Equitable restrained domination number of some graphs

S. K. Vaidya[1]* and P. D. Ajani[2]

Abstract

A dominating set $S \subseteq V$ is said to be a restrained dominating set of graph G if every vertex not in S is adjacent to a vertex in S and also to a vertex in $V - S$. A set $S \subseteq V$ is called an equitable dominating set if for every vertex $v \in V - S$, there exist a vertex $u \in S$ such that $uv \in E$ and $|deg(u) - deg(v)| \leq 1$. A dominating set S is called an equitable restrained dominating set if it is both restrained and equitable. The minimum cardinality of an equitable restrained dominating set is called equitable restrained domination number of G, denoted by $\gamma_r^e(G)$. We investigate $\gamma_r^e(G)$ parameter for some standard graphs and also establish some characterizations.

Keywords

Dominating set, equitable dominating set, equitable restrained dominating set, equitable restrained domination number.

AMS Subject Classification

05C69, 05C76.

[1] *Department of Mathematics, Saurashtra University, Rajkot-360005, Gujarat, India.*
[2] *Department of Mathematics, Atmiya University, Rajkot-360005, Gujarat, India.*
Corresponding author: [1] samirkvaidya@yahoo.co.in; [2] paragajani@gmail.com

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1. Introduction

The domination in graphs is one of the fastest developing areas among various concepts in graph theory. It has received remarkable attention of many researchers because it has various applications such as linear algebra to social services and design analysis of network to military surveillance.

Let $G = (V, E)$ be a finite, undirected graph without multiple edges and loops, where V and E denote the set of vertices and edges of G respectively. Cardinality of set A denoted by $|A|$, means the number of elements of set A. The order and size of a graph G are defined as the cardinality of its vertex set and edge set respectively. If u is an end vertex of an edge f then f is said to be incident with vertex u. Two vertices u and v are said to be connected if the is a path from u to v in G. Two vertices are adjacent if they are connected by an edge in G and two edges are adjacent if they shared

a common vertex in G. $\Delta(G)$ denotes the maximum degree of graph G, defined as $\Delta(G) = max\{deg(u)|u \in V\}$ while $\delta(G)$ denotes the minimum degree of graph G, defined as $\delta(G) = min\{deg(u)|u \in V\}$.

For any real number z, $\lceil z \rceil$ represents the least integer which is more than or equal to z and $\lfloor z \rfloor$ represents the greatest integer which is less than or equal to z. A vertex with degree one is said to be a pendant vertex while an edge with degree one is said to be a pendant edge in graph G. We rely on Harary [5] for graph graph theoretic terminology and notation.

A subset S of vertex set V in a graph G is called a dominating set if every vertex not in S is adjacent to at least one vertex of S. If no proper subset $S' \subset S$ is a dominating set then dominating set S is called a minimal dominating set. The domination number of G, denoted by $\gamma(G)$ is defined as the minimum cardinality of a dominating set of G and the corresponding dominating set is called a γ-set of G.

Variety of dominating sets are available in the existing literature. These variants are introduced by identifying one or more characteristics of elements of vertex subset or edge subset. A brief account on dominating set and related concepts can be found in Haynes *et al* [6]. One such a variant is restrained domination, a set S subset of vertex set V is a

restrained dominating set if every vertex $v \in V - S$ is adjacent to a vertex in S as well as another vertex $V - S$. The restrained domination number of G, denoted by $\gamma_{re}(G)$, is the minimum cardinality of a restrained dominating set S of G. The concept of restrained domination was introduced by Telle and Proskurowski [10] as a vertex partitioning problem. The restrained domination of complete graph, multipartite graphs and the graphs with minimum degree two is explored by Domke *et al.* [3, 4] while Vaidya and Ajani [11–13] have studied the restrained domination in the context of path, cycle and wheel.

