin{cases} B, & \text{if } B = \{b\} \\ X, & \text{if } B \neq \{b\}. \end{cases} \tag{3.1}$$

Then the set $\{a\}$ is b-open but not b-γ-open.

Remark 3.8. *The notion of b-open and b-γ-open sets are independent. A space X is γ-regular space $\Leftrightarrow$ The sets b-open and b-γ-open are equal.*

Definition 3.9. *In the above Example 3.7, the set of all γ-open sets $\tau_\gamma = \{X, \phi, \{b\}\}$. Here, b-$\gamma$-open and b-open sets are not equal. Again, suppose we define γ on τ by $\gamma(B) = B$, then the sets b-γ-open and b-open are equal.*

Proposition 3.10. *Let B and C be two subsets of a space (X,τ) with an operation γ on the topology τ. Then the below statements hold:*

(i) $bcl_\gamma(\emptyset) = \emptyset$ and $bcl_\gamma(X) = X$;

(ii) B is a b-γ-closed $\Leftrightarrow bcl_\gamma(B) = B$;

(iii) $bcl_\gamma(B)$ is a b-γ-closed set of (X,τ) and $B \subseteq bcl_\gamma(B)$;

(iv) If $B \subset C$, then $bcl_\gamma(B) \subset bcl_\gamma(C)$;

(v) $bcl_\gamma(B) \cup bcl_\gamma(C) \subset bcl_\gamma(B \cup C)$;

(vi) $bcl_\gamma(B \cap C) \subset bcl_\gamma(B) \cap bcl_\gamma(C)$.

Proof. Evident. □

Proposition 3.11. *Let B and C be two subsets of a space (X,τ) with an operation γ on the topology τ. Then the below statements hold:*

(i) $bint_\gamma(\emptyset) = \emptyset$ and $bint_\gamma(X) = X$;

(ii) B is a b-γ-open $\Leftrightarrow bint_\gamma(B) = B$;

(iii) $bint_\gamma(B)$ is a b-γ-open set of (X,τ) and $bint_\gamma(B) \subseteq B$;

(iv) If $B \subset C$, then $bint_\gamma(B) \subset bint_\gamma(C)$;

(v) $bint_\gamma(B) \cup bint_\gamma(C) \subset bint_\gamma(B \cup C)$;

(vi) $bint_\gamma(B \cap C) \subset bint_\gamma(B) \cap bint_\gamma(C)$.

Proof. Evident. □

Proposition 3.12. *Let $B \subseteq X$ with an operation γ on the topology τ. Then the below statements holds:*

(i) $int_\gamma(B) \subseteq int(B) \subseteq bint(B) \subseteq B \subseteq bcl(B) \subseteq cl(B) \subseteq cl_\gamma(B)$;

(ii) $int_\gamma(B) \subseteq bint_\gamma(B) \subseteq bint(B) \subseteq B \subseteq bcl(B) \subseteq bcl_\gamma(B) \subseteq cl_\gamma(B)$.

Proposition 3.13. *Let $B \subseteq X$ with an operation γ on the topology τ. Then the below statements are equivalent:*

(i) B is a b-γ-open set in (X,τ);

(ii) $X \setminus B$ is a b-γ-closed set in (X,τ);

(iii) $bcl_\gamma(X \setminus B) = X \setminus B$.

Proof. Evident. □

Proposition 3.14. *Let $B \subseteq X$ with an operation γ on the topology τ. Then the below statements are equivalent:*

(i) B is a b-γ-closed set in (X,τ);

(ii) $X \setminus B$ is a b-γ-open set in (X,τ);

(iii) $bint_\gamma(X \setminus B) = X \setminus B$;

Proof. Evident. □

4. b-γ-boundary and b-γ-exterior

Definition 4.1. *Let C be a subset of a space (X,τ). Then the b-γ-boundary of C (briefly, b-$\gamma bd(C)$) is given by $b\text{-}\gamma bd(C) = bcl_\gamma(C) \cap bcl_\gamma(X \setminus C)$.*

Theorem 4.2. *Let (X,τ) be a space and $B \subseteq X$. Then the below statements are hold:*

(1) $b\text{-}\gamma bd(B) = b\text{-}\gamma bd(X \setminus B)$;

(2) $b\text{-}\gamma bd(B) = bcl_\gamma(B) \setminus bint_\gamma(B)$;

(3) $b\text{-}\gamma bd(B) \cap bint_\gamma(B) = \phi$;

(4) $b\text{-}\gamma bd(B) \cup bint_\gamma(B) = bcl_\gamma(A)$.

