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Convergence of L-fuzzy nets via bitopological semiopen sets

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Abstract
In this paper, we introduce the concept of an (i, j)-semi-remote neighbourhood of fuzzy points and establish the Moore-Smith (i, j)-semi-convergence theory of L-fuzzy nets.

Keywords
Fuzzy (i, j)-semi-closed sets, fuzzy (i, j)-semi-open sets, (i, j)-semi-remote neighoburhood, (i, j)-semi-convergence.

AMS Subject Classification
54A40.

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Contents

1. Introduction

The usual notion of a set was generalized with the introduction of fuzzy sets by Zadeh in the classical paper [9] of 1965. Since then many authors have expansively developed the theory of fuzzy sets and its applications to several sectors of both pure and applied sciences, such as [2, 5, 7, 8]. As it is known now that the traditional neighbourhood method is not effective any longer in fuzzy topology, in order to overcome this deficiency Pu and Liu introduced the concepts of the fuzzy point and the Q-neighbourhood and established a systematic Moore-Smith convergence theory of fuzzy nets [5]. It paved a new way for the study of the fuzzy topology. Later on, Wang introduced the concept of remote neighbourhood systems [7]. Q-neighbourhood and remote neighbourhood can be used in wide aspect [1, 3–7]. In this paper, we introduce and study the concept of an (i, j)-semi-remote neighbourhood of fuzzy points with the concept of remote neighbourhood and using the concepts fuzzy (i, j)-semi-closed sets and L-(i, j)-semi-remote neighbourhood.

2. preliminaries

Throughout this paper, $L = L(\leq, \vee, \wedge, \prime)$ will denote a fuzzy lattice, that is, a completely distributive lattice with a smallest element 0 and largest element 1 $(0 \neq 1)$ and with an order reversing involution $a \to a'(A \in L)$. Let X be a nonempty crisp set and we shall denote by L^X the lattice of all L-subsets of X and if $A \subseteq X$ by χ_A the characteristic function of A. An element p of L is called prime if, and only if $p \neq 1$ and whenever $a, b \in L$ with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$[8]. The set of all prime elements of L will be denoted by $Pr(L)$. An element α of L is called union irreducible or coprime if, and only if whenever $a, b \in L$ with $\alpha \leq a \vee b$ then $\alpha \leq a$ or $\alpha \leq b$ [8]. The st of all non zero union irreducible elements of L will be denoted by $M(L)$. It is obvious that $p \in pr(L)$ if, and only if $p' \in M(L)$. We denote $M^*(L^X) = \{x_\alpha : x \in X$ and $\alpha \in M(L)\}$. For the definition of a fuzzy point x_α we follow x_α we follow Pu and Liu [5]. When the support and value of a fuzzy point are trivial, we use briefly the symbols e to denote fuzzy point. A fuzzy point $x_\alpha \in A$, where A is an L-fuzzy set in X if, and only if $\alpha \leq A(x)$. The constant L-fuzzy sets taking on the values 0 and 1 on X are designated by 0_X and 1_X, respectively. An fuzzy net $S = \{S(n), n \in D\}$ is a function $S : D \to \zeta$ where D is directed set with order relation $\geq$ and ζ the collection of all the fuzzy points in X [8]. A net S is called an α-net $(\alpha \in M(L))$ if for each $\lambda \in \beta'(\alpha)$ (where $\beta'(\alpha)$ denotes the union of all minimal sets relative to α), there is $n_0 \in D$ such that $V(S(n)) \geq \lambda$ whenever $n \geq n_0$, where $V(S(n))$ is the height of point $S(n)$.

Definition 2.1. *Let L be a fuzzy lattice, X be a nonempty crisp set and $\tau \subseteq L^X$. An L-fuzzy topology is a family of τ of L-subsets of X which satisfies the following conditions:*

1. $0, 1 \in \tau$,
2. *If* $A, B \in \tau$, *then* $A \wedge B \in \tau$,
3. *If* $A_i \in \tau$ *for each* $i \in I$, *then* $\vee_{i \in I} A_i \in \tau$.

The pair (L^X, τ) is called an L-fuzzy topological space. Every member of τ is called a L-fuzzy open set.