A set $S \subseteq V$ is called an equitable dominating set if for every vertex $v \in V - S$, there exist a vertex $u \in S$ such that $uv \in E$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set is called an equitable domination number of G, denoted by $\gamma^e(G)$. This concept was conceived by Swaminathan and Dharmalingam [9]. A vertex $v \in V$ is called equitably adjacent with a vertex $u \in V$ if $|deg(u) - deg(v)| \leq 1$ and $uv \in E$. A vertex $u \in V$ is called equitable isolate if $|deg(u) - deg(v)| \geq 2$ for all $v \in N(u)$. Analogous to the characteristic of an isolated vertex in a dominating set, an equitable isolate must belong to every equitable dominating set of G. Clearly, the isolated vertices are the equitable isolates. Therefore, $I_s \subseteq I_e \subseteq S$ for every equitable domiating set S, where I_s is the set of all isolated vertices and I_e is the set of all equitable isolates of G.

Many domination models are introduced by combining two or more domination characteristic. Restrained edge domination [8], equitable total domination [1], global equitable domination [2], total restrained domination [10] are among worth to mention. Motivated through the concepts of restrained domination and equitable domination, a new concept of equitable restrained domination was introduced by Kulli [7]. A dominating set which is both restrained and equitable is called equitable restrained dominating set. The minimum cardinality of an equitable restrained dominating set is called equitable restrained domination number of G, denoted by $\gamma_r^e(G)$.

In this paper we characterize equitable restrained dominating set and investigate equitable restrained domination number of some standard graphs.

Definition 1.1. *The wheel graph W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as the apex and the vertices corresponding to cycle C_n are known as rim vertices while the edges corresponding to cycle are known as rim edges.*

Definition 1.2. *The helm H_n is the graph obtained from wheel W_n by attaching a pendant edge to each rim vertex.*

Definition 1.3. *The closed helm CH_n is the graph obtained from helm H_n by joining each pendant vertex to form a cycle.*

Proposition 1.4. [3] For complete graph K_n, $\gamma_r(K_n) = 1$.

Proposition 1.5. [3] For star $K_{1,n}$, $\gamma_r(K_{1,n}) = n + 1$.

Proposition 1.6. [3] For cycle C_n, $\gamma_r(C_n) = n - 2 \left\lfloor \dfrac{n}{3} \right\rfloor$.

Proposition 1.7. [3] For path P_n, $\gamma_r(P_n) = n - 2 \left\lfloor \dfrac{n-1}{3} \right\rfloor$.

Proposition 1.8. [3] For complete bipartite graph $K_{m,n}$, $\gamma_r(K_{m,n}) = 2$.

2. Main Results

Theorem 2.1. For any graph G, $\gamma(G) \leq \gamma_r(G) \leq \gamma_r^e(G)$.

Proof: By definition of restrained dominating set of graph G, every restrained dominating set is a dominating set. Similarly every equitable restrained dominating set is a restrained as well as an equitable dominating set. So, it is obvious that for any graph G, $\gamma(G) \leq \gamma_r(G) \leq \gamma_r^e(G)$.

Theorem 2.2. For any graph G, $\gamma(G) \leq \gamma^e(G) \leq \gamma_r^e(G)$.

Proof: Clearly every equitable dominating set of graph G is a dominating set and further every equitable restrained dominating set is a restrained as well as an equitable dominating set. Hence $\gamma(G) \leq \gamma^e(G) \leq \gamma_r^e(G)$.

Theorem 2.3. If G is regular or bi-regular $(k, k+1)$ graph, for $k > 0$ then $\gamma_r(G) = \gamma_r^e(G)$.

Proof: Let G be a regular graph. Then degree of every vertex in G is same (say) k. Let S be a restrained dominating set of graph G with minimum cardinality, which implies that $|S| = \gamma_r(G)$. For any vertex $u \in V - S$ there exist a vertex $v \in S$ such that $uv \in E(G)$, as S is a restrained dominating set. Also $deg(u) = deg(v) = k$. It follows that $|deg(u) - deg(v)| = 0 \leq 1$. Therefore S is an equitable restrained dominating set of G so that $\gamma_r^e(G) \leq |S| = \gamma_r(G)$. But by Theorem 2.1, $\gamma_r(G) \leq \gamma_r^e(G)$. Therefore $\gamma_r(G) = \gamma_r^e(G)$.

