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# Measure of fuzzy $(i, j)$-$s$-compactness

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**Abstract**

In this paper, the notion of fuzzy $(i, j)$-$s$-compactness degrees is introduced in $L$-fuzzy topological spaces by means of the implication operation of $L$. Characterizations of fuzzy $(i, j)$-$s$-compactness degrees in $L$-fuzzy topological spaces are obtained, and some properties of fuzzy $(i, j)$-$s$-compactness degrees are researched.

**Keywords**

$L$-bitopological spaces, fuzzy $(i, j)$-$s$-compactness, Fuzzy $(i, j)$-$s$-compactness degree.

**AMS Subject Classification**

54A40, 54D30, 03E72.

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## Contents



## 1. Introduction

It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [4], various kinds of fuzzy compactness [4, 7] have been established. However, these concepts of fuzzy compactness rely on the structure of $L$ and $L$ is required to be completely distributive. In [10], for a complete De Morgan algebra $L$, author introduced a new definition of fuzzy compactness in $L$-topological spaces using open $L$-sets and their inequality. This new definition does not depend on the structure of $L$. In this paper, the notion of fuzzy $(i, j)$-$s$-compactness degrees is introduced in $L$-fuzzy topological spaces by means of the implication operation of $L$. Characterizations of fuzzy $(i, j)$-$s$-compactness degrees in $L$-fuzzy topological spaces are obtained, and some properties of fuzzy $(i, j)$-$s$-compactness degrees are researched.

## 2. preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, $X$ a nonempty set and $L^X$ the set of all $L$-fuzzy sets (or $L$-sets for short) on $X$. The smallest element and the largest element in $L$ are denoted by 0 and 1. The smallest element and the largest element in $L^X$ are denoted by $\underline{0}$ and $\underline{1}$. An element $a$ in $L$ is called a prime element if $b \wedge c \leq a$ implies that $b \leq a$ or $c \leq a$. $a$ in $L$ is called a co-prime element if $a'$ is a prime element [6]. The set of nonunit prime elements in $L$ is denoted by $P(L)$ and the set of nonzero co-prime elements in $L$ by $M(L)$. The binary relation $\prec$ in $L$ is defined as follows: for $a, b \in L, a \prec b$ if and only if for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [5]. In a completely distributive De Morgan algebra $L$, each element $b$ is a sup of $\{a \in L | a \prec b\}$. The set $s(b) = \{a \in L | a \prec b\}$ is called the greatest minimal family of $b$ in the sense of [7, 13]. Now, for $b \in L$, we define $s^*(b) = s(b) \cap M(L)$, $\alpha(b) = \{A \in L | a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$. In a complete DeMorgan frame $L$, there exists a binary operation $\rightarrow$. Explicitly the implication is given by $a \rightarrow b = \vee\{c \in L : a \wedge c \leq b\}$. We interpret $[a \leq b]$ as the degree to which $a \leq b$, then $[a \leq b] = a \rightarrow b$.

**Definition 2.1.** *[15] An L-topology on a set X is a mapping $\tau : L^{\rightarrow} L$ which satisfying the following conditions:*

1. $\tau(\underline{1}) = \tau(\underline{0}) = 1$*;*
2. *for any* $A, B, \tau(A \cap B) \geq \tau(A) \wedge \tau(B)$*;*
3. *for any* $A_\lambda \in L^X, \lambda \in \Delta$, $\tau(\bigvee_{\lambda \in \Delta} A_\lambda) \geq \bigwedge_{\lambda \in \Delta} \tau(A_\lambda)$.

The pair $(X, \tau)$ is called an $L$-fuzzy topological space. $\tau(U)$ is called the degree of openness of $U$, $\tau^\star(U) = \tau(U')$

is called the degree of closedness of $U$, where $U'$ is the $L$-complement of $U$. For any family $\mathscr{U} \subset L^X$, $\tau(\mathscr{U}) = \bigwedge_{A\in\mathscr{U}} \tau(A)$ is called the degree of openness of $U$.

For a subfamily $\Phi \subset L^X$, $2^{(\Phi)}$ denotes the set of all finite subfamilies of $\Phi$. For any $a \in L$, $a$ denotes $\underline{a}$ constant value mapping from $X$ to $L$, its value is $a$.

**Definition 2.2.** *An L-bitopological space (or L-bts for short) is an ordered triple $(X,\tau_1,\tau_2)$, where $\tau_1$ and $\tau_2$ are subfamilies of $L^X$ which contains $\underline{0},\underline{1}$ and is closed for any suprema and finite infima.*

**Definition 2.3.** *[24] An L-fuzzy inclusion on X is a mapping $\tilde{\subset} : L^X \times L^X \to L$ defined by the equality $\tilde{\subset}(A,B) = \bigwedge_{x\in X} (A'(x) \vee B(x))$.*

In this paper, we will write $[A\tilde{\subset}B]$ instead of $\tilde{\subset}(A,B)$.