Let (L^X, τ_1, τ_2) be an L-fuzzy topological space, e be an fuzzy point and P an L-fuzzy closed set in (L^X, τ_1, τ_2). Then P is called a remote neighbourhood of e if $e \notin P$. The set of all remote neighbourhoods of e will be denoted by $\eta(e)$. $i\operatorname{Int}(A), i\operatorname{Cl}(A)$ and A' will denote the τ_i-interior, the τ_i-closure and the complement of the L-fuzzy set A in X, respectively.

Example 2.2. *Let $x_1, x_2, \ldots$ be a sequence in a set X. Then it is a net with an index set $D = \{1, 2, \ldots\}$. So the concept of a net is a generalization of the concept of a sequence.*

Definition 2.3. *Let L_1 and L_2 be fuzzy lattices. A mapping $f : L_1 \to L_2$ is called an order homomorphism if the following conditions hold:*

1. $f(0) = 0$.
2. $f(\vee A_i) = \vee f(A_i)$ *for* $\{A_i\} \subset L_1$.
3. $f^{-1}(B') = (f^{-1}(B))'$ *for each* $B \in L_2$.

Definition 2.4. *[6] A fuzzy set A of an L-fuzzy bitopological space (L^X, τ_1, τ_2) is said to be (i, j)-semi-open if $A \leq j\operatorname{Cl}(i\operatorname{Int}(A))$. The complement of an (i, j)-semi-open set is called an (i, j)-semi-closed set.*

Proposition 2.5. *[6] Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and A an L-set of (L^X, τ_1, τ_2). Then (i, j)-$s\operatorname{Int}(A) = \cup\{B : B \in (i, j)\text{-}SO(L^X), B \leq A\}$, (i, j)-$s\operatorname{Cl}(A) = \cap\{B : B \in (i, j)\text{-}SC(L^X), A \leq B\}$ are called the (i, j)-semi-interior and the (i, j)-semi-closure of A, respectively, where (i, j)-$SO(L^X)$ and (i, j)-$SC(L^X)$ will always denote the family of (i, j)-semi-open sets and the family of (i, j)-semi-closed sets of an L-fuzzy bitopological space (L^X, τ_1, τ_2), respectively.*

3. Convergence of L-fuzzy nets

Definition 3.1. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, x_α a fuzzy point and $P \in (i, j)$-$SC(L^X)$. Then P is called an L-fuzzy (i, j)-semi-remote neighbourhood, or briefly, (i, j)-SC-RN of x_α if $x_\alpha \notin P$. The set of all (i, j)-SC-RNs of x_α will be denoted by ζ_{x_α}.*

Definition 3.2. *Let A be an L-set of an L-fuzzy bitopological space (L^X, τ_1, τ_2). Then a fuzzy point x_α is called an (i, j)-semi-adhere point of A if $A \nleq P$ for each $P \in \zeta_{x_\alpha}$. If x_α is a (i, j)-semi-adherence point of A and $x_\alpha \notin A$, or $x_\alpha \in A$ and for each fuzzy point x_μ satisfying $x_\alpha \leq x_\mu \in A$ we have $A \nleq x_\mu \vee P$, then x_α is called an (i, j)-semi-accumulation point of A. The union of all (i, j)-semi-accumulation points of A will be called (i, j)-semi-derived set of A and denoted $A^{d((i,j)-s)}$.*

Definition 3.3. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. Then*

1. *e is said to be an (i, j)-semi-limit point of S (or S (i, j)-semi-converges to e; in symbols, $S \to e(\alpha)$) if for each $P \in \zeta(e)$, $S(n) \notin P$ is eventually true (that is, if there exists $n_0 \in D$ such that for every $n \in D, n \geq n_0$, always possess $S(n) \notin P$).*
2. *e is said to be an (i, j)-semi-cluster point of S (or S (i, j)-semi-accumulates to e; in symbols, $S \infty e(\alpha)$) if for each $P \in \zeta(e), S(n) \notin P$ is frequently true (that is, if for every $n_0 \in D$, there always exists $n \in C, n \geq n_0$ such that $S(n) \notin P$).*

The union of all (i, j)-semi-limit points and all (i, j)-semi-cluster points of S will be denoted by (i, j)-semi-$\lim S$ and (i, j)-semi-ad S, respectively. Obviously, (i, j)-semi-$\lim S \leq (i, j)$-semi-ad S.