Now, let G be a bi-regular graph. Then degree of every vertex in G is either k or $k+1$. Let S be a restrained dominating set of graph G with minimum cardinality, which implies that $|S| = \gamma_r(G)$. For any vertex $u \in V - S$ there exists a vertex $v \in S$ such that $uv \in E(G)$, as S is a restrained dominating set. Also $deg(u) = deg(v) = k$ or $k+1$. It follows that $|deg(u) - deg(v)| \leq 1$. Therefore S is an equitable restrained dominating set of G so that $\gamma_r^e(G) \leq |S| = \gamma_r(G)$. But by Theorem 2.1, $\gamma_r(G) \leq \gamma_r^e(G)$. Therefore $\gamma_r(G) = \gamma_r^e(G)$.

Theorem 2.4. For complete graph K_n, $\gamma_r^e(K_n) = 1$.

Proof: For complete graph K_n, clearly every singleton set of n vertices will form an equitable restrained dominating set of K_n. Hence, $\gamma_r^e(K_n) = 1$.

Theorem 2.5. For star $K_{1,n}$, $\gamma_r^e(K_{1,n}) = n+1$.

Proof: For star $(K_{1,n})$, by Proposition 1.5, $\gamma_r(K_{1,n}) = n+1$. Hence $\gamma_r^e(K_{1,n}) = n+1$.

Theorem 2.6. For cycle C_n, $\gamma_r^e(C_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor$.

Proof: Here C_n is a 2-regular graph. For regular graph, restrained dominating set is clearly an equitable restrained dominating set. So $\gamma_r^e(C_n) = \gamma_r(C_n)$. But according to Proposition 1.6, $\gamma_r(C_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor$. Hence $\gamma_r^e(C_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor$.

Theorem 2.7. For path P_n, $\gamma_r^e(P_n) = n - 2\left\lfloor\dfrac{n-1}{3}\right\rfloor$.

Proof: Here P_n is a bi-regular graph, as degree of every vertex in P_n is either 1 or 2. For bi-regular graph, restrained dominating set is clearly an equitable restrained dominating set. So $\gamma_r^e(P_n) = \gamma_r(P_n)$. But by Proposition 1.7, $\gamma_r(P_n) = n - 2\left\lfloor\dfrac{n-1}{3}\right\rfloor$. Hence $\gamma_r^e(P_n) = n - 2\left\lfloor\dfrac{n-1}{3}\right\rfloor$.

Theorem 2.8. For complete bipartite graph $K_{m,n}$,

$$\gamma_r^e(K_{m,n}) = \begin{cases} 2 & \text{; if } |m-n| \leq 1 \\ m+n & \text{; if } |m-n| \geq 2 \end{cases}$$

Proof: Let $K_{m,n}$ be the complete bipartite graph with bi-partition V_1 and V_2. Where order of vertex set V_1 and V_2 are m and n respectively. Which implies that $deg(u) = n, \forall u \in V_1$ and $deg(v) = m, \forall v \in V_2$.

Suppose $|m-n| \leq 1$. Then $K_{m,n}$ is a bi-regular graph. Then by Proposition 1.8 and Theorem 2.3, $\gamma_r(K_{m,n}) = \gamma_r^e(K_{m,n})$.

Now, suppose $|m-n| \geq 2$. Let $S \subseteq V(K_{m,n})$ be an equitable restrained dominating set with minimum cardinality of $K_{m,n}$. Then $|S| = m+n$.

If possible let $|S| < m+n$. Let $u \in V(K_{m,n}) - S$ be any vertex, which follows that either $u \in V_1$ or $u \in V_2$. Let $u \in V_1$ (if $u \in V_2$, it follows the same argument). Since S is equitable restrained dominating set, there exists $v \in S$ such that $uv \in E(K_{m,n})$ and $|deg(u) - deg(v)| \leq 1$. Note that V_1 and V_2 are independent vertex sets. Since $uv \in E(K_{m,n})$ and $u \in V_1$, we get $v \in V_2$. Therefore $deg(u) = n$ and $deg(v) = m$. Now, $|deg(u) - deg(v)| = |n-m| = |m-n| \geq 2$, which is a contradiction. Therefore $|S| = m+n$. Hence $\gamma_r^e(K_{m,n}) = m+n$, for $|m-n| \geq 2$.