Proof. (1) Evident from Definition 4.1

(2) By definition, b-γbd$(B) = bcl_\gamma(B) \cap bcl_\gamma(X \setminus B) = bcl_\gamma(B) \cap [X \setminus bint_\gamma(B)] = [bcl_\gamma(B) \cap X] \setminus [bcl_\gamma(B) \cap bint_\gamma(B)] = bcl_\gamma(B) \setminus bint_\gamma(B)$.

(3) Also, by using (2), b-γbd$(B) \cap bint_\gamma(B) = [bcl_\gamma(B) \setminus bint_\gamma(B)] \cap bint_\gamma(B) = [bcl_\gamma(B) \cap bint_\gamma(B)] \setminus bint_\gamma(B) = bint_\gamma(B) \setminus bint_\gamma(B) = \phi$.

(4) By using (3), b-γbd$(B) \cup bint_\gamma(B) = [bcl_\gamma(B) \setminus bint_\gamma(B)] \cup bint_\gamma(B) = bcl_\gamma(B)$. □

Theorem 4.3. *Let (X,τ) be a space and $B \subseteq X$. Then the below statements are hold:*

(i) The set B is a b-γ-open $\Leftrightarrow B \cap b$-$\gamma bd(B) = \phi$;

(ii) The set B is a b-γ-closed $\Leftrightarrow b$-$\gamma bd(B) \subset B$;

(iii) The set B is a b-γ-clopen $\Leftrightarrow b$-$\gamma bd(B) = \phi$.

Proof. (i) Suppose that B be a b-γ-open set. Then $B = bint_\gamma(B)$, Thus $B \cap b$-γbd$(B) = bint_\gamma(B) \cap b$-$\gamma$bd$(B) = \phi$. Conversely, let $B \cap b$-γbd$(B) = \phi$. Then by Theorem 4.2, $B \cap [bcl_\gamma(B) \setminus bint_\gamma(B)] = [B \cap bcl_\gamma(B)] \setminus [B \cap bint_\gamma(B)] = B \setminus bint_\gamma(B) = \phi$. So, $B = bint_\gamma(B)$ and hence B is b-γ-open.

(ii) Suppose that B be a b-γ-closed set. Then $B = bcl_\gamma(B)$. But b-γbd(B)= $bcl_\gamma(B) \setminus bint_\gamma(B) = B \setminus bint_\gamma(B)$. Therefore b-γbd$(B) \subset B$. Conversely, consider b-γbd$(B) \subset B$. By Theorem 4.2, $bcl_\gamma(B) = b$-γbd$(B) \cup bint_\gamma(B) \subset B \cup bint_\gamma(B) = B$. Therefore $bcl_\gamma(B) \subset B$ and $B \subset bcl_\gamma(B)$. Hence, $B = bcl_\gamma(B)$. Thus B is b-γ-closed.

(iii) Suppose that B be a b-γ-clopen set. Then $B = bint_\gamma(B)$ and also $B = bcl_\gamma(B)$. Then by Theorem 4.2, b-γbd$(B) = bcl_\gamma(B) \setminus bint_\gamma(B) = B \setminus B = \phi$. Conversely, assume that b-γbd$(B) = \phi$. Then b-γbd$(B) = bcl_\gamma(B) \setminus bint_\gamma(B) = \phi$ and hence, B is b-γ-clopen. □

Definition 4.4. *Let (X,τ) be a space and B be a subset of a space X. Then the set $X \setminus bcl_\gamma(B)$ is said to be b-γ-exterior of B and is denoted by b-$\gamma ext(B)$. Every point $x \in X$ is said to be a b-γ-exterior point of B, if it is a b-γ-interior point of $X \setminus B$.*

Definition 4.5. *Let (X,τ) be a space and N be a subset of a space X. N is said to be a b-γ-neighborhood of a point $x \in X$ if $\exists$ a b-γ-open set P such that $x \in P \subseteq N$.*

The class of all b-γ-nbds of $x \in X$ is called the b-γ-neighborhood system of x and it is denoted by b-γ-N_x.