Proposition 3.4. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. Then the following statements are valid:*

1. *If $S = \{S(n) : n \in D\} \to e(\alpha)$, $T = \{T(n) : n \in D\}$ is an L-fuzzy net with the same domain as S for each $n \in D, T(n) \geq S(n)$ holds. Then $T = \{T(n) : n \in D\} \to e(\alpha)$.*
2. *If $S = \{S(n) : n \in D\} \infty e(\alpha)$, $T = \{T(n) : n \in D\}$ is an L-fuzzy net with the same domain as S for each $n \in D, T(n) \geq S(n)$ holds. Then $T = \{T(n) : n \in D\} \infty e(\alpha)$.*
3. *If $S = \{S(n) : n \in D\} \to e(\alpha)$ and $d \leq e$. Then $S = \{S(n) : n \in D\} \to d(\alpha)$.*
4. *If $S = \{S(n) : n \in D\} \infty e(\alpha)$ and $d \leq e$. Then $S = \{S(n) : n \in D\} \infty d(\alpha)$.*

Proof. The proof follows from the respective definitions. □

Theorem 3.5. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. Then*

1. *$S \to e(\alpha)$ if, and only if $e \in (i, j)$-semi-$\lim S$.*
2. *$S \infty e(\alpha)$ if, and only if $e \in (i, j)$-semi-ad S.*

Proof. (1). Suppose that $S \to e(\alpha)$, then by Definition 3.3, e is said to be (i,j)-semi-limit points of S. And (i,j)-semi-$\lim S$ is the union of all (i,j)-semi-limit point of S, then we have $e \in (i,j)$-semi-$\lim S$. Conversely, suppose that $e \in (i,j)$-semi-$\lim S$ and $P \in \zeta(e)$. Then $e \notin P$, and so (i,j)-semi-$\lim S \nleq P$. By the definition of (i,j)-semi-$\lim S$, there must exists an (i,j)-semi-limit point d of S such that $d \notin P$, that is $P \in \zeta(d)$. Hence, S is eventually not in P, that is, $S \to e(\alpha)$. (2). Suppose that $S \infty e(\alpha)$, then by Definition 3.3, e is said to be (i,j)-semi-cluster point of S. And (i,j)-semi-ad S is the Union of all (i,j)-semi-cluster points of S, then we have $e \in (i,j)$-semi-ad S. Conversely, suppose that $e \in (i,j)$-semi-ad S and $P \in \zeta(e)$. Then $e \notin P$, and so (i,j)-semi-ad $S \nleq P$. By the definition of (i,j)-semi-ad S, there must exist an (i,j)-semi-cluster point d of S such that $d \notin P$, that is $P \notin \zeta(d)$. Hence, $S \notin P$ is frequently true, that is, $S \infty e(\alpha)$. □

Theorem 3.6. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. Then (i,j)-semi-$\lim S$ and (i,j)-semi-ad S and (i,j)-semi-ad S are (i,j)-semi-closed.*

Proof. Let $e \in (i,j)$-$s\mathrm{Cl}((i,j)$-semi-$\lim S)$. Then (i,j)-semi-$\lim S \nleq P$ for each $P \in \zeta(e)$. Hence there exists $d \in M^*(L^X)$ such that $d \in (i,j)$-semi-lim S and $d \notin P$. Then $P \in \zeta(d)$. By Theorem 3.5 (1), $S \to d(\alpha)$, that is, $S(n) \notin P$ is eventually true. Thus, $e \in (i,j)$-semi-$\lim S$. This implies that (i,j)-semi-$\lim S$ is (i,j)-semi-closed. Similarly (i,j)-semi-ad S is (i,j)-semi-closed. □

Theorem 3.7. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $A \in L^X$.*

1. *If there exists in A an L-fuzzy net $S = \{S(n) : n \in D\}$ such that $S \infty e(\alpha)$, then e is an (i,j)-semi-adherence point of A.*

2. *If e is an (i,j)-semi-adherence point of A, then there exists in A an L-fuzzy net $S = \{S(n) : n \in D\}$ such that $S \to e(\alpha)$.*

Proof. (1) Let $S \infty e(\alpha)$ and $S(n) \in A$ for each $n \in D$. Then for each $P \in \zeta(e)$. $A \nleq P$ because of the fact that $S(n) \notin P$ is frequently true. Hence, e is an (i,j)-semi-adherence point of A. (2) If e is an (i,j)-semi-adherence point of A, then for each $P \in \zeta(e)$ there exists a point $S(P)$ such that $S(P) \leq A$ and $S(P) \nleq P$. Define $S = \{S(P), P \in \zeta(e)\}$, then S is an L-fuzzy net in A because of the fact that $\zeta(e)$ is a directed set in which the order is defined by inclusion. Clearly, $S \to e(\alpha)$. □

