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# On faintly $b$-$\mathscr{I}$-continuous multifunctions

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**Abstract**
The aim of this paper is to introduce and study upper and lower faintly $b$-$\mathscr{I}$-continuous multifunctions as a generalization of upper and lower $b$-$\mathscr{I}$-continuous multifunctions, respectively.

**Keywords**
Ideal topological spaces, $b$-$\mathscr{I}$-open sets, $b$-$\mathscr{I}$-closed sets, faintly $b$-$\mathscr{I}$-continuous multifunctions.

**AMS Subject Classification**
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## 1. Introduction

It is well known that various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good number of them have been extended to the setting of multifunctions [4, 8, 10, 12, 13]. This implies that both, functions and multifunctions are important tools for studying other properties of spaces and for constructing new spaces from previously existing ones. The concept of ideals in topological spaces has been introduced and studied by Kuratowski [7] and Vaidyanathaswamy, [16]. An ideal $\mathscr{I}$ on a topological space $(X,\tau)$ is a nonempty collection of subsets of $X$ which satisfies (i) $A\in\mathscr{I}$ and $B\subset A$ implies $B\in\mathscr{I}$ and (ii) $A\in\mathscr{I}$ and $B\in\mathscr{I}$ implies $A\cup B\in\mathscr{I}$. Given a topological space $(X,\tau)$ with an ideal $\mathscr{I}$ on $X$ and if $\mathscr{P}(X)$ is the set of all subsets of $X$, a set operator $(\cdot)^\star$: $\mathscr{P}(X)\to\mathscr{P}(X)$, called the local function [16] of $A$ with respect to $\tau$ and $\mathscr{I}$, is defined as follows: for $A\subset X$, $A^\star(\tau,\mathscr{I})=\{x\in X|U\cap A\notin\mathscr{I}$ for every $U\in\tau(x)\}$, where $\tau(x)=\{U\in\tau|x\in U\}$. A. Kuratowski closure operator $\mathrm{Cl}^\star(.)$ for a topology $\tau^\star(\tau,\mathscr{I})$ called the $*$-topology, finer than $\tau$ is defined by $\mathrm{Cl}^\star(A)=A\cup A^\star(\tau,\mathscr{I})$ when there is no chance of confusion, $A^\star(\mathscr{I})$ is denoted by $A^\star$. If $\mathscr{I}$ is an ideal on $X$, then $(X,\tau,\mathscr{I})$ is called an ideal topological space. In 2007, Akadag [1] introduced the notion of $b$-$\mathscr{I}$-open sets in ideal topological space. In this paper, we introduce and study upper and lower faintly $b$-$\mathscr{I}$-continuous multifunctions on ideal topological space.