Theorem 4.6. *Let B and C are two subsets of a space (X,τ). Then the below statements are hold:*

(i) b-$\gamma ext(\phi) = X$ and b-$\gamma ext(X) = \phi$;

(ii) b-$\gamma ext(B) = bint_\gamma(X \setminus B)$;

(iii) b-$\gamma ext(B) \cap b$-$\gamma bd(B) = \phi$;

(iv) b-$\gamma ext(B) \cup b$-$\gamma bd(B) = bcl_\gamma(X \setminus B)$;

(v) $\{bint_\gamma(B)$, b-$\gamma bd(B)$ and b-$\gamma ext(B)\}$ form a partition of X;

(vi) If $B \subset C$, then b-$\gamma ext(C) \subset b$-$\gamma ext(B)$;

(vii) b-$\gamma ext(B \cup C) \subset b$-$\gamma ext(B) \cup b$-$\gamma ext(C)$;

(viii) b-$\gamma ext(B \cap C) \supset b$-$\gamma ext(B) \cap b$-$\gamma ext(C)$.

Proof. (i) Evident.
(ii) Evident from Definition 4.4

(iii) From statement (ii) and Theorem 4.2, we have b-$\gamma ext(B) \cap b$-γbd$(B) = bint_\gamma(X \setminus B) \cap b$-$\gamma$bd$(X \setminus B) = \phi$.

(iv) Also, From statement (ii) and Theorem 4.2, we have b-$\gamma ext(B) \cup b$-γbd$(B) = bint_\gamma(X \setminus B) \cup b$-$\gamma$bd$(X \setminus B) = bcl_\gamma(X \setminus B)$.

(v) and (vi) Evident.

(vii) By definition, b-$\gamma ext(B \cup C) = X \setminus bcl_\gamma(B \cup C) \subset X \setminus [bcl_\gamma(B) \cup bcl_\gamma(C)] = [X \setminus bcl_\gamma(B)] \cap [X \setminus bcl_\gamma(C)] = b$-$\gamma ext(B) \cap b$-$\gamma ext(C) \subset b$-$\gamma ext(B) \cup b$-$\gamma ext(C)$.

(viii) Also by definition, b-$\gamma ext(B \cap C) = X \setminus bcl_\gamma(B \cap C) \supset X \setminus [bcl_\gamma(B) \cap bcl_\gamma(C)] = [X \setminus bcl_\gamma(B)] \cup [X \setminus bcl_\gamma(C)] = b$-$\gamma ext(B) \cup b$-$\gamma ext(C) \supset b$-$\gamma ext(B) \cap b$-$\gamma ext(C)$. □

Remark 4.7. *In the above Theorem 4.6, the inclusion relation of the statement (vi), (vii) cannot be replaced by equality as shown in the below example.*

Example 4.8. *Let $X = \{a,b,c\}$ with topology $\tau_X = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$. Define an operation γ on τ_X by*

$$\gamma(B) = \begin{cases} int(cl(B)), & \text{if } a \in B \\ cl(B), & \text{if } a \notin B. \end{cases} \tag{4.1}$$

Let $C = \{a,b\}$ and $D = \{b,c\}$. Then b-$\gamma ext(C) = \phi$ and b-$\gamma ext(D) = \{a\}$. But b-$\gamma ext(C \cup D) = \phi$, So, b-$\gamma ext(C) \cup b$-$\gamma ext(D) \not\subset b$-$\gamma ext(C \cup D)$. Also, b-$\gamma ext(C \cap D) = \{a\}$. Therefore, b-$\gamma ext(C \cap D) \not\subset b$-$\gamma ext(C) \cap b$-$\gamma ext(D)$.