Definition 3.8. *Let $S = \{S(n) : n \in D\}$ and $T = \{T(m) : m \in E\}$ be two nets in L^X. Then T is said to be a subset of S, if there exists a mapping $N : E \to D$ such that*

1. *$T = SN$;*

2. *For every $n_0 \in D$, there exists $m_0 \in E$ such that $N(m) \geq n_0$ for $m \geq m_0$.*

Theorem 3.9. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. Then S has a subnet T such that $T \to e(\alpha)$ if, and only if $S \infty e(\alpha)$.*

Proof. Suppose that $T = \{T(m) : m \in E\}$ is a subnet of S, $T \to e(\alpha), P \in \zeta(e)$ and $n_0 \in D$. By the definition of subnet, there exists a mapping $N : E \to D$ and $m_0 \in E$ such that $N(m) \geq n_0 (N(m) \in D)$ when $m \geq m_0$ $(m \in E)$. Since T (i,j)-semi-converges to e, there is $m_1 \in E$. When $m \geq m_1 (m \in E), T(m) \notin P$. Since E is a directed set, there exists $m_2 \in E$ such that $m_2 \geq m_0$ and $m_2 \geq m_1$. Hence, $T(m_2) \notin P$ and $N(m_2) \geq n_0$. Let $n = N(m_2)$. Then $S(n) = S(N(m_2)) = T(m_2) \notin P$ and $n \geq n_0$. This means that $S(n) \notin P$ is frequently true. Thus $S \infty e(\alpha)$. Conversely, suppose that $S \infty e(\alpha)$. Then for each $P \in \zeta(e)$ and $n \in D$, there exists $N(P,n) \in D$ such that $N(P,n) \geq n$ and $S(N(P,n)) \notin P$. Let $E = \{(N(P,n),P) : P \in \zeta(e), n \in D\}$, and define $(N(P_1,n_1),P_1) \leq (N(P_2,n_2),P_2)$ if, and only if $n_1 \leq n_2$ and $P_1 \leq P_2$. Thus E is a directed set because: (a) For each $(N(P(,n),P))$, since $n \in D$ and D is a directed set, we have $n \leq n$. Also, since $P \in \zeta(e)$ and $\zeta(e)$ is a directed set, $P \leq P$. Hence $n \leq n$ and $P \leq P$ which equivalent that $(N(P(,n),P)) \leq (N(P(,n),P))$. Thus $\leq$ is reflexive on E. (b) Let $(N(P_1,n_1),P_1), (N(P_2,n_2),P_2)$ and $(N(P_3,n_3),P_3)$ belong to E such that $(N(P_1,n_1),P_1) \leq (N(P_2,n_2),P_2)$ and $(N(P_2,n_2),P_2) \leq (N(P_3,n_3),P_3)$. Thus $n_1 \leq n_2, P_1 \leq P_2$ and $n_2 \leq n_3$ and $P_2 \leq P_3$. Since D and $\zeta(e)$ are directed sets, we get $n_1 \leq n_3$ and $P_1 \leq P_3$ which equivalent that $(N(P_1,n_1),P_1) \leq (N(P_3,n_3),P_3)$. Thus $\leq$ is transitive on E. (c) Let $(N(P_1,n_1),P_1)$ and $(N(P_2,n_2),P_2)$ belong to E. Since $n_1, n_2 \in D$ and D is a directed set, there is $n \in D$ such that $n_1 \leq n$ and $n_2 \leq n$. Also, since $P_1, P_2 \in \zeta(e)$, we have $P = P_1 \vee P_2 \in \zeta(e)$ and $P_1 \leq P, P_2 \leq P$. Hence there exists $(N(P,n),P) \in E$ with $(N(P_1,n_1),P_1) \leq (N(P(,n),P))$ and $(N(P_2,n_2),P_2) \leq (N(P(,n),P))$. Hence $(E, \leq)$ is a directed set. Let $T(N(P,n),P) = S(N(P,n))$. Then T is a subnet of S and $T \to e(\alpha)$. □

Theorem 3.10. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space, $e \in M^*(L^X)$ and $S = \{S(n) : n \in D\}$ an L-fuzzy net in L^X. If T is a subnet of S, then:*

