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Complementary role of ideals in TSBF−algebras

M. Jansi[1]* and V. Thiruveni[2]

Abstract

In this paper we introduced the notions of ideals related to subalgebras of BF-algebras and subalgebras of topological soft BF-algebras and we discuss the complementry role of these ideals on topological soft BF-algebras collaborated with some cardinal concepts - interior and closure of a topological space.

Keywords

TSBF-algebra, $S-$ Ideal of $BF-$ algebra, Interior and Closure of a subset of a $BF-$ algebra.

AMS Subject Classification

06F35, 54A10, 54A05.

[1] *Department of Mathematics, M.S. University Constituent College, Nagalapuram-628904, Tamil Nadu, India.*

[2] *PG and Resarch Department of Mathematics, Saiva Bhanu Kshatriya College, Aruppukottai-626101, Tamil Nadu, India.*

***Corresponding author**: [2] thiriveni2009@gmail.com; [1] mjansi92@gmail.com

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Contents

1 Introduction 1037

2 Preliminaries 1037

3 Complementry Role of Ideals on TSBF- Algebras 1038

4 Conclusion 1039

References 1039

1. Introduction

In [1] Walendziak introduced the notion of BF-algebras, which is a generalization of $B-$algebras. He also introduced the notions of Ideals and normal ideal in $BF-$algebras and studied their properties and characterization. In [2] R.A.Borzooei, G.R. Rezaei and N. Kouhietan studied the topological BL-algebras and discussed their properties. In [7], Y. B. Jun and C. H. Park introduced the notion of ideals reflated to subalgebras on BCK/BCI- algebras. In [6], we introduced the notion of topological $BF-$algebras and in [13], we introduced the notion of topological soft $BF-$ algebras. Motivated by this, in this paper we introduced the notion of $S-$ideal on topological and topological soft $BF-$ algebras and studied their properties. The aim of this article is to lay a foundation for providing a soft algebaric tool in considering problems that contain uncertainties and study the connection between soft BF-algebras and topology.

2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. *[11] A BF-algebra is an algebra $(X,*,0)$ of type $(2,0)$ (that is, a non-empty set X with a binary operation $*$ and a constant 0) satisfying the following conditions*

1. $x*x=0$,
2. $x*0=x$,
3. $0*(x*y)=y*x, \forall x,y\in X$.

Definition 2.2. *[11] Let $(X,*,0)$ be a BF-algebra. A nonempty subset N of X is called a subalgebra of X if $x*y\in N$, for all $x,y\in N$.*

Definition 2.3. *[9] A pair (F,E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U. where, U is an initial universal set and E is a set of parameters.*

Definition 2.4. *[13] Let (F,A) be a soft BF-algebra over a BF-algebra X and τ be a topology on X. Let $x\in X$. Then (F,A,τ) is said to be a topological soft BF-algebra (TSBF-algebra) over X with respect to $F(x)$, if for every $a,b\in F(x)$ and any open set W of $a*b$, there exist open sets U and V of a and b respectively such that $U*V\subseteq W$.*

Definition 2.5. *[9] Let (F,A) be a soft set over a BF-algebra X. Then (F,A) is called a soft BF-algebra over X if $F(x)$ is a subalgebra of X, $\forall x\in A$.*

Theorem 2.6. *[13] Let (F,A,τ) be a TSBF-algebra with respect to $F(a)$. If $\{0\}$ is open and for every distinct $x,y\in X$, the value $x*y\neq 0$ then all the subsets of $F(a)$ becomes open.*

Definition 2.7. *[4] An ideal of a BF-algebra X is a subset I containing 0 such that if $x*y \in I$ and $y \in I$ then $x \in I$.*

Definition 2.8. *[5] For a subset of a topological space X, the interior of A is defined as the union of all open sets contained in A and the closure of A is defined as the intersection of all closed sets containing A.*

3. Complementry Role of Ideals on TSBF- Algebras

In this section, we define $S-$ideal on $TSBF-$algebras and discuss their roles on $TSBF-$algebras.

Definition 3.1. *Let (F,A,τ) be a TSBF-algebra over a $BF-$ algebra X. A non-empty subset N of X is called a subalgebra of X if $x*y \in N$, for all $x,y \in N$.*

Definition 3.2. *An ideal of a TSBF-algebra (F,A,τ) over a $BF-$ algebra X. is a subset I of X, containing 0 such that if $x*y \in I$ and $y \in I$ then $x \in I$.*

Definition 3.3. *Let $(X,0,*)$ be a BF-algebra and S be a subalgebra of X. A subset I of X is called an ideal of X related to S ($S-$ideal of X) if it satisfies the condition*

$$\forall x \in S \text{ and } \forall y \in I, x*y \in I \implies x \in I$$

Example 3.4. *Consider the BF-algebra $X = \{0,1,2,3,4\}$ with the following Cayley table,*

*	0	1	2	3	4
0	0	1	2	3	4
1	1	0	1	0	1
2	2	1	0	1	0
3	3	0	1	0	1
4	4	1	0	1	0

Considering the subalgebra $S = \{0,1,2\}$, the subset $I = \{0,2,4\}$ is an $S-$ideal.