Definition 4.9. *Let X be a space and $B \subseteq X$. Then a point $x \in X$ is said to be a b-γ-limit point of a set $B \subset X$ if every b-γ-open set $P \subset X$ containing x contains a point of B other than x.*

The collection of all b-γ-limit points of B is said to be a b-γ-derived set of B and it is mentioned by b-$\gamma Ds(B)$.

Proposition 4.10. *Let B be a subset of a space (X,τ). Then, the below statements are hold:*

(i) The set B is b-γ-closed $\Leftrightarrow b$-$\gamma Ds(B) \subset B$;

(ii) The set B is b-γ-open $\Leftrightarrow B$ is b-γ-neighborhood, $\forall$ point $x \in B$;

(iii) $bcl_\gamma(B) = B \cup b$-$\gamma Ds(B)$.

Proof. (i) Let B be a b-γ-closed set and $x \in B$. Then $x \in X \setminus B$, which is open. Thus $\exists$ a b-γ-open set $(X \setminus B)$ such that $(X \setminus B) \cap B = \phi$. Therefore $x \notin b$-$\gamma Ds(B)$. Thus, b-$\gamma Ds(B) \subset B$.

Conversely, assume that b-$\gamma Ds(B) \subset B$ and $x \notin B$. Then $x \notin b$-$\gamma Ds(B)$. Thus $\exists$ a b-γ-open set V containing x such that

$V \cap B = \phi$. Therefore $X \setminus B = \bigcup_{x \in B} \{K, K$ is b-γ-open $\}$. Hence, B is b-γ-closed.

(ii) Let B be a b-γ-open set. Then B is a b-γ-neighborhood, $\forall\, x \in B$.

Conversely, let B be a b-γ-neighborhood, $\forall\, x \in G$. Then $\exists$ a b-γ-open set V_x containing x such that $x \in V_x \subseteq B$. Therefore $B = \bigcup_{x \in G} V_x$. Thus, B is a b-γ-open.

(iii) Since, b-$\gamma Ds(B) \subset bcl_\gamma(B)$ and $B \subset bcl_\gamma(B)$, $B \cup b$-$\gamma Ds(B) \subset bcl_\gamma(B)$.

Conversely, assume that $x \notin b$-$\gamma Ds(B) \cup B$. Then $x \notin b$-$\gamma Ds(B)$, $x \notin B$. Then $\exists$ a b-γ-open set V containing x such that $V \cap B = \phi$. Therefore $x \notin bcl_\gamma(B)$ which implies that $bcl_\gamma(B) \subset B \cup b$-$\gamma Ds(B)$. Thus, $bcl_\gamma(B) = B \cup b$-$\gamma Ds(B)$. □

Theorem 4.11. *Let B and C be two subsets of a space (X, τ). Then the below statements are hold:*

(i) *If $B \subset C$, then b-$\gamma Ds(B) \subset b$-$\gamma Ds(C)$.*

(ii) *B is a b-γ-closed set $\Leftrightarrow$ B contains each of its b-γ-limit points.*

(iii) $bcl_\gamma(B) = B \cup b$-$\gamma Ds(B)$.

Proof. (i) Evident.

(ii) If B be a b-γ-closed set, then $X \setminus B$ is b-γ-open. If $x \notin B$, then $x \in X \setminus B$. Then $\exists$ a b-γ-open $(X \setminus B)$ such that $(X \setminus B) \cap B = \phi$. Therefore $x \notin b$-$\gamma Ds(B)$. Hence, b-$\gamma Ds(B) \subset B$.

Conversely, assume that b-$\gamma Ds(B) \subset B$ and $x \notin B$. Then $x \notin b$-$\gamma Ds(B)$. Then $\exists$ a b-γ-open set M containing x such that $M \cap B = \phi$ and therefore

$$X \setminus B = \bigcup_{x \in B} \{M,\ M \text{ is } b\text{-}\gamma\text{-open }\}.$$

Hence B is b-γ-closed.