1. *If $S \to e(\alpha)$, Then $T \to e(\alpha)$.*

2. *If $T \infty e(\alpha)$, then $S \infty e(\alpha)$.*

3. *(i,j)-semi-$\lim S \leq (i,j)$-semi-$\lim T$.*

4. *(i,j)-semi-ad $T \leq (i,j)$-semi-ad S.*

Proof. (1). Suppose $T = \{T(m) : m \in E\}$ is a subnet of S, $S \to e(\alpha)$ and $P \in \zeta(e)$, then $S(n) \notin P$ is eventually true. From the definition of subnet, there exists a mapping $N : E \to D$ and for every $m \in E$, there exists $n \in D$ such that $T(m) = S(N(m)) = S(n)$. That is to say, every element of the net T is actually the element of the net S. So $T(m) \notin P$ is eventually true. Thus we have $T \to e(\alpha)$. (2). Suppose that $T = \{T(m) : m \in E\}$ is a subnet of S, $T \infty e(\alpha), P \in \zeta(e)$ and $n_0 \in D$. By the definition of subnet, there exists a mapping $N : E \to D$ and $m_0 \in E$ such that $N(m) \geq n_0(N(m) \in D)$ when $m \geq m_0(m \in E)$. Since T (i, j)-semi-accumulates to e for $m_0 \in E$ there is $m_1 \in E$ there is $m_1 \in E$. When $m_1 \geq m_0(m_1 \in E)$, $T(m_1) \notin P$. Let $n = N(m_1)$. Then $S(n) = S(N(m_1)) = T(m_1) \notin P$ and $n \geq n_0$. This means that $S(n) \notin P$ is frequently true. Thus $S\infty e(\alpha)$. (3). By Theorem 3.5, $S \to e(\alpha)$ means $e \in (i, j)$-semi-lim$S$ and $T \to e(\alpha)$ means $e \in (i, j)$-semi-lim$T$. Thus by (1), we have (i, j)-semi-lim$S \leq (i, j)$-semi-lim$T$. (4) By Theorem 3.5, $S\infty e(\alpha)$ means $e \in (i, j)$-semi-ad S and $T\infty e(\alpha)$ means $e \in (i, j)$-semi-ad T. Thus by (2), we have (i, j)-semi-ad $T \leq (i, j)$-semi-ad S. □

Definition 3.11. *An OH $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ is said to be (i, j)-semi-irresolute if $f^{-1}(B) \in (i, j)$-$SO(L_1^X)$ for each $B \in (i, j)$-$SO(L_2^Y)$.*

Theorem 3.12. *For an OH $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ the following are equivalent:*

1. *f is (i, j)-semi-irresolute.*
2. *$f^{-1}(B) \in (i, j)$-$SC(L_1^X)$ for each $B \in (i, j)$-$SC(L_2^Y)$.*
3. *(i, j)-$s\text{Cl}(f^{-1}(B)) \leq f^{-1}((i, j)$-$s\text{Cl}(B))$ for each $B \in L_2^Y$.*

Proof. (1)⇒(2): f is (i, j)-semi-irresolute if $f^{-1}(A) \in (i, j)$-$SO(L_1^X)$ for each $A \in (i, j)$-$SO(L_2^Y)$. For each $B \in (i, j)$-$SC(L_2^Y)$, $B' \in (i, j)$-$SO(L_2^Y)$. So we have $(f^{-1}(B))' = f^{-1}(B') \in (i, j)$-$SO(L_1^X)$. This shows $f^{-1}(B) \in (i, j)$-$SC(L_1^X)$. (2)⇒(1): For each $A \in (i, j)$-$SO(L_2^Y)$, $A' \in (i, j)$-$SC(L_2^Y)$. Then by (2) we have $(f^{-1}(A))' = f^{-1}(A') \in (i, j)$-$SC(L_1^X)$. This shows that $f^{-1}(A) \in (i, j)$-$SO(L_1^X)$. Hence by Definition 3.11, f is (i, j)-semi-irresolute. (2)⇒(3): For each $B \in L_2^Y, (i, j)$-$s\text{Cl}(B) \in (i, j)$-$SC(L_2^Y)$. Then by (2) we have $f^{-1}((i, j)$-$s\text{Cl}(B)) \in (i, j)$-$SC(L_1^X)$. And $B \leq (i, j)$-$s\text{Cl}(B)$ implies $f^{-1}(B) \leq f^{-1}((i, j)$-$s\text{Cl}(B))$. From the definition of (i, j)-semi-closure we have (i, j)-$s\text{Cl}(f^{-1}(B)) \leq f^{-1}((i, j)$-$s\text{Cl}(B))$. (3)⇒(1): Let $B \in (i, j)$-$SC(L_2^Y)$, then $B = (i, j)$-$s\text{Cl}(B)$. By (3) we have $f^{-1}(B) \leq (i, j)$-$s\text{Cl}(f^{-1}(B)) \leq f^{-1}((i, j)$-$s\text{Cl}(B) = f^{-1}(B)$, that is, $f^{-1}(B) = (i, j)$-$s\text{Cl}(f^{-1}(B))$. Hence $f^{-1}(B) \in (i, j)$-$SC(L_2^X)$ and consequently, f is (i, j)-semi-irresolute. □

