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On the quasi-central elements of Banach algebras

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Abstract
In the eighties of the last century Rennison studied and characterized the set of quasi-central elements of some unital Banach algebras. In this paper, we improve one of Rennison's result assert that for given unital Banach algebra *A* the set of quasi-central elements *Q(A)* , need not be equal to centre of *A* and need not be closed under addition or multiplication. Further, we prove that if *A* is unital ultraprime Banach algebra, then *Q(A) need not be equal to centre of* A *and need not be closed under addition or multiplication.*

Keywords
quasi-central elements, ultraprime algebra.

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1. Introduction and Preliminaries

In this paper, we will denote by A to be a unital Banach algebra over the complex field $\mathbb{C}$, and *Z(A)* to be the centre of A. Le page in ([5] Proposition 3) proved that,

$Z(A)=\{a\in A:\| x(\lambda-a)\|\leq\|(\lambda-a)x\|$ for all $x\in A$ and $\lambda\in\mathbb{C}\}$.

In [3] Rennison defined the set of all quasi-central elements of A by

$Q(A)=\bigcup_{K\geq 1}Q(K,A)$,

where $Q(K,A)=\{a\in A:\| x(\lambda-a)\|\leq K\|(\lambda-a)x\|$ for all $x\in A$and $\lambda\in\mathbb{C}\}$. In [8] they studied and proved some properties of *Q(A)* with different definition.

In general $Q(A)\neq Z(A)$, and its clear that in ([1] corollary 2), where $Q(1+\varepsilon,A)\neq Z(A)$ for each $\varepsilon>0$. However in [3–5] showed that for A is a semi-simple Banach algebra (for example C^*-algebra), or a semi-prime Banach algebra with dense socle, and for all semi-prime Banach algebra with *Q(A)* is closed under addition or multiplication or with all pairs of elements of *Q(A)* commute then $Q(A)=Z(A)$ it is true. Now, we describe in a general setting, the quasi-central elements as the same way as Rennison described in [1].

Let $\Delta=\{z\in\mathbb{C}:|z|\leq 1\}$ and let $A(\Delta)$ be the disc algebra of all complex valued functions continuous on Δ and analytic on its interior, with pointwise algebraic operations and uniform norm

$\|f\|_\Delta=\sup_{z\in\Delta}|f(z)|$.

And, let $B=A(\Delta)\oplus h$ where h is indeterminate and introduce an associative commutative product and a norm on B by defining

$\|f+\alpha h\|_B=\|f\|_\Delta+|\alpha|$ and $h^2=0, fh=f(0)h$ for all $f\in A(\Delta)$ and $\alpha\in\mathbb{C}$. We can easily show that B becomes a unital commutative Banach algebra with radical Rad $B=\mathbb{C}h$.

Now, assume that

$$T=\left\{x=\begin{pmatrix}x_1&x_2&x_3\\0&x_4&x_5\\0&0&x_6\end{pmatrix}:x_i\in B\right\}$$

and a norm on T defined by $\|x\|_T=\sum_{i=1}^{6}\|x_i\|_B$. It's clear that T is a unital Banach algebra and *Z(T)* may identified with B.

2. The Main Results

Now we can present our theorem.

Theorem 2.1. *The quasi-central of T are precisely those of* *the form* $a = \begin{pmatrix} f+\alpha_1 h & \beta_1 h & \beta_2 h \\ 0 & f+\alpha_2 h & \beta_3 h \\ 0 & 0 & f+\alpha_3 h \end{pmatrix}$

where $f \in A(\Delta)$, $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$, *and* $\beta_3 \in \mathbb{C}$ *and either* f *is non-constant or* f *is constant with* $\beta_1 = \beta_2 = \beta_3 = 0$ *and* $\alpha_1 = \alpha_2 = \alpha_3$.

Proof. If f is constant, $\beta_1 = \beta_2 = \beta_3 = 0$ and $\alpha_1 = \alpha_2 = \alpha_3$ then $a \in Z(T) \subseteq Q(T)$.
Now suppose that f is non-constant and let M be chosen as in (Lemma 2, [1]).