(iii) Since, b-$\gamma Ds(B) \subset bcl_\gamma(B)$ and $B \subset bcl_\gamma(B)$, b-$\gamma Ds(B) \cup B \subset bcl_\gamma(B)$. Conversely, assume that $x \notin b$-$\gamma Ds(B) \cup B$. Then $x \notin b$-$\gamma Ds(B)$, $x \notin B$. Then $\exists$ a b-γ-open set M containing x such that $M \cap B = \phi$. Thus $x \notin bcl_\gamma(B)$. This gives that $bcl_\gamma(B) \subset b$-$\gamma Ds(B) \cup B$. Hence, $bcl_\gamma(B) = b$-$\gamma Ds(B) \cup B$. □

Theorem 4.12. *Let X be a space and $B \subseteq X$. B is b-γ-open $\Leftrightarrow$ B is b-γ-neighborhood, $\forall$ point $x \in H$.*

Proof. Let B be a b-γ-open set. Then clearly B is a b-γ-neighborhood, $\forall\, x \in B$. Conversely, let B be a b-γ-neighborhood, $\forall\, x \in B$. Then $\exists$ a b-γ-open set U_x containing x such that $x \in U_x \subseteq B$. Therefore, $B = \bigcup_{x \in B} U_x$. Hence, B is a b-γ-open. □

Theorem 4.13. *Let (X, τ) be space. If b-γ-N_x be the b-γ-neighborhood systems of a point $x \in X$, then the below statements are hold:*

(1) *Every member of b-γ-N_x contains a point x and b-γ-N_x is not empty;*

(2) *Every superset of members of N_x belongs b-γ-N_x;*

(3) *Every member $N \in b$-γ-N_x is a superset of a member $V \in b$-γ-N_x, where V is b-γ-neighborhood of every point $x \in V$.*

Proof. Evident. □

Definition 4.14. *Let X be a space. $B \subseteq X$ is called locally b-γ-closed if $B = V \cap K$, $\forall$ open set V and K is b-γ-closed set in X.*

Theorem 4.15. *Let X be a space and $B \subseteq X$. The set B is locally b-γ-closed $\Leftrightarrow B = V \cap bcl_\gamma(B)$.*

Proof. Suppose that B is a locally b-γ-closed set. Then $B = V \cap K$, $\forall$ open set V and K is b-γ-closed set in X. Thus, $B \subseteq bcl_\gamma(B) \subseteq bcl_\gamma(K) = K$. Therefore $B \subseteq V \cap bcl_\gamma(B) \subseteq V \cap bcl_\gamma(K) = B$. Hence $B = V \cap bcl_\gamma(B)$. Conversely, since the set $bcl_\gamma(B)$ is b-γ-closed and $B = U \cap bcl_\gamma(B)$. Then, clearly B is locally b-γ-closed. □

Theorem 4.16. *Let X be a space and B be a locally b-γ-closed subset of X. Then the below statements are hold:*

(i) *The set $bcl_\gamma(A) \setminus B$ is a b-γ-closed set;*

(ii) *The set $B \cup (X \setminus bcl_\gamma(B))$ is a b-γ-open set;*

(iii) $B \subseteq bint_\gamma(B \cup (X \setminus bcl_\gamma(B)))$.

Proof. (i) If B is a locally b-γ-closed set, then $\exists$ an open set V such that $B = V \cap bcl_\gamma(B)$. Therefore, $bcl_\gamma(B) \setminus B = bcl_\gamma(B) \setminus [V \cap bcl_\gamma(B)] = bcl_\gamma(B) \cap [X \setminus (V \cap bcl_\gamma(B))]$ $bcl_\gamma(B) \cap [(X \setminus V) \cup (X \setminus bcl_\gamma(B))]$ $= bcl_\gamma(B) \cap (X \setminus V)$, which is b-γ-closed.

(ii) By statement (i), we have $X \setminus [(bcl_\gamma(B) \setminus B)]$ is a b-γ-open set and $X \setminus [(bcl_\gamma(B) \setminus B)] = X \setminus bcl_\gamma(B) \cup (X \cap B) = B \cup [X \setminus bcl_\gamma(B)]$. Thus $B \cup [X \setminus bcl_\gamma(B)]$ is b-γ-open.