Definition 3.13. *An order-homomorphism $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ is said to be (i, j)-semi-irresolute at a point $e \in M^*(L_1^X)$ if (i, j)-$s\text{Cl}((f^{-1}(P)) \in \zeta_1(e)$ for each $P \in \zeta_2(f(e))$, where $\zeta_1(e)$ and $\zeta_2(f(e))$ denote the set of all (i, j)-SC-RNs of e and $f(e)$, respectively.*

Theorem 3.14. *An order-homomorphism $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ is (i, j)-semi-irresolute if, and only if f is (i, j)-semi-irresolute for each point $e \in M^*(L_1^X)$.*

Proof. Suppose that f is (i, j)-semi-irresolute and $e \in M^*(L_1^X)$. Then $f^{-1}(P)$ is (i, j)-semi-closed for each $P \in \zeta_2(f(e))$. So $e \notin f^{-1}(P)$. Hence $f^{-1}(P) = (i, j)$-$s\text{Cl}(f^{-1}(P)) \in \zeta_1(e)$ and so f is (i, j)-semi-irresolute at e. Conversely, suppose that f is (i, j)-semi-irresolute for each $e \in M^*(L_1^X)$ and $P \in (i, j)$-$SC(L_2^Y)$. We may assume that $f^{-1}(P) \neq 1_X$ and suppose that $e \notin f^{-1}(P)$. Then $f(e) \notin P$ and so $P \in \zeta_2(f(e))$. Hence, (i, j)-$s\text{Cl}(f^{-1}(P)) \in \zeta_1(e)$, that is $e \notin f^{-1}(P)$ implies that $e \notin (i, j)$-$s\text{Cl}(f^{-1}(P))$ or (i, j)-$s\text{Cl}(f^{-1}(P)) \leq f^{-1}(P)$. Thus, $f^{-1}(P)$ is (i, j)-semi-closed in (L_1^X, δ), that is, f is (i, j)-semi-irresolute. □

Theorem 3.15. *Let $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ be (i, j)-semi-irresolute at $e \in M^*(L_1^X)$ and S an L-fuzzy net in L_1^X. If $S \to e(\alpha)$ we have $f(S)$ (i, j)-semi-converges to $f(e)$ where $f(S) = \{f(S(n)), n \in D\}$ is an L-fuzzy net in L_2^Y.*

Proof. Suppose that f is (i, j)-semi-irresolute at $e \in M^*(L_1^X)$ and $S \to e(\alpha)$. Let $P \in \zeta_2(f(e))$. Then S is eventually not in (i, j)-$s\text{Cl}(f^{-1}(P)) \in \zeta_1(e)$, and hence $f(S)$ is eventually not in P, that is, $f(S) \to f(e)(\alpha)$. □

Theorem 3.16. *Let $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ be (i, j)-semi-irresolute. Then for each L-fuzzy net S in L_1^X we have $f((i, j)$-semi-lim$S) \leq (i, j)$-semi-lim $f(S)$.*

Proof. Suppose that $e \in M^*(L_1^X)$, S is an L fuzzy net in L_1^X and $f(e) \in f((i, j)$-semi-lim$S)$. Then $e \in \alpha$-g-lim$S$. By Theorem 3.5 we have $S \to e(\alpha)$. Since f is (i, j)-semi-irresolute, $f(S) \to f(e)(\alpha)$ base on Theorem 3.14 and 3.15. And by Theorem 3.5 we have $f(e) \in (i, j)$-semi-lim $f(S)$. Thus, $f((i, j)$-semi-lim$S) \leq (i, j)$-semi-lim $f(S)$. □