## 2. Preliminaries

For a subset $A$ of a topological space $(X,\tau)$, $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of $A$ with respect to $\tau$ and the interior of $A$ with respect to $\tau$, respectively. A subset $S$ of an ideal topological space $(X,\tau,\mathscr{I})$ is said to be $b$-$\mathscr{I}$-open [1] if $S\subset\mathrm{Int}(\mathrm{Cl}^*(S))\cup\mathrm{Cl}^*(\mathrm{Int}(S))$. The complement of a $b$-$\mathscr{I}$-open set is called a $b$-$\mathscr{I}$-closed set. The $b$-$\mathscr{I}$-closure [1] and the $b$-$\mathscr{I}$-interior [1], that can be defined in the same way as $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively, will be denoted by $b\mathscr{I}\,\mathrm{Cl}(A)$ and $b\mathscr{I}\,\mathrm{Int}(A)$, respectively. The family of all $b$-$\mathscr{I}$-open (resp. $b$-$\mathscr{I}$-closed) sets of $(X,\tau,\mathscr{I})$ is denoted by $B\mathscr{I}O(X)$ (resp. $B\mathscr{I}C(X)$). The family of all $b$-$\mathscr{I}$-open (resp. $b$-$\mathscr{I}$-closed) sets of $(X,\tau,\mathscr{I})$ containing a point $x\in X$ is denoted by $B\mathscr{I}O(X,x)$ (resp. $B\mathscr{I}C(X,x)$). A subset $N$ of an ideal topological space $(X,\tau,\mathscr{I})$ is said to be $b$-$\mathscr{I}$-neighborhood of a point $x\in X$, if there exists a $b$-$\mathscr{I}$-open set $V$ such that $x\in V\subset N$. A point $x\in X$ is called a $\theta$-cluster point of $A$ [15] if $\mathrm{Cl}(V)\cap A\neq\varnothing$ for every open set $V$ of $X$ containing $x$. The set of all $\theta$-cluster points of $A$ is called the $\theta$-closure of $A$ and is denoted by $\mathrm{Cl}_\theta(A)$. If $A=\mathrm{Cl}_\theta(A)$, then $A$ is said to be $\theta$-closed [15]. The complement of a $\theta$-closed set is said to be a $\theta$-open set [15]. The union of all $\theta$-open sets contained in a subset $A$ is called the $\theta$-interior of $A$ and is denoted by $\text{Int}_\theta(A)$. It follows from [15] that the collection of $\theta$-open sets in a topological space $(X,\tau)$ forms a topology $\tau_\theta$ on $X$. By a multifunction $F:(X,\tau)\to(Y,\sigma)$, we mean a point-to-set correspondence from $X$ into $Y$, also we always assume that $F(x)\neq\varnothing$ for all $x\in X$. For a multifunction $F:(X,\tau)\to(Y,\sigma)$, the upper and lower inverse of any subset $A$ of $Y$ by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A)=\{x\in X: F(x)\subseteq A\}$ and $F^-(A)=\{x\in X: F(x)\cap A\neq\varnothing\}$. In particular, $F^-(y)=\{x\in X: y\in F(x)\}$ for each point $y\in Y$. A multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is said to be lower $b$-$\mathscr{I}$-continuous [4] (resp. upper $b$-$\mathscr{I}$-continuous) multifunction if $F^-(V)\in B\mathscr{I}O(X)$ (resp. $F^+(V)\in B\mathscr{I}O(X)$) for every $V\in\sigma$.

## 3. Faintly $b$-$\mathscr{I}$-continuous multifunctions

**Definition 3.1.** *A multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is said to be :*

1. *upper faintly b-$\mathscr{I}$-continuous at $x\in X$ if for each $V\in\sigma_\theta$ containing $F(x)$, there exists $U\in B\mathscr{I}O(X)$ containing $x$ such that $F(U)\subset V$;*
2. *lower faintly b-$\mathscr{I}$-continuous at $x\in X$ if for each $V\in\sigma_\theta$ such that $F(x)\cap V\neq\varnothing$, there exists $U\in B\mathscr{I}O(X)$ containing $x$ such that $F(u)\cap V\neq\varnothing$ for every $u\in U$;*
3. *upper (lower) faintly b-$\mathscr{I}$-continuous if it has this property at each point of $X$.*

**Remark 3.2.** *It is clear that every upper b-$\mathscr{I}$-continuous multifunction is upper faintly b-$\mathscr{I}$-continuous. But the converse is not true in general, as the following example shows.*

**Example 3.3.** *Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{b\},X\}$, $\sigma=\{\emptyset,\{a\},X\}$ and $\mathscr{I}=\{\emptyset,\{a\}\}$. The multifunction $F:(X,\tau,\mathscr{I})\to(X,\sigma)$ defined by $F(x)=\{x\}$ for all $x\in X$ is upper faintly b-$\mathscr{I}$-continuous but is not upper b-$\mathscr{I}$-continuous.*

**Definition 3.4.** *A sequence $(x_\alpha)$ is said to b-$\mathscr{I}$-converge to a point $x$ if for every $V\in B\mathscr{I}O(X,x)$, there exists an index $\alpha_0$ such that for $\alpha\geq\alpha_0$, $\alpha_n\in V$. This is denoted by $x_\alpha \underset{\longrightarrow}{b\mathscr{I}} x$.*

**Theorem 3.5.** *For a multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$, the following statements are equivalent :*

1. *$F$ is upper faintly b-$\mathscr{I}$-continuous;*
2. *For each $x\in X$ and for each $V\in\sigma_\theta$ such that $x\in F^+(V)$, there exists $U\in B\mathscr{I}O(X,x)$ such that $U\subset F^+(V)$;*
3. *For each $x\in X$ and for each $\theta$-closed set $V$ such that $x\in F^+(Y\backslash V)$, there exists $H\in B\mathscr{I}C(X)$ such that $x\in X\backslash H$ and $F^-(V)\subset H$;*
4. *$F^+(V)\in B\mathscr{I}O(X)$ for any $V\in\sigma_\theta$;*
5. *$F^-(V)\in B\mathscr{I}C(X)$ for any $\theta$-closed set $V$ of $Y$;*
6. *$F^-(Y\backslash V)\in B\mathscr{I}C(X)$ for any $V\in\sigma_\theta$;*
7. *$F^+(Y\backslash V)\in B\mathscr{I}O(X)$ for any $\theta$-closed set $V$ of $Y$;*
8. *For each $x\in X$ and for each net $(x_\alpha)$ which b-$\mathscr{I}$-converges to $x\in X$ and for each $V\in\sigma_\theta$ such that $x\in F^+(V)$, the net $(x_\alpha)$ is eventually in $F^+(V)$.*