Theorem 2.9. For wheel W_n,

$$\gamma_r^e(W_n) = \begin{cases} 1 & \text{; if } n = 3,4 \\ n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1 & \text{; if } n \geq 5 \end{cases}$$

Proof: Let $v_1, v_2, ..., v_n$ be the rim vertices and u be the apex of wheel W_n with $|V(W_n)| = n+1$. Note that $deg(v_i) = 3$ for $1 \leq i \leq n$ and $deg(u) = n = \Delta(W_n)$. We divide the proof in following two different cases,

Case 1: For $n = 3,4$

Let $S \subseteq V(W_n)$ be an equitable restrained dominating set. Then $u \in S$, as vertex u is adjacent to all the rim vertices v_i, for $1 \leq i \leq n$. Also every vertex not in S is adjacent to a vertex in S as well as to a vertex in $V - S$. So set $S = \{u\}$ is a restrained dominating set. In this case, $deg(u) = 3$ or 4 and $deg(v_i) = 3$, for $1 \leq i \leq n$. Moreover, $|deg(u) - deg(v_i)| \leq 1$, for $1 \leq i \leq n$. Therefore set $S = \{u\}$ is an equitable dominating set. Which implies that $S = \{u\}$ is an equitable restrained dominating set with minimum cardinality. Hence $\gamma_r^e(W_n) = 1$, for $n = 3,4$.

Case 2: For $n \geq 5$

Let $S \subseteq V(W_n)$ be an equitable restrained dominating set. In this case, $deg(u) = n = \Delta(W_n)$ and $deg(v_i) = 3$, for $1 \leq i \leq n$. Then $S = \{u\}$ is a restrained dominating set but it is not an equitable dominating set, as $|deg(u) - deg(v_i)| \geq 2$, for $1 \leq i \leq n$. We observe that rim vertices v_i, for $1 \leq i \leq n$ of W_n forms a cycle C_n and by Theorem 2.6, $\gamma_r^e(C_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor$. It follows that at least $|S| = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$ vertices including apex u are required to dominate all the vertices of W_n, such that S is an equitable restrained dominating set. By removal of any of the vertex from set S will not satisfy the property of either restrained dominating set or equitable dominating set. Which implies that $|S| = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$ is an equitable restrained dominating set of W_n with minimum cardinality. Hence $\gamma_r^e(W_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$, for $n \geq 5$.

Illustration 2.10. The wheel W_6 is shown in Figure 1 where the set of solid vertices $\{v, v_1, v_4\}$ is its equitable restrained dominating set of minimum cardinality.
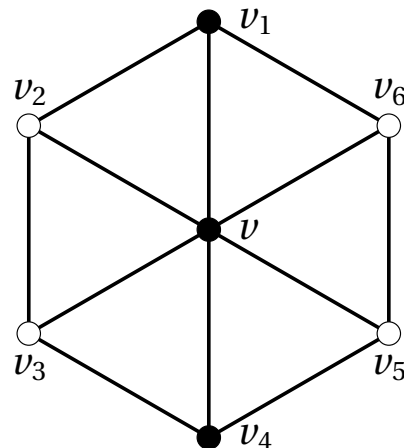
Figure 1: $\gamma_r^e(W_6) = 3$

Theorem 2.11. For helm H_n,

$$\gamma_r^e(H_n) = \begin{cases} n+1 & ; \text{if } n = 3,4,5 \\ \\ 2n-2\left\lfloor\dfrac{n}{3}\right\rfloor+1 & ; \text{if } n \geq 6 \end{cases}$$

Proof: Let $V(H_n) = \{v, u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ be the vertex set of H_n. Where $u_1, u_2, ..., u_n$ are pendant vertices, $v_1, v_2, ..., v_n$ are rim vertices and v is the apex of helm H_n. Obviously $|V(H_n)| = 2n+1$. Note that $deg(v) = n = \Delta(H_n)$, $deg(v_i) = 4$, for $1 \leq i \leq n$ and $deg(u_j) = 1$, for $1 \leq j \leq n$.