Theorem 3.17. *Let $f : (L_1^X, \tau_1, \tau_2) \to (L_2^Y, \sigma_1, \sigma_2)$ be (i, j)-semi-irresolute. Then for each L-fuzzy net T in L_2^Y we have (i, j)-semi-lim $f^{-1}(T) \leq f^{-1}((i, j)$-semi-lim$T)$.*

Proof. Let $T = \{T(n) : n \in D\}$ be an L-fuzzy net in L_2^Y. Then $f^{-1}(T) = \{f^{-1}(T(n)) : n \in D\}$ and L-fuzzy net in L_1^X. Since f is (i, j)-semi-irresolute, according to Theorem 3.16 we have $f((i, j)$-semi-lim $f^{-1}(T)) \leq (i, j)$-semi-lim $f(f^{-1}(T)) \leq (i, j)$-semi-lim$T$. Hence, (i, j)-semi-lim $f^{-1}(T) \leq f^{-1}((i, j)$-semi-lim$T)$. □

Definition 3.18. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $g \in L^X, r \in L$.*

1. *A collection $\mu = \{f_i\}_{i \in J}$ of L-subsets is called an r-level cover of g if, and only if $(\bigvee_{i \in J} f_i)(x) \nleq r$ for all $x \in X$ with $g(x) \geq r'$. If each f_i is open then μ is called an r-level open cover of g. If g is the whole space 1_X, then μ is called an r-level cover of 1_X if, and only if $(\bigvee_{i \in J} f_i)(x) \nleq r$ for all $x \in X$.*
2. *An r-level cover $\mu = \{f_i\}_{i \in J}$ of g is said to have a finite r-level subcover if there exists a finite subset F of J such that $(\bigvee_{i \in F} f_i)(x) \nleq r$ for all $x \in X$ with $g(x) \geq r'$.*

Definition 3.19. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $g \in L^X$. The L-fuzzy subset g is said to be compact if, and only if for every prime $p \in L$ and every collection*

$\mu = \{f_i\}_{i\in J}$ of open L-subsets with $(\bigvee_{i\in J} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p'$ there exists a finite subset F of J such that $(\bigvee_{i\in F} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p'$, that is every p-level oepn cover of g has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then the L-fuzzy bitopological space (L^X, τ_1, τ_2) is called compact.

Definition 3.20. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $g \in L^X$. The L-fuzy subset g is called (i, j)-semi-compact if and only every p-lever cover of g consiting of (i, j)-semi-open L-subsets has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then we say that the L-fuzzy bitopological space (X, δ) is (i, j)-semi-compact.*

Theorem 3.21. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $g \in L^X$. The L-fuzzy subset g is said to be (i, j)-semi-compact if, and only if for every $\alpha \in M(L)$ and every collection $\{f_i\}_{i\in J}$ of (i, j)-semi-closed L-fuzzy sets with $(\bigwedge_{i\in J} f_i)(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, there exists a fintie subset F of J with $(\bigwedge_{i\in F} f_i)(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, that is, L-fuzzy points $x_\alpha \in M(L^X)$ such that $x_\alpha \leq g$.*

Proof. This follows immediately from Definition 3.20 and the duality of p and α. □

Definition 3.22. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $x_\alpha \in M^*(L^X)$ and $S = (S_m)_{m\in D}$ be a net. Then x_α is called an (i, j)-semi-cluster of S if, and only if for ech (i, j)-semi-closed L-subset f with $f(x) \ngeq \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geq n$ and $S_m \nleq f$, that is $h(S_m) \nleq f(SuppS_m)$.*

Theorem 3.23. *Let (L^X, τ_1, τ_2) be an L-fuzzy bitopological space and $g \in L^X$. The L-fuzzy subset g is said to be (i, j)-semi-compact if, and only if for every constant α-net $(S_m)_{m\in D}$ contained in $g(S_m) \leq g$ for every $m \in D$ has an (i, j)-semi-cluster point with height α, $x_\alpha \in M^*(L^X)$, contained in $g(x_\alpha) \leq g$ for each $\alpha \in M(L)$.*