*Proof.* (1)⇔(2): Clear.
(2)⇔(3): Let $x\in X$ and $V\in\sigma_\theta$ such that $x\in F^+(Y\backslash V)$. By (2), there exists $U\in B\mathscr{I}O(X,x)$ such that $U\subset F^+(Y\backslash V)$. Then $F^-(V)\subset X\backslash U$. Take $H=X\backslash U$. We have $x\in X\backslash H$ and $H\in B\mathscr{I}O(X)$. The converse is similar.
(1)⇔(4): Let $x\in F^+(V)$ and $V\in\sigma_\theta$. By (1), there exists $U_x\in B\mathscr{I}O(X,x)$ such that $U_x\subset F^+(V)$. It follows that $F^+(V)=\bigcup_{x\in F^+(V)} U_x$. Since any union of $b$-$\mathscr{I}$-open sets is $b$-$\mathscr{I}$-open, $F^+(V)\in B\mathscr{I}O(X)$. The converse can be shown similarly.
(4)⇔(5)⇔(6)⇔(7)⇔(8) : Clear.
(1)⇒(8): Let $(x_\alpha)$ be a net which $b$-$\mathscr{I}$-converges to $x$ in $X$ and let $V\in\sigma_\theta$ such that $x\in F^+(V)$. Since $F$ is an upper faintly $b$-$\mathscr{I}$-continuous multifunction, there exists $U\in B\mathscr{I}O(X,x)$ such that $U\subset F^+(V)$. Since $(x_\alpha)$ $b$-$\mathscr{I}$-converges to $x$, there exists an index $\alpha_0\in J$ such that $x_\alpha\in U$ for all $\alpha\geq\alpha_0$. From here, we obtain that $x_\alpha\in U\subset F^+(V)$ for all $\alpha\geq\alpha_0$. Thus, the net $(x_\alpha)$ is eventually in $F^+(V)$.
(8) ⇒ (1): Suppose that (1) is not true. There exists a point $x$ and $V\in\sigma_\theta$ with $x\in F^+(V)$ such that $U\not\subseteq F^+(V)$ for each $U\in B\mathscr{I}O(X,x)$. Let $x_U\in U$ and $x_U\notin F^+(V)$ for each $U\in B\mathscr{I}O(X,x)$. Then for each $b$-$\mathscr{I}$-neighborhood net $(x_U)$, $x_U \underset{\longrightarrow}{b\mathscr{I}} x$, but $(x_U)$ is not eventually in $F^+(V)$. This is a contradiction. Thus, $F$ is an upper faintly $b$-$\mathscr{I}$-continuous multifunction. □

**Theorem 3.6.** *For a multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$, the following statements are equivalent :*

1. *$F$ is lower faintly b-$\mathscr{I}$-continuous;*
2. *For each $x\in X$ and for each $V\in\sigma_\theta$ such that $x\in F^-(V)$, there exists $U\in B\mathscr{I}O(X,x)$ such that $U\subset F^-(V)$;*
3. *For each $x\in X$ and for each $\theta$-closed set $V$ such that $x\in F^-(Y\backslash V)$, there exists $H\in B\mathscr{I}C(X)$ such that $x\in X\backslash H$ and $F^+(V)\subset H$;*
4. *$F^-(V)\in B\mathscr{I}O(X)$ for any $V\in\sigma_\theta$;*
5. *$F^+(V)\in B\mathscr{I}C(X)$ for any $\theta$-closed set $V$ of $Y$;*
6. *$F^+(Y\backslash V)\in B\mathscr{I}C(X)$ for any $V\in\sigma_\theta$;*
7. *$F^-(Y\backslash V)\in B\mathscr{I}O(X)$ for any $\theta$-closed set $V$ of $Y$;*
8. *For each $x\in X$ and for each net $(x_\alpha)$ which b-$\mathscr{I}$-converges to $x\in X$ and for each $V\in\sigma_\theta$ such that $x\in F^-(V)$ the net $(x_\alpha)$ is eventually in $F^-(V)$.*