Case 1: For $n = 3,4,5$

We observe that pendant vertices $u_1, u_2, ..., u_n$ are mutually non-adjacent which must be in every restrained dominating set. Moreover pendant vertices $u_1, u_2, ..., u_n$ dominate the rim vertices $v_1, v_2, ..., v_n$ and apex v is dominated by iself. If $S \subseteq V(H_n)$ is an equitable restrained dominating set then $\{v, u_1, u_2, ..., u_n\} \in S$. Note that S is a restrained dominating set of H_n, as every vertex in $V - S$ is adjacent to vertex in S and to a vertex in $V - S$. Also, for every vertex (say) $w \in V - S$ there exist a vertex (say) $u \in S$ such that $uw \in E(H_n)$ and $|deg(u) - deg(w)| \leq 1$. It follows that S is also equitable dominating set of H_n.

Also $S = \{v, u_1, u_2, ..., u_n\}$ is an equitable restrained dominating set with minimum cardinality $|S| = n+1$ because removal of any of the vertex from set S will not dominate all the vertices of H_n. Hence $\gamma_r^e(H_n) = n+1$, for $n = 3,4,5$.

Case 2: For $n \geq 6$

We observe that rim vertices $v_1, v_2, ..., v_n$ together with apex v forms a wheel W_n in helm H_n. By Theorem 2.9, $\gamma_r^e(W_n) = n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$, for $n \geq 5$. If $S \subseteq V(H_n)$ is an equitable restrained dominating set then $\{u_1, u_2, ..., u_n\} \in S$, as pendant vertices $u_1, u_2, ..., u_n$ are mutually non-adjacent which must be in every restrained dominating set. So S is an equitable restrained dominating set of H_n with minimum cardinality, $|S| = 2n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$. Hence $\gamma_r^e(H_n) = 2n - 2\left\lfloor\dfrac{n}{3}\right\rfloor + 1$, for $n \geq 6$.

Illustration 2.12. The helm H_4 is shown in Figure 2 where the set of solid vertices $\{v, u_1, u_2, u_3, u_4\}$ is its equitable restrained dominating set of minimum cardinality.
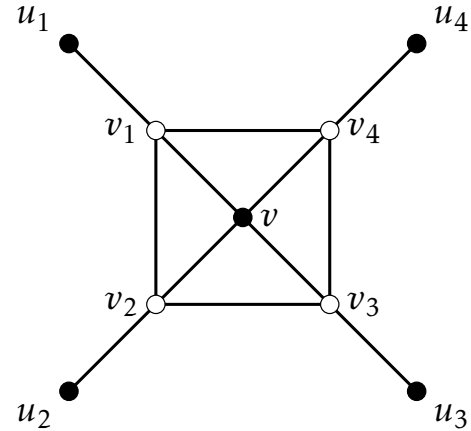

Figure 2: $\gamma_r^e(H_4) = 5$

Theorem 2.13. For closed helm CH_n,

$$\gamma_r^e(CH_n) = \begin{cases} 2 & ; \text{if } n = 3,4 \\ \\ 3 & ; \text{if } n = 5 \\ \\ \left\lceil\dfrac{n+12}{4}\right\rceil + \left\lceil\dfrac{n-9}{4}\right\rceil & ; \text{if } n \geq 6 \end{cases}$$

Proof: The closed helm CH_n contains wheel W_n and outer cycle C_n. Let $\{v, u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\} \in V(CH_n)$, where v is the apex, $u_1, u_2, ..., u_n$ are rim vertices of W_n in CH_n and $v_1, v_2, ..., v_n$ are corresponding adjacent vertices of outer cycle in CH_n. We note that $|V(CH_n)| = 2n+1$ and $\Delta(CH_n) = n = deg(v)$.

Case 1: For $n = 3,4$

Here $|V(CH_3)| = 7$, for $n = 3$ and $|V(CH_4)| = 9$, for $n = 4$. Moreover $\Delta(CH_3) = 3$ and $\Delta(CH_4) = 4$. This implies that at least two vertices are required to dominate all the vertices of CH_3 and CH_4 respectively. If $S \subseteq V(CH_n)$, for $n = 3,4$ is an equitable restrained dominating set then $|S| = 2$, which is minimum. Hence $\gamma_r^e(CH_n) = 2$, for $n = 3,4$.