Proof. Let $\alpha \in M(L)$ and $S = (S_m)_{m\in D}$ be a constant α-net in g without any (i, j)-semi-cluster point with height α in g. Then for each $x \in X$ with $g(x) \geq \alpha$,x_α is not an (i, j)-semi-cluster point of S, that is, there are $n_x \in D$ and an (i, j)-semi-closed L-subset f_x with $f_x(x) \ngeq \alpha$ and $X_m \leq f_x$ for each $m > n_x$. Let $x^1, ..., x^k$ be elements of X with $g(x^i) \geq \alpha$ for each $i \in \{1, ..., k\}$. Then there are $n_{x_1}, ..., n_{x_k} \in D$ and (i, j)-semi-closed L-subset f_{xi} with $f_{x_i}(x^i) \ngeq \alpha$ and $S_m \leq f_{x_i}$ for each $m \geq n_{x_i}$ and for each $i \in \{1, ..., k\}$. Since D is a directed set, there is $n_0 \in D$ such that $n_0 \geq n_{x_i}$ fore ach $i \in \{1, ..., k\}$ and $S_m \leq f_{x_i}$ for $i \in \{1, ..., k\}$ and each $m \geq n_0$. Now, consider $\mu = \{f_x\}_{x\in X}$ with $g(x) \geq \alpha$. Then $(\bigwedge_{f_X\in\mu} f_x)(y) \ngeq \alpha$ for all $y \in X$ with $g(y) \geq \alpha$, because ${}_y(y) \ngeq \alpha$. We also have that for any finit subfamily $\upsilon = \{f_{x_1}, ..., f_{x_k}\}$ of μ, there is $y \in X$ with $g(y) \geq \alpha$ and $(\bigwedge_{k=1}^{n} f_{x_i})(y) \geq \alpha$ since $X_m \leq \bigwedge_{k=1}^{n} f_{xi}$ for ach $m \geq n_0$ because $S_m \leq f_{xi}$ for each $i \in \{1, ..., k\}$ and for each $m \geq n_0$. Hence by Theorem 3.21, g is not (i, j)-semi-compact. Conversly, suppose that g is not (i, j)-semi-compact. Then, by Theorem 3.21, there exist $\alpha \in M(L)$ and a collection $\mu = \{f_i\}_{i\in J}$ fo (i, j)-semi-closed L-subsets with $(\bigwedge_{i\in J} f_i)(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, but for any fintie subfamily υ of μ there is $x \in X$ with $g(x) \geq \alpha$ and $(\bigwedge_{f\in\upsilon} f)(x) \geq \alpha$. Consider the family of all finite subsets of $\mu, 2^\mu$, with the order $\upsilon_1 \leq \upsilon_2$ if, and only if $\upsilon_1 \subseteq \upsilon_2$. Then 2^μ is a directed set. So, writing x_α as S_υ for every $V \in 2^\mu, (S_\upsilon)_{\upsilon\in 2^\mu}$ is a constant α-net in g because the height of X_υ for all $\upsilon \in 2^\mu$ is α and $S_\upsilon \leq g$ for all $\upsilon \in 2^\mu$, that is, $g(x) \geq \alpha$. $(S_\upsilon)_{\upsilon\in 2^\mu}$ also satisfies the condition that for each (i, j)-semi-closed L-subset $f_i \in \upsilon$ we have $x_\alpha = S_\upsilon \leq f_i$. Let $y \in X$ with $g(y) \geq \alpha$. Then $(\bigwedge_{i\in J} f_i)(y) \ngeq \alpha$, that is, there exists $j \in J$ with $f_j(y) \ngeq \alpha$. Let $\upsilon_0 = \{f_j\}$. So, for any $\upsilon \geq \upsilon_0$, $S_\upsilon \leq \bigwedge_{f\in\upsilon} f_i \leq \bigwedge_{f\in\upsilon_0} f_i = f_j$. Thus, we get an (i, j)-semi-closed L-subset f_j with $f_j(y) \nleq \alpha$ and $\upsilon_0 \in 2^\mu$ such that for any $\upsilon \geq \upsilon_0$, $S_\upsilon \leq f_j$. That means that $y_\alpha \in M^*(L^X)$ is not an (i, j)-semi-cluster point of $(X_\upsilon)_{\upsilon\in 2^\mu}$ for all $y \in X$ with $g(y) \geq \alpha$. Hence,the constant α-net $(S_\upsilon)_{\upsilon\in 2^\mu}$ has no (i, j)-semi-cluster point in g with height α. □

Corollary 3.24. *An L-fuzzy bitopological space is an (i, j)-semi-compact if, and only if every constant α-net in (L^X, τ_1, τ_2) has an (i, j)-semi-cluster point with height α, where $\alpha \in M(L)$.*

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