Take any $\lambda \in \mathbb{C}, x = \begin{pmatrix} x_1 & x_2 & x_3 \\ 0 & x_4 & x_5 \\ 0 & 0 & x_6 \end{pmatrix} \in T, x_i \in B$, and write

$$a = \begin{pmatrix} f+\alpha_1 & \beta_1 h & \beta_2 h \\ 0 & f+\alpha_2 h & \beta_3 h \\ 0 & 0 & f+\alpha_3 h \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{pmatrix}.$$

Then $ax - xa =$

$$\begin{pmatrix} 0 & (a_1-a_4)x_2 - a_2(x_1-x_4) & (a_1-a_6)x_3 - a_3(x_1-x_6) + a_2x_5 - a_5x_2 \\ 0 & 0 & (a_4-a_6)x_5 - a_5(x_4-x_6) \\ 0 & 0 & 0 \end{pmatrix} \tag{2.1}$$

and,
$(\lambda - a)x =$

$$\begin{pmatrix} (\lambda-a_1)x_1 & (\lambda-a_1)x_2 - a_2x_4 & (\lambda-a_1)x_3 - a_2x_5 - a_3x_6 \\ 0 & (\lambda-a_4)x_4 & (\lambda-a_4)x_5 - a_5x_6 \\ 0 & 0 & (\lambda-a_6)x_6 \end{pmatrix}. \tag{2.2}$$

Thus

$$\begin{aligned} &\| ax - xa \|_T = \| (a_1-a_4)x_2 - a_2(x_1-x_4) \|_B \\ &+ \| (a_1-a_6)x_3 - a_3(x_1-x_6) + a_2x_5 - a_5x_2 \|_B \\ &+ \| (a_4-a_6)x_5 - a_5(x_4-x_6) \|_B \le \| (a_1-a_4)x_2 \|_B \\ &+ \| a_2x_1 \|_B + \| a_2x_4 \|_B + \| (a_1-a_6)x_3 \|_B + \| a_3x_1 \|_B + \| a_3x_6 \|_B + \| a_2x_5 \|_B + \| a_5x_2 \|_B \\ &+ \| (a_4-a_6)x_5 \|_B + \| a_5x_4 \|_B + \| a_5x_6 \|_B \end{aligned} \tag{2.3}$$

and,
$\| (\lambda-a)x \|_T = \| (\lambda-a_1)x_1 \|_B + \| (\lambda-a_1)x_2 - a_2x_4 \|_B + \| (\lambda-a_1)x_3 - a_2x_5 - a_3x \|_B + \| (\lambda-a_4)x_4 \|_B + \| (\lambda-a_4)x_5 - a_5x_6 \|_B + \| (\lambda-a_6)x_6 \|_B$.

Taking into account Lemma 2 in [1] in the following steps:

$$\begin{aligned} &\| a_2x_1 \|_B = \| \beta_1 hx_1 \|_B = \mid \beta_1 \mid \| hx_1 \|_B \le M \mid \beta_1 \mid \| (\lambda-a_1)x_1 \|_B \\ &\le M \mid \beta_1 \mid \| (\lambda-a)x \|_T = L_1 \| (\lambda-a)x \|_T, \end{aligned} \tag{2.4}$$

where $L_1 = M \mid \beta_1 \mid$.
Similarly

$$\| a_2x_4 \|_B \le L_1 \| (\lambda-a)x \|_T \tag{2.5}$$

$$\begin{aligned} &\| a_3x_1 \|_B = \| \beta_2 hx_1 \|_B = \mid \beta_2 \mid \| hx_1 \|_B \le M \mid \beta_2 \mid \| (\lambda-a_1)x_1 \|_B \\ &\le M \mid \beta_2 \mid \| (\lambda-a)x \|_T = L_2 \| (\lambda-a)x \|_T, \end{aligned} \tag{2.6}$$

where $L_2 = M \mid \beta_2 \mid$.

$$\| a_3x_6 \|_B \le L_2 \| (\lambda-a)x \|_T. \tag{2.7}$$

$$\begin{aligned} &\| a_5x_4 \|_B = \| \beta_3 hx_4 \|_B = \mid \beta_3 \mid \| hx_4 \|_B \le M \mid \beta_3 \mid \| (\lambda-a_4)x_4 \|_B \\ &\le M \mid \beta_3 \mid \| (\lambda-a)x \|_T \\ &= L_3 \| (\lambda-a_4)x_4 \|_T, \end{aligned} \tag{2.8}$$

where $L_3 = M \mid \beta_3 \mid$.