*Proof.* The proof is similar to that of Theorem 3.5. □

**Lemma 3.7.** *[1] Let $A$ and $B$ be subsets of an ideal topological space $(X,\tau,\mathscr{I})$. If $A \in B\mathscr{I}O(X)$ and $B \in \tau$, then $A \cap B \in B\mathscr{I}O(B)$.*

**Theorem 3.8.** *Let $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a multifunction and $U \in \tau$. If $F$ is a lower (upper) faintly b-$\mathscr{I}$-continuous multifunction, then multifunction $F_{|U}:(U,\tau|U,\mathscr{I}|U) \to (Y,\sigma)$ is a lower (upper) faintly b-$\mathscr{I}|U$-continuous multifunction.*

*Proof.* Let $V \in \sigma_\theta$, $x \in U$ and $x \in F_{|U}^-(V)$. Since $F$ is a lower faintly $b$-$\mathscr{I}$-continuous multifunction, there exists $G \in B\mathscr{I}O(X,x)$ such that $G \subset F^-(V)$. From here by Lemma 3.7, we obtain that $x \in G \cap U \in B\mathscr{I}O(U)$ and $G \cap U \subset F_{|U}^-(V)$. This shows that the restriction multifunction $F_{|U}$ is a lower faintly $b$-$\mathscr{I}|U$-continuous. The proof of the upper faintly $b$-$\mathscr{I}|U$-continuity of $F_{|U}$ can be done by the similar manner. □

**Lemma 3.9.** *[11] The following hold for a multifunction $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$:*

1. *$G_F^+(A \times B) = A \cap F^+(B)$ and*
2. *$G_F^-(A \times B) = A \cap F^-(B)$*

*for each subsets $A \subset X$ and $B \subset Y$.*

**Theorem 3.10.** *Let $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a multifunction. If the graph multifunction of $F$ is an upper faintly b-$\mathscr{I}$-continuous, then $F$ is upper faintly b-$\mathscr{I}$-continuous.*

*Proof.* Let $x \in X$ and $V \in \sigma_\theta$ such that $x \in F^+(V)$. We obtain that $x \in G_F^+(X \times V)$ and that $X \times V$ is a $\theta$-open set. Since the graph multifunction $G_F$ is upper faintly $b$-$\mathscr{I}$-continuous, it follows that there exists $U \in B\mathscr{I}O(X,x)$ such that $U \subset G_F^+(X \times V)$. Since $U \subset G_F^+(X \times V) = X \cap F^+(V) = F^+(V)$, $U \subset F^+(V)$. Thus, $F$ is upper faintly $b$-$\mathscr{I}$-continuous. □

**Theorem 3.11.** *A multifunction $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$ is lower faintly b-$\mathscr{I}$-continuous if $G_F:(X,\tau,\mathscr{I}) \to (X \times Y, \tau \times \sigma)$ is lower faintly b-$\mathscr{I}$-continuous.*

*Proof.* Suppose that $G_F$ is lower faintly $b$-$\mathscr{I}$-continuous. Let $x \in X$ and $V \in \sigma_\theta$ such that $x \in F^-(V)$. Then $X \times V$ is $\theta$-open in $X \times Y$ and $G_F(x) \cap (X \times V) = (\{x\} \times F(x)) \cap (X \times V) = \{x\} \times (F(x) \cap V) \neq \varnothing$. Since $G_F$ is lower faintly $b$-$\mathscr{I}$-continuous, there exists $U \in B\mathscr{I}O(X,x)$ such that $U \subset G_F^-(X \times V)$; hence $U \subset F^-(V)$. This shows that $F$ is lower faintly $b$-$\mathscr{I}$-continuous. □

**Theorem 3.12.** *Suppose that $(X,\tau,\mathscr{I})$ is an ideal topological space and $(X_\alpha,\tau_\alpha)$ are topological spaces where $\alpha \in J$. Let $F:(X,\tau,\mathscr{I}) \to \prod_{\alpha\in J} X_\alpha$ be a multifunction from $(X,\tau,\mathscr{I})$ to the product space $\prod_{\alpha\in J} X_\alpha$ and let $P_\alpha: \prod_{\alpha\in J} X_\alpha \to X_\alpha$ be the projection multifunction for each $\alpha \in J$ which is defined by $P_\alpha((x_\alpha)) = \{x_\alpha\}$. If $F$ is an upper (lower) faintly b-$\mathscr{I}$-continuous multifunction, then $P_\alpha \circ F$ is an upper (lower) faintly b-$\mathscr{I}$-continuous multifunction for each $\alpha \in J$.*