(iii) It is obvious that, $B \subseteq (B \cup [X \setminus bcl_\gamma(B)]) = bint_\gamma[B \cup (X \setminus bcl_\gamma(B))]$. □

5. b-γ-g-open and b-γ-g-closed sets

Definition 5.1. *Let (X, τ) be a space and $B \subseteq X$ is said to be b-γ-generalized closed set (for shortly, b-γ-g-closed) in (X, τ), if $bcl_\gamma(B) \subset V$ whenever $B \subset V$ and V is a b-γ-open set of (X, τ).*

The complement of b-γ-generalized closed set is called b-γ-generalized open (for shortly, b-γ-g-open) set.

Remark 5.2. *Let (X, τ) be a space and $B \subseteq X$. Then:*

(i) *The set B is b-γ-generalized open $\Leftrightarrow B^c$ is b-γ-generalized closed;*

(ii) *The set B is b-γ-generalized closed $\Leftrightarrow B^c$ is b-γ-generalized open.*

Theorem 5.3. *Let (X, τ) be a space. $B \subseteq X$. is said to be b-γ-g-open $\Leftrightarrow C \subseteq bint_\gamma(B)$, whenever C is b-γ-closed set and $C \subseteq B$.*

Proof. Let B be a b-γ-generalized open set in X. Then B^c is b-γ-generalized closed in X. Let C be a b-γ-closed set in X such that $C \subseteq B$. Then $B^c \subseteq C^c$, $C^c \in b$-$\gamma O(X)$. Since B^c is b-γ-generalized closed, $bcl_\gamma(B^c) \subseteq C^c$, which gives $[bint_\gamma(B)]^c \subseteq C^c$. Hence $C \subseteq bint_\gamma(B)$.

Conversely, suppose that $C \subseteq bint_\gamma(B)$, whenever $C \subseteq B$ and C is b-γ-closed set in X. Then $[bint_\gamma(B)]^c \subseteq C^c = D$, where D is b-γ-open set in X. That is $bcl_\gamma(B^c) \subseteq D$, which gives B^c is b-γ-generalized closed . Thus B is b-γ-generalized open. □

Theorem 5.4. *Let X be a space with an operation γ on the topology τ. Then each b-γ-closed set is b-γ-g-closed.*

Proof. Let B be a b-γ-closed set in a space X and $B \subseteq C$, where C is b-γ-open in X. Since B is b-γ-closed, $bcl_\gamma(B) = B \subseteq C$. Thus $bcl_\gamma(B) \subseteq C$. Hence, B is b-γ-g-closed. □

The converse of the above Theorem 5.4 may not be true as shown in the below example.

Example 5.5. *Let $X = \{a,b,c\}$ and τ_X be the discrete topology. Define an operation γ on τ_X by $\gamma(B) = X$. Here the set $\{a,b\}$ is b-γ-generalized closed but not b-γ-closed.*

Proposition 5.6. *Let X be a space. $B \subseteq X$ is b-γ-generalized closed $\Leftrightarrow B \cap bcl_\gamma(\{y\}) = \phi$ holds, $\forall\, y \in bcl_\gamma(B)$.*

Proof. Suppose that V be a b-γ-open set such that $B \subseteq V$. Take a point $y \in bcl_\gamma(B)$. By supposition $\exists$ a $x \in bcl_\gamma(\{y\})$ and $x \in B \subseteq V$. Then $V \cap \{y\} \neq \phi$. This implies $y \in V$. Therefore $bcl_\gamma(B) \subseteq V$. Hence, B is b-γ-generalized closed set.

Conversely, Suppose that B be a b-γ-generalized closed subset of X and take $y \in bcl_\gamma(B)$ such that $B \cap bcl_\gamma(\{y\}) = \phi$. Since $bcl_\gamma(\{y\})$ is a b-γ-closed in (X,τ), $X \setminus bcl_\gamma(\{y\})$ is a b-γ-open set. Since $B \subseteq X \setminus bcl_\gamma(\{y\})$ and B is b-γ-generalized closed, we have $bcl_\gamma(B) \subseteq X \setminus bcl_\gamma(\{y\})$ holds. Therefore $y \notin bcl_\gamma(B)$, which is a contradiction. Thus, $B \cap bcl_\gamma(\{y\}) \neq \phi$. □