$$\| a_5x_6 \|_B \le L_3 \| (\lambda-a)x \|_T \tag{2.9}$$

Next, by (2.9)

$$\begin{aligned} &\| a_2x_5 \|_B = \| \beta_1 hx_5 \|_B = \mid \beta_1 \mid \| hx_5 \|_B \le M \mid \beta_1 \mid \| (\lambda-a_4)x_5 \|_B \\ &\le M \mid \beta_1 \mid [\| (\lambda-a_4)x_5 - a_5x_6 \|_B + \| a_5x_6 \|_B] \\ &\le M[1 + M \mid \beta_3 \mid] \mid \beta_1 \mid \| (\lambda-a)x \|_T = L_4 \| (\lambda-a)x \|_T, \end{aligned} \tag{2.10}$$

where $L_4 = M[1 + M \mid \beta_3 \mid] \mid \beta_1 \mid$.
By (2.5),

$$\begin{aligned} &\|a_5x_2\|_B = \|\beta_3 hx_2\|_B = |\beta_3| \, \|hx_2\|_B \\ &\le M |\beta_3| \, \|(\lambda-a_1)x_2\|_B \\ &\le M |\beta_3| \, [\|(\lambda-a_1)x_2 - a_2x_4\| + \|a_2x_4\|_B] \\ &\le M[1 + M|\beta_1|] \, |\beta_3| \, \|(\lambda-a)x\|_T = L_5 \|(\lambda-a)x\|_T, \end{aligned} \tag{2.11}$$

where $L_5 = M[1 + M \mid \beta_1 \mid] \mid \beta_3 \mid$.

$$\begin{aligned} &\| (a_1-a_4)x_2 \|_B = \| ((f+\alpha_1 h) - (f+\alpha_2 h))x_2 \|_B = \\ &\mid \alpha_1 - \alpha_2 \mid \| hx_2 \|_B \\ &\le M \mid \alpha_1 - \alpha_2 \mid \| (\lambda-a_1)x_2 \|_B \\ &\le M \mid \alpha_1 - \alpha_2 \mid \| (\lambda-a_1)x_2 - a_2x_4 \|_B + \| a_2x_4 \|_B \\ &\le M[1 + M \mid \beta_1 \mid] \mid \alpha_1 - \alpha_2 \mid \| (\lambda-a)x \|_T = L_6 \| (\lambda-a)x \|_T, \end{aligned} \tag{2.12}$$

where $L_6 = M[1 + M \mid \beta_1 \mid] \mid \alpha_1 - \alpha_2 \mid$.
Now, by (2.7) and (2.10)

$$\begin{aligned} &\| (a_1-a_6)x \|_B = \mid \alpha_1 - \alpha_3 \mid \\ &\| hx_3 \|_B \le M \mid \alpha_1 - \alpha_3 \mid \| (\lambda-a_1)x_3 \|_B \\ &\le M \mid \alpha_1 - \alpha_3 \mid [\| (\lambda-a_1)x_3 - a_2x_5 - a_3x_6 \|_B + \\ &\| a_2x_5 \|_B + \| a_3x_6 \|_B \\ &\le M[1 + M(1 + M \mid \beta_1 \mid) \mid \beta_3 \mid + M \mid \beta_2 \mid] \mid \alpha_1 - \alpha_3 \mid, \end{aligned} \tag{2.13}$$

where $L_7 = M[1 + M(1 + M \mid \beta_1 \mid) \mid \beta_3 \mid + M \mid \beta_2 \mid] \mid \alpha_1 - \alpha_3 \mid$.