*Proof.* Take any $\alpha_0 \in J$. Let $V_{\alpha_0}$ be a $\theta$-open set in $(X_{\alpha_0},\tau_{\alpha_0})$. Then $(P_{\alpha_0} \circ F)^+(V_{\alpha_0}) = F^+(P_{\alpha_0}^+(V_{\alpha_0})) = F^+(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$ (resp. $(P_{\alpha_0} \circ F)^-(V_{\alpha_0}) = F^-(P_{\alpha_0}^-(V_{\alpha_0})) = F^-(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$). Since $F$ is an upper (lower) faintly $b$-$\mathscr{I}$-continuous multifunction and since $V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha$ is a $\theta$-open set, $F^+(V_{\alpha_0} \times \prod_{\alpha\neq 0} X_\alpha)$ (resp. $F^-(V_{\alpha_0} \times \prod_{\alpha\neq\alpha_0} X_\alpha)$) is a $b$-$\mathscr{I}$-open set in $(X,\tau,\mathscr{I})$. This shows that $P_\alpha \circ F$ is an upper (lower) faintly $b$-$\mathscr{I}$-continuous multifunction. Hence $P_{\alpha_0} \circ F$ is an upper (lower) faintly $b$-$\mathscr{I}$-continuous multifunction for each $\alpha \in J$. □

**Theorem 3.13.** *Suppose that for each $\alpha \in J$, $(X,\tau,\mathscr{I})$ is an ideal topological space and $(Y_\alpha,\sigma_\alpha)$ is a topological space. Let $F_\alpha: X_\alpha \to Y_\alpha$ be a multifunction for each $\alpha \in J$ and let $F: \prod_{\alpha\in J} X_\alpha \to \prod_{\alpha\in J} Y_\alpha$ be defined by $F((x_\alpha)) = \prod_{\alpha\in J} F_\alpha(x_\alpha)$ from the product space $\prod_{\alpha\in J} X_\alpha$ to the product space $\prod_{\alpha\in J} Y_\alpha$. If $F$ is an upper (lower) faintly b-$\mathscr{I}$-continuous multifunction, then each $F_\alpha$ is an upper (lower) faintly b-$\mathscr{I}$-continuous multifunction for each $\alpha \in J$.*

*Proof.* Let $V_\alpha$ be a $\theta$-open set of $Y_\alpha$. Then $V_\alpha \times \prod_{\alpha\neq\beta} Y_\beta$ is a $\theta$-open set. Since $F$ is an upper (lower) faintly $b$-$\mathscr{I}$-continuous multifunction, it follows that $F^+(V_\alpha \times \prod_{\alpha\neq\beta} Y_\beta) = F_\alpha^+(V_\alpha) \times \prod_{\alpha\neq\beta} X_\beta$ (resp. $F^-(V_\alpha \times \prod_{\alpha\neq\beta} Y_\beta) = F_\alpha^-(V_\alpha) \times \prod_{\alpha\neq\beta} X_\beta$) is a $b$-$\mathscr{I}$-open set. Consequently, we obtain that $F_\alpha^+(V_\alpha)$ (resp. $F_\alpha^-(V_\alpha)$) is a $b$-$\mathscr{I}$-open set. Thus, we show that $F_\alpha$ is an upper (lower) faintly $b$-$\mathscr{I}$-continuous multifunction. □

**Definition 3.14.** *A topological space $(X,\tau)$ is said to be $\theta$-$T_2$ [14] if for each pair of distinct points $x$ and $y$ in $X$, there exist disjoint $\theta$-open sets $U$ and $V$ in $X$ such that $x \in U$ and $y \in V$.*

**Definition 3.15.** *An ideal topological space $(X,\tau,\mathscr{I})$ is said to be b-$\mathscr{I}$-$T_2$ if for each pair of distinct points $x$ and $y$ in $X$, there exist $U,V \in B\mathscr{I}O(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