Theorem 5.7. *If $B \cap bcl_\gamma(\{y\}) \neq \phi$ holds, $\forall\, y \in bcl_\gamma(B)$, then $bcl_\gamma(B) \setminus B$ does not contain a non empty b-γ-closed set.*

Proof. Assume that $\exists$ a non empty b-γ-closed set G such that $G \subseteq bcl_\gamma(B) \setminus B$. Take $y \in G$, $y \in bcl_\gamma(B)$ holds. It follows that $B \cap G = B \cap bcl_\gamma(G) \supseteq B \cap bcl_\gamma(\{y\}) \neq \phi$. Therefore, $B \cap G \neq \phi$, which is a contradiction. □

Corollary 5.8. *A subset B of (X,τ) is b-γ-generalized closed $\Leftrightarrow B = G \setminus H$, where G is b-γ-closed and H contains no non-empty b-γ-closed subsets.*

Proof. Necessity follows from Theorem 5.7 and Proposition 5.6, with $G = bcl_\gamma(B)$ and $H = bcl_\gamma(B) \setminus B$.

Conversely, suppose that $B = G \setminus H$ and $B \subseteq Q$ with Q is b-γ-open. Therefore, $G \cap (X \setminus Q)$ is a b-γ-closed subset of H and hence is empty. Therefore, $bcl_\gamma(B) \subseteq G \subseteq Q$. □

Theorem 5.9. *Let B be a subset of X and $B \subseteq C \subset bcl_\gamma(B)$. If B is b-γ-generalized closed, then C is also a b-γ-generalized closed set of X.*

Proof. Let B be a b-γ-generalized closed set and $B \subseteq C \subset bcl_\gamma(B)$. Let V be a b-γ-open set of X such that $B \subseteq V$. Since B is b-γ-generalized closed, $bcl_\gamma(B) \subseteq V$. Now $bcl_\gamma(B) \subseteq bcl_\gamma(C) \subseteq bcl_\gamma(bcl_\gamma(B)) = bcl_\gamma(B) \subseteq V$. Therefore, $bcl_\gamma(B) \subseteq V$, V is b-γ-open. Thus, B is a b-γ-generalized closed set in X. □

Theorem 5.10. *Let (X,τ) be a space and γ be an operation on τ. Then $\forall\, y \in X$, either $\{y\}$ is b-γ-closed or the set $X \setminus \{y\}$ is b-γ-generalized closed in (X,τ).*

Proof. Assume that $\{y\}$ is not b-γ-closed. By Remark 3.1, we have $X \setminus \{y\}$ is not b-γ-open set. Let V be any b-γ-open set such that $X \setminus \{y\} \subseteq V$. Therefore $V = X$. Thus $bcl_\gamma(X \setminus \{y\}) \subseteq V$. Hence, $X \setminus \{y\}$ is b-γ-generalized closed set. □

6. Conclusion

In this paper, the ideas of b-γ-boundary, b-γ-exterior and locally b-γ-closed sets are presented. Also some concepts and lemmas of b-γ-g-open and b-γ-g-closed sets are also investigated. The results are illustrated with a well-analyzed examples. For future study, some other fields such as Fuzzy topology, Intuitionistic topology, Nano topology and etc., can be considered for studying these sets.

Acknowledgment

The author is thankful to the reviewers for their helpful comments and effective suggestions to improve the quality of this paper.

References

[1] D. Andrijevic, On b-open sets, *Matematicki Vesnik*, 48(1996), 59–64.

[2] S. Kasahara, Operation-Compact Spaces, *Math. Japon*, 24(1979), 97–105.

[3] H. Z. Ibrahim, Weak forms of γ-open sets and New separation axioms, *Int. J. Sci. Eng. Res.*, 3(5)(2012), 1–4.

[4] H. Ogata, Operation on topological spaces and associated topology, *Math. Jap.*, 36(1)(1991), 175–184.

[5] C. Sivashanmugaraja and A. Vadivel, Weak forms of fuzzy γ-open sets, *Global Journal of Pure and Applied Mathematics*, 13(2)(2017), 251–261.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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