Again by (2.9), we have

$$\| (a_4-a_6)x_5\|_B =\mid \alpha_2-\alpha_3 \mid\| hx_5\|_B \le M \mid \alpha_2-\alpha_3 \mid \| (\lambda-a_4)x_5\|_B \le M \mid \alpha_2-\alpha_3 \mid [\| (\lambda-a_4)x_5-a_5x_6\|_B+ \| a_5x_6\|_B] \le M[1+M \mid \beta_3 \mid] \mid \alpha_2-\alpha_3 \mid\| (\lambda-a)x\|_T = L_8 \| (\lambda-a)x\|_T. \tag{2.14}$$

The inequalities (2.3) – (2.14), show that for some constant L, where $L=\sum_{i=1}^{8} L_i$, then

$$\| ax-xa\|_T \le L \| (\lambda-a)x\|_T, \text{for all } x \text{ in } T \text{ and } \lambda \text{ in } \mathbb{C}. \tag{2.15}$$

So, by the Remark in [1], $a\in Q(T)$. Now conversely, let that $a\in Q(T)$. By [1] for some constant L,

$\| ax-xa\|_T \le L \| (\lambda-a)x\|_T$, for all x in T and λ in $\mathbb{C}$.

Taking $x_2=x_3=\ldots=x_6=0$ in (2.3), we have $\| a_2x_1\|_B+ \| a_3x_1\|_B \le L \| (\lambda-a_1)x_1\|_B$, for all x_1 in B and λ in $\mathbb{C}$, so we have $\| a_2x_1\|_B \le L \| (\lambda-a_1)x_1\|_B$, and $\| a_3x_1\|_B \le L \| (\lambda-a_1)x_1\|_B$, for all x_1 in B and λ in $\mathbb{C}$, and so by Lemma 1 in [1] we get,$a_2,a_3 \in Rad\ B=\mathbb{C}h$

$So, a_2=\beta_1 h$ and $a_3=\beta_2 h$ for some $\beta_1,\beta_2\in\mathbb{C}$.

Taking $x_1=x_3=\ldots=x_6=0$ and $x_2\neq 0$. This implies that $\| (a_1-a_4)x_2\|_B+ \| a_5x_2\|_B \le L \| (\lambda-a_1)x_2\|_B$, for all x_2 in B and λ in $\mathbb{C}$.

So, we have $\| (a_1-a_4)x_2\|_B \le L \| (\lambda-a)x_2\|_B$, and $\| a_5x_2\|_B \le L \| (\lambda-a_1)x_2\|_B$ by lemma 1 again, gives that

$a_1-a_4, a_5 \in Rad\ B=\mathbb{C}h$.

Hence we can express a_1 and a_4 in the form

$a_1=f+\alpha_1 h$ and $a_4=f+\alpha_2 h$, for some $f\in A(\Delta)$ and $\alpha_1,\alpha_2\in\mathbb{C}$

also we can express a_5 in the form $a_5=\beta_3 h, \beta_3\in\mathbb{C}$

Finally, taking $x_1=x_2=\ldots=x_6=0$ and $x_3\neq 0$, this implies that

$\| (a_1-a_6)x_3\|_B \le L \| (\lambda-a_1)x_3\|_B$, for all x_3 in B and λ in $\mathbb{C}$.

again by Lemma 1, we have

$a_1-a_6 \in Rad B=\mathbb{C}h$

hence we can express a_1 and a_6 in the form

$a_1=f+\alpha_1 h$ and $a_6=f+\alpha_3 h$, for some $f\in A(\Delta)$ and $\alpha_1,\alpha_3\in\mathbb{C}$.

Now, we get the final form of a

$$a=\begin{pmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{pmatrix} = \begin{pmatrix} f+\alpha_1 h & \beta_1 h & \beta_2 h \\ 0 & f+\alpha_2 h & \beta_3 h \\ 0 & 0 & f+\alpha_3 h \end{pmatrix}$$

If, f = μ is constant then $(\mu-a)^2=0$

$$\mu=\mu I=\begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix}, a=\begin{pmatrix} \mu+\alpha_1 h & \beta_1 h & \beta_2 h \\ 0 & \mu+\alpha_2 h & \beta_3 h \\ 0 & 0 & \mu+\alpha_3 h \end{pmatrix}$$

$$\mu-a=\begin{pmatrix} \alpha_1 h & -\beta_1 h & -\beta_2 h \\ 0 & \alpha_2 h & -\beta_3 h \\ 0 & 0 & \alpha_3 h \end{pmatrix}, so,\ (\mu-a)^2=0,$$

because $h^2=0$.