**Theorem 3.16.** *Let $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$ be an upper faintly b-$\mathscr{I}$-continuous multifunction and punctually closed from a topological space $X$ to a $\theta$-normal space $Y$ and let $F(x) \cap F(y) = \varnothing$ for each pair of distinct points $x$ and $y$ of $X$. Then $(X,\tau,\mathscr{I})$ is a b-$\mathscr{I}$-$T_2$ space.*

*Proof.* Let $x$ and $y$ be any two distinct points in $X$. Then we have $F(x) \cap F(y) = \varnothing$. Since $Y$ is $\theta$-normal, it follows that there exist disjoint $\theta$-open sets $U$ and $V$ containing $F(x)$ and $F(y)$, respectively. Thus $F^+(U)$ and $F^+(V)$ are disjoint $b$-$\mathscr{I}$-open sets containing $x$ and $y$, respectively and hence $(X,\tau,\mathscr{I})$ is $b$-$\mathscr{I}$-$T_2$. □

**Definition 3.17.** *Let $F:(X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a multifunction. The multigraph $G(F)$ is said to be b-$\mathscr{I}$-$\theta$-closed if for each $(x,y) \notin G(F)$, there exist b-$\mathscr{I}$-open set $U$ and $V \in \sigma_\theta$*

*containing x and y, respectively such that* $(U\times V)\cap G(F)=\varnothing$.

**Theorem 3.18.** *If a multifunction* $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ *is an upper faintly b-*$\mathscr{I}$*-continuous such that* $F(x)$ *is* $\theta$*-compact relative to Y for each* $x\in X$ *and Y is a* $\theta$*-*$T_2$ *space, then the multigraph* $G(F)$ *of F is b-*$\mathscr{I}$*-*$\theta$*-closed in* $X\times Y$.

*Proof.* Let $(x,y)\in(X\times Y)\backslash G(F)$. That is $y\notin F(x)$. Since $Y$ is $\theta$-$T_2$ for each $z\in F(x)$, there exist disjoint $\theta$-open sets $V(z)$ and $U(z)$ of $Y$ such that $z\in U(z)$ and $y\in V(y)$. Then $\{U(z):z\in F(x)\}$ is a $\theta$-open cover of $F(x)$ and since $F(x)$ is $\theta$-compact subset relative to $Y$, there exists a finite number of points, say, $z_1,z_2,...,z_n$ in $F(x)$ such that $F(x)\subset\cup\{U(z_i):i=1,2,...,n\}$. Put $U=\cup\{U(z_i):i=1,2,...,n\}$ and $V=\cap\{V(y_i):i=1,2,...,n\}$. Then $U,V\in\sigma_\theta$ such that $F(x)\subset U$, $y\in V$ and $U\cap V=\varnothing$. Since $F$ is upper faintly $b$-$\mathscr{I}$-continuous multifunction, there exists $W\in B\mathscr{I}O(X,x)$ such that $F(W)\subset U$. We have $(x,y)\in W\times V\subset(X\times Y)\backslash G(F)$. We obtain that $(W\times V)\cap G(F)=\varnothing$ and hence $G(F)$ is $b$-$\mathscr{I}$-$\theta$-closed in $X\times Y$. □

**Theorem 3.19.** *Let* $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ *be a multifunction having b-*$\mathscr{I}$*-*$\theta$*-closed multigraph* $G(F)$*. If B is a* $\theta$*-compact subset relative to Y, then* $F^-(B)$ *is b-*$\mathscr{I}$*-closed in X.*

*Proof.* Let $x\in X\backslash F^-(B)$. For each $y\in B$, $(x,y)\notin G(F)$ and there exist $U(y)\in B\mathscr{I}O(X)$ and $V(y)\in\sigma_\theta$ containing $x$ and $y$, respectively, such that $F(U(y))\cap V(y)=\varnothing$. That is, $U(y)\cap F^-(V(y))=\varnothing$. Then $\{V(y)\colon y\in B\}$ is a $\theta$-open cover of $B$ and since $B$ is $\theta$-compact relative to $Y$, there exists a finite subset $B_0$ of $B$ such that $B\subset\cup\{V(y):y\in B_0\}$. Put $U=\cap\{U(y):y\in B_0\}$. Then $U\in B\mathscr{I}O(X,x)$ and $U\cap F^-(B)=\varnothing$; that is, $x\in U\subset X\backslash F^-(B)$. This shows that $F^-(B)\in B\mathscr{I}C(X)$. □

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