Definition 3.5. *Let (F,A,τ) be a TSBF-algebra with respect to $F(a)$ over a $BF-$ algebra X and S be a subalgebra of X. A subset I of X is called an ideal of X related to S ($S-$ideal of X) if it satisfies the condition*

$$\forall x \in S \text{ and } \forall y \in I, x*y \in I \implies x \in I$$

Example 3.6. *Consider the BF-algebra X in example 3.4. Let $A = \{1,2\}$.*

Define a set $F : A \to \mathbb{P}(X)$ by $F(1) = \{0,1\}$, $F(2) = \{0,1,2\}$. Then $F(1)$ and $F(2)$ are $BF-$ subalgebras of X. $\implies (F,A)$ is a soft $BF-$ algebra.
Consider the topology $\tau = \{X, \phi, \{0,2\}, \{0,1,2\}, \{1\}, \{0,1,2,3\}, \{0,1,2,4\}\}$
Then (F,A,τ) is a TSBF-algebra with respect to $F(2)$.
We can verify that $I = \{0,2,4\}$ is an $F(2)-$ ideal.

Definition 3.7. *Let (F,A,τ) be a TSBF-algebra with respect to $F(a)$ and S be a subalgebra of X. For a subset B of X, the interior of B is defined as the union of all open sets contained in B and the closure of B is defined as the intersection of all closed sets containing B.*

The interior of B is denoted by Int B and the closure of B is denoted by Cl B or $\bar{B}$.

Example 3.8. *In example 3.4, if we consider $B = \{0,1\}$, then Int $B = \{1\}$ and $\bar{B} = X$.*

Remark 3.9. *As a TSBF-algebra (F,A,τ) is naturally a topological space (X,τ), the following properties follows trivially for every subset B of X*

1. *$IntB \subseteq B \subseteq \bar{B}$*
2. *$B = IntB$ whenever B is open*
3. *$B = \bar{B}$ whenever B is closed*

Theorem 3.10. *Let $(X,*,0)$ be a BF-algebra and S be any subalgebra of X, then $\{0\}$ an $S-$ ideal.*

Proof: Let S be a subalgebra of X and $I = \{0\}$.
Since $x*0 = x$ and $x \notin I$, we must have $\{0\}$ is an $S-$ideal.

Remark 3.11. *$I = \{0\}$ is an $S-$ideal for any subalgebra S of X and referred as an improper $S-$ideal of X.*

Definition 3.12. *Let (F,A,τ) is a TSBF-algebra with respect to $F(a)$ over X and $B \subseteq X$. Then H is the least open set of B if H is the intersection of all open sets containing B.*

Theorem 3.13. *Let (F,A) be a soft BF-algebra over X and B be a subalgebra of X which contains $F(a)$, for some $a \in A$. If $\tau = \{X, \phi, B\}$ is a topology for X then (F,A,τ) is a TSBF-algebra with respect to $F(a)$.*

Proof. Let $x,y \in F(a)$. Then $x*y \in F(a)$.
Since B contains $F(a)$, we have x,y and $x*y \in B$.
Now, B is the least open set for $x*y$.
Therefore, we can choose open sets U and V of x and y respectively such that $U*V \subseteq B$.
Since B is minimal, we have $U = V = B$. And hence $U*V = B*B$.
Since B is a subalgebra of X, $U*V = B$.
Therefore, (F,A,τ) is a TSBF-algebra with respect to $F(a)$. ☐

Theorem 3.14. *Let (F,A,τ) be a TSBF-algebra with respect to $F(a)$. If $F(a)$ is the least open set containing 0 then $F(a)$ is also the least open set for every x in $F(a)$.*

Proof. We want to prove that, $F(a)$ is a minimal open set for all $x \in F(a)$.
Suppose there exists an open set $B \subset F(a)$ of $x \in F(a)$. Since $F(a)$ is the least open set of 0, $0 \notin B$.
Since $x*0 = x$, there exists open sets U and V of x and 0 respectively for every open set W of x such that $U*V \subseteq W$.
Therefore, $U*V \subseteq B$.....(1)

Since $F(a)$ is the least open set of $0, V$ contains $F(a) \Rightarrow x, 0 \in V$.

Now, (1)implies $x * x \in B$. But $x * x = 0$ *and* $0 \notin B$, which implies $U * V \nsubseteq B$.

This contradicts the fact that (F, A, τ) is a TSBF-algebra with respect to $F(a)$.