However, by **Theorem (3.7)** of [3], any quasi-central element with finite spectrum is necessarily central and so, in this case we must have $\beta_1=\beta_2=\beta_3=0$ and $\alpha_1=\alpha_2=\alpha_3=0$. □

Corollary 2.2. *$Q(T)$ is not a closed subset of T.*

Proof. If $u(z)=z$ for all z in δ then

$$a_n=\begin{pmatrix} u/n & h_1 & h_2 \\ 0 & u/n & h_3 \\ 0 & 0 & u/n \end{pmatrix}\in Q(T),$$

for all n because, for any $\lambda\in\mathbb{C}$ and

$$x=\begin{pmatrix} x_1 & x_2 & x_3 \\ 0 & x_4 & x_5 \\ 0 & 0 & x_6 \end{pmatrix}\in T, x_i\in B,$$

then

$$a_nx-xa_n=\begin{pmatrix} 0 & h_1(x_4-x_1) & h_1x_5-h_2(x_1-x_6) \\ 0 & 0 & h_3(x_6-x_4) \\ 0 & 0 & 0 \end{pmatrix} \tag{2.16}$$

and,

$$(\lambda-a_n)x= \begin{pmatrix} (\lambda-u/n)x_1 & (\lambda-u/n)x_2-h_1x_2 & (\lambda-u/n)x_3-h_1x_5-h_2x_6 \\ 0 & (\lambda-u/n)x_4 & (\lambda-u/n)x_5-h_3x_6 \\ 0 & 0 & (\lambda-u/n)x_6 \end{pmatrix} \tag{2.17}$$

Thus

$$\| a_nx-xa_n\|_T =\| h_1(x_4-x_1)\|_B + \| h_1x_5-h_2(x_1-x_6)\|_B + \| h_3(x_6-x_4)\|_B \le \| h_1x_1\|_B + \| h_1x_4\|_B+ \| h_1x_5\|_B+ \| h_2x_1\|_B + \| h_2x_6\|_B+ \| h_3x_4\|_B+ \| h_3x_6\|_B. \tag{2.18}$$

and,

$$\| (\lambda-a_n)x\|_T =\| (\lambda-u/n)x_1\|_B + \| (\lambda-u/n)x_2-h_1x_4\|_B + \| (\lambda-u/n)x_3-h_1x_5-h_2x_6\|_B + \| (\lambda-u/n)x_4\|_B+ \| (\lambda-u/n)x_5 -h_3x_6\|_B+ \| (\lambda-u/n)x_6\|_B.$$

Now, by Lemma 2 in [1] and for some constant L, we have $\| a_nx-xa_n\|_T \le L \| (\lambda-a_n)x\|_T$ for all x in T and λ in $\mathbb{C}$.

Hence, by the Remark in [1], it's show that an $a_n\in Q(T)$ for all n but $a_n\to \begin{pmatrix} 0 & h_1 & h_2 \\ 0 & 0 & h_3 \\ 0 & 0 & 0 \end{pmatrix}\notin Q(T)$. □

Corollary 2.3. *For each* $\varepsilon > 0, Q(1+\varepsilon, T) \neq Z(T)$.

Proof. Let $u(z) = z$ for all z in Δ , Then a =
$$\begin{pmatrix} u & 0 & 0 \\ 0 & u+\varepsilon h & 0 \\ 0 & 0 & u \end{pmatrix} \in Q(T)$$
for any λ in $\mathbb{C}$ and x =
$$\begin{pmatrix} x_1 & x_2 & x_3 \\ 0 & x_4 & x_5 \\ 0 & 0 & x_6 \end{pmatrix} \in T, x_i \in B$$

$$ax - xa = \begin{pmatrix} 0 & -\varepsilon h x_2 & 0 \\ 0 & 0 & \varepsilon h x_5 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.19}$$

and
$(\lambda - a)x =$

$$\begin{pmatrix} (\lambda-u)x_1 & (\lambda-u)x_2 & (\lambda-u)x_3 \\ 0 & (\lambda-u-\varepsilon h)x_4 & (\lambda-u-\varepsilon h)x_5 \\ 0 & 0 & (\lambda-u)x_6 \end{pmatrix} \tag{2.20}$$