Case 2: For $n = 5$

Here $|V(CH_5)| = 11$ and $\Delta(CH_5) = 5$. Which implies that at least three vertices are required to dominate all the vertices of CH_5. If $S \subseteq V(CH_5)$ is an equitable restrained dominating set then $|S| = 3$. If possible, suppose $S' \subseteq S$ is an equitable restrained dominating set such that $|S'| = 2 < 3 = |S|$. Now, $\Delta(CH_5).2 = (5).(2) = 10 < 11 = |V(CH_5)|$. Therefore S' can not be an equitable restrained dominating set of CH_5. It follows that S is an equitable restrained dominating set of CH_5 with minimum cardinality $|S| = 3$. Hence $\gamma_r^e(CH_n) = 3$, for $n = 5$.

Case 3: For $n \geq 6$

Here $|V(CH_n)| = 2n+1$. Moreover $deg(u_i) = 4$, $deg(v_j) = 3$, for $1 \le i, j \le n$ and $\Delta(CH_n) = n = deg(v)$, for $n \ge 6$. Note that apex vertex v is an equitable isolate in CH_n and remaining vertices are equitably adjacent in CH_n. As apex vertex v being an equitable isolate, it must belong to every equitable restrained dominating set of CH_n. Now, we construct a vertex set $S \subseteq V(CH_n)$ as follows,

$$S = \{v, u_3, u_7, ..., u_{4i+3}, v_1, v_5, ..., v_{4j+1}\},$$

where $0 \le i \le \left\lfloor \dfrac{n-2}{4} \right\rfloor$ and $0 \le j \le \left\lfloor \dfrac{n-1}{4} \right\rfloor$ with $|S| = \left\lceil \dfrac{n+12}{4} \right\rceil + \left\lceil \dfrac{n-9}{4} \right\rceil$. In above set S, every vertex in $V - S$ is adjacent to vertex in S and to a vertex in $V - S$, so S is a restrained dominating set of CH_n. Also, for every vertex (say) $w \in V - S$ there exist a vertex (say) $u \in S$ such that $uw \in E(CH_n)$ and $|deg(u) - deg(w)| \le 1$. It follows that S is also equitable dominating set of CH_n. So set S is a restrained as well as an equitable dominating set of CH_n. Therefore, S is an equitable restrained dominating set of CH_n. Moreover, due to adjacency nature of vertices of CH_n, it is easy to observe that set S is of minimum cardinality because by removal of any of the vertex from set S will not satisfy the property of either restrained dominating set or equitable dominating set. Thus, set S is an equitable restrained dominating set of CH_n with minimum cardinality $|S| = \left\lceil \dfrac{n+12}{4} \right\rceil + \left\lceil \dfrac{n-9}{4} \right\rceil$.

Hence $\gamma_r^e(CH_n) = \left\lceil \dfrac{n+12}{4} \right\rceil + \left\lceil \dfrac{n-9}{4} \right\rceil$, for $n \ge 6$.

Illustration 2.14. The closed helm CH_8 is shown in Figure 2 where the set of solid vertices $\{v, u_3, u_7, v_1, v_5\}$ is its equitable restrained dominating set of minimum cardinality.
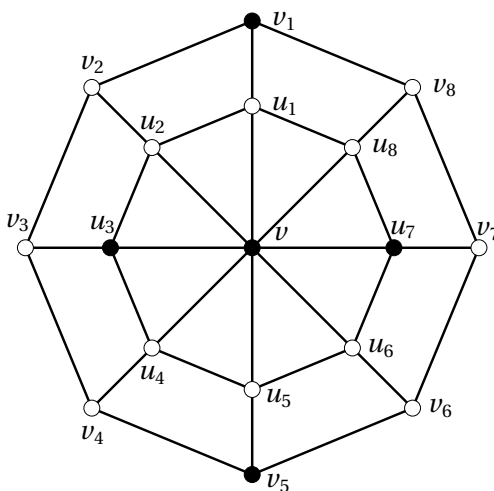

Figure 3: $\gamma_r^e(CH_8) = 5$

Conclusion

The concept of equitable restrained dominating set is a variant of equitable dominating set and restrained dominating set. We have characterized equitable restrained dominating set and also obtained equitable restrained domination number of some standard graphs. To derive similar results in the context of other variants of domination is an open area of research.

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