Therefore, we conclude that there is no such B. That is, $F(a)$ is the least open set for every x in $F(a)$. □

Theorem 3.15. *Let (F, A, τ) be a TSBF-algebra with respect to $F(a)$ over X and for every distinct $x, y \in F(a)$, $x * y \neq 0$. If the ideals related to $F(a)$ are open then all the subsets of $F(a)$ becomes open.*

Proof. Since $x * 0 = x$, $\{0\}$ is an ideal related to $F(a)$. As, $\{0\}$ is open, from theorem 2.6, all the subsets of $F(a)$ becomes open. □

Theorem 3.16. *Let (F, A, τ) be a TSBF-algebra with respect to $F(a)$. If 0 is an interior point of an ideal $I \subseteq F(a)$ of X then $IntI = I$.*

Proof. We want to prove that, every $x \in I$ is an interior point of I.Let $x \in I$.

Since $x * x = 0$, for every open set W of 0, there exists open sets U and V of x such that $U * V \subseteq W$.

As 0 is an interior point of I, $U * V \subseteq I$....(1)

If $U \subseteq I$, then there is nothing to prove.

Suppose $U \nsubseteq I$, there exists an element y in X such that $y \in U$ *and* $y \notin I$....(2)

Now, (1) implies, $y * x \in I, \forall x \in V$.

Since I is an ideal of X, $y \in I$, which is a contradiction to (2).

$\Rightarrow U \subseteq I$.

Therefore we conclude that,for every $x \in I$,there exits an open set U which is contained in I. Therefore, Int $I = I$. □

Definition 3.17. *Let (F, A, τ) is a TSBF-algebra with respect to $F(a)$ overX. If $0 * x = x, \forall x \in X$ then (F, A, τ) is called a commutative TSBF-algebra with respect to $F(a)$.*

Theorem 3.18. *Let (F, A, τ) be a commutative TSBF-algebra with respect to $F(a)$ and I be an ideal contained in $F(a)$. If 0is not an interior point of I then Int $I = \phi$.*

Proof. Let $I \subseteq F(a)$. Since 0 is not an interior point of I, every open set H of 0 contains at least one element y (say) such that $y \notin I$....(1)

Let $x \in I$. Since (F, A, τ) is commutative, $0 * x = x$.

Hence for every open set W of x, there exists an open sets U and V of 0 and x respectively such that $U * V \subseteq W$.

Suppose x is an interior point of I. Then $U * V \subseteq I$.

From (1), $y \in U, 0 \in U$ *and* $x \in V$, which implies $y * x \in I$.

Since I is an ideal and $x \in I$ we have $y \in I$, which is a contradiction to (1).

Therefore, x is not an interior point of I and this is true for all $x \in I$.

Therefore, *Int* $I = \phi$. □

Theorem 3.19. *Let (F, A, τ) be a TSBF-algebra with respect to $F(a)$. If any ideal related to $F(a)$ is open and if I is an open ideal of X, then $I^c \cap F(a)$ is open.*

Proof. Let $x \in I^c \cap F(a) \implies x \in F(a)$ *and* $x \in I^c \implies x \in F(a)$ and $x \notin I$......(1)

Since $x * x = 0$, for every open set W of 0, there exists an open sets U and V of x such that $U * V \subseteq W$.

Since $0 \in I$ and I is open, $U * V \subseteq I$.

If $V \subseteq I^c \cap F(a)$, then $I^c \cap F(a)$ is open.

Suppose $V \nsubseteq I^c \cap F(a)$, there exists an element $y \in V$ *and* $y \notin I^c \cap F(a)$

Case(i): Suppose, $y \in V, y \notin I^c$ *and* $y \notin F(a)$ or $y \in V, y \notin I^c$ *and* $y \in F(a)$.

Then $y \in I$. Then since $x * y \in I$ and I is an ideal, we have $x \in I$, which contradicts (1).

Hence, $V \subseteq I^c \cap F(a)$.

Case(ii): Suppose, $y \in V, y \in I^c$ *and* $y \notin F(a)$. Since ideals related to $F(a)$ are open, $F(a)$ is open, which implies $V \cap F(a)$ is open and contains x. Also $V \cap F(a)$is contained in $I^c \cap F(a)$.

Therefore in both the cases, for every x in $I^c \cap F(a)$ there exists an open set contained in $I^c \cap F(a)$. Therefore $I^c \cap F(a)$ is open. □

With the similar arguments in the above theorem, we can prove the following:

Theorem 3.20. *Let (F, A, τ) be a TSBF-algebra with respect to $F(a)$. If any ideal related to $F(a)$ is open and if I is an ideal of X such that 0 is an interior point of I, then $I^c \cap F(a)$ is open.*

4. Conclusion

As a result of this study, we got the idea of building topologies on TSBF – algebras with respect to F(a) over a BF- algebra. Based on this study, we can identify some basic sets such as open set, closed set, interior of a set and the closure of a set, which plays main role in the construction of TSBF-algebra with respect to F(a) over a BF- algebra.

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