$x(\lambda - a) =$
$$\begin{pmatrix} x_1(\lambda-u) & x_2(\lambda-u-\varepsilon h) & x_3(\lambda-u) \\ 0 & x_4(\lambda-u-\varepsilon h) & x_5(\lambda-u) \\ 0 & 0 & x_6(\lambda-u). \end{pmatrix} \tag{2.21}$$

Thus

$$\| ax - xa\|_T = \| \varepsilon h x_2\|_B + \| \varepsilon h x_5\|_B \tag{2.22}$$

and
$\| (\lambda - a)x\|_T = \| (\lambda-u)x_1\|_B + \| (\lambda-u)x_2 \|_B + \| (\lambda-u)x_3\|_B + \| (\lambda-u-h)x_4\|_B + \| (\lambda-u-h)x_5\|_B + \| (\lambda-u)x_6\|_B$.
Now, by Lemma 2 in [1]

$$\| \varepsilon h x_2\|_B \leq \varepsilon \| (\lambda-u)x_2\|_B \leq \varepsilon \| (\lambda-a)x\|_T. \tag{2.23}$$

$$\| \varepsilon h x_5\|_B \leq \varepsilon \| (\lambda-u-\varepsilon h)x_5\|_B \leq \varepsilon \| (\lambda-a)x\|_T. \tag{2.24}$$

The inequalities (2.22) – (2.24) show that
$\| ax - xa\|_T \leq \varepsilon \| (\lambda-a)x\|_T$, so that
$\| x(\lambda-a)\|_T = \| (\lambda-a)x + (ax-xa)\|_T$
$\leq \| (\lambda-a)x\|_T + \| ax-xa\|_T$
$\leq (1+\varepsilon) \| (\lambda-a)x\|_T$.
Hence, $a \in Q(1+\varepsilon, T)$ but $a \notin Z(T)$. □

Remark 2.4. *The above theorem holds also for dimension greater than* 3×3.

3. The quasi-central elements of ultraprime Banach algebra

Recall from [2] that a normed algebra A is called ultraprime if there exists a positive constant $L > 0$, such that

$$L \| a \| \; \| b \| \; \leq \; \| M_{a,b} \| \; \forall a, b \in A, \tag{3.1}$$

where $M_{a,b}$ is two-sided multiplication operator defined by:
$M_{a,b} : A \to A$
$x \to M_{a,b} = axb \forall x \in A$
if $a = b$, then A is called ultrasemiprime (see also [6]). From now on A is ultraprime Banach algebra with identity.

Theorem 3.1. *Every element in the centre of A is quasi-central.*

Proof. Let (A, ‖ . ‖) be an ultraprime Banach algebra with identity, and let
$L = inf\{\| M_{a,b} \| : a, b \in A\}$
be the constant of ultraprimeness of A.
Fix $a \in Z(A)$, clearly that $\lambda - a \in Z(A)$. Now, for $0 \neq x \in A$ we have

$$L \| x \| \; \| x(\lambda - a) \| \leq \| M_{x(\lambda-a),x} \|. \tag{3.2}$$

Also, for all $y \in A$
$M_{x(\lambda-a),x}(y) = x(\lambda-a)yx = xy(\lambda-a)x = M_{x,(\lambda-a)x}(y)$,
this implies that $\| M_{x(\lambda-a),x} \| = \| M_{x,(\lambda-a)x} \|$.
Therefore from (3.2),
$L \| x \| \; \| x(\lambda-a) \| \leq \| x \| \; \| (\lambda-a)x \|$.
So, $\| x(\lambda-a) \| \leq \frac{1}{L} \| (\lambda-a)x \|$. Since for $0 < L < 1$ the inequality (3.1) is always true, it follows that if $K = \frac{1}{L}$, then we have $a \in Q(K, A)$ which complete the proof. □

Corollary 3.2. *$Q(A)$ is not closed under addition or multiplication.*

Proof. Clear from ([7] Theorem 2.4) and above theorem. □

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