**Definition 2.4.** *[9] Let $(X,\tau)$ be an L-ts, $a \in L\backslash\{1\}$, and $A \in L^X$. A family $\mu \subseteq L^X$ is called*

1. *an a-shading of A if for any $x \in X$, $A'(x) \vee \bigvee_{B\in\mu} B(x) \not\leq a$.*
2. *a strong a-shadining of A if $\bigwedge_{x\in X} (A'(x) \vee \bigvee_{B\in\mu} B(x)) \not\leq a$.*

**Definition 2.5.** *[9] Let $(X,\tau)$ be an L-ts, $a \in L\backslash\{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is called*

1. *an a-remote neighborhood family of A if for any $x \in X$, $(A(x) \wedge \bigwedge_{B\in\mu} B(x)) \not\geq a$.*
2. *a strong a-remote neighbourhood family of A if $\bigvee_{x\in X} (A(x) \wedge \bigwedge_{B\in\mu} B(x)) \not\geq a$.*
3. *a $s_a$-cover of A if for any $x \in X$, it follows that $a \in s(A'(x) \vee \bigvee_{B\in\mu} B(x))$.*
4. *a strong $s_a$-cover of A if for any $x \in X$, it follows that $a \in s(\bigwedge_{x\in X} (A'(x) \vee \bigvee_{B\in\mu} B(x)))$.*
5. *a $Q_a$-cover of A if for any $x \in X$, it follows that $\bigvee_{x\in X} (A'(x) \vee \bigvee_{B\in\mu} B(x)) \geq a$.*

**Definition 2.6.** *[11] Let $(X,\tau_1,\tau_2)$ be an L-bts, $A \in L^X$. Then A is called an $(i,j)$-semi-open set if $A \leq j\mathrm{Cl}(i\mathrm{Int}(A))$. The complement of an $(i,j)$-semi-open set is called an $(i,j)$-semi-closed set. Also, $(i,j)$-$SO(L^X)$ and $(i,j)$-$SC(L^X)$ will always denote the family of all $(i,j)$-semi-open sets and $(i,j)$-semi-closed sets respectively. Obviously, $A \in (i,j)$-$SO(L^X)$ if and only if $A' \in (i,j)$-$SC(L^X)$.*

**Definition 2.7.** *[11] Let $(L^X,\tau_1,\tau_2)$ be an L-bitopological space, $A,B \in L^X$. Let $(i,j)$-$s\mathrm{Int}(A) = \vee\{B \in L^X | B \leq A, B \in (i,j)\text{-}SO(L^X)\}$, $(i,j)$-$s\mathrm{Cl}(A) = \wedge\{B \in L^X | A \leq B, B \in (i,j)\text{-}SC(L^X)\}$. Then $(i,j)$-$s\mathrm{Int}(A)$ and $(i,j)$-$s\mathrm{Cl}(A)$ are called the $(i,j)$-semi-interior and $(i,j)$-semi-closure of A, respectively.*

**Definition 2.8.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopology on X. For any $A \in L^X$, define a mapping $\tau_{(i,j)s} : L^X \to L$ by $\tau_{(i,j)s}(A) = \bigvee_{B\leq A} (\tau_i(B) \wedge \bigwedge_{x_\lambda \prec A} \bigwedge_{x_\lambda \not\leq D \geq B} (\tau_i(D'))')$. Then $\tau_{(i,j)s}$ is called the L-fuzzy $(i,j)$-semi-open operator induced by $\tau_1$ and $\tau_2$, where $\tau_{(i,j)s}(A)$ can be regarded as the degree to which A is $(i,j)$-semi-open and $\tau^{\star}_{(i,j)s}(B) = \tau_{(i,j)s}(B')$ can be regarded as the degree to which B is $(i,j)$-semi-closed. For any family $\mathscr{U} \subset L^X$, $\tau_{(i,j)s}(\mathscr{U}) = \bigwedge_{A\in\mathscr{U}} \tau_{(i,j)s}(A)$ is called the degree of $(i,j)$-semi-openness of $\mathscr{U}$.*

**Definition 2.9.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopology on X and let $\tau_{(i,j)s}$ be the L-fuzzy $(i,j)$-semi-open operator induced by $\tau_1$ and $\tau_2$. Then $\tau_i(A) \leq \tau_{(i,j)s}(A)$ for any $A \in L^X$.*

**Definition 2.10.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space. $G \in L^X$ is said to be L-fuzzy $(i,j)$-s-compact if for every family $\mathscr{U} \subset L^X$, it follows that $\bigwedge_{A\in\mathscr{U}} \tau_{(i,j)s}(A) \wedge \bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{U}} A(x)) \leq \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} \bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{V}} A(x))$.*

## 3. Measures of fuzzy $(i,j)$-$s$-compactness

Let $(X,\tau_1,\tau_2)$ be an $L$-bitopological space and $G \in L^X$. Then $G$ is fuzzy $(i,j)$-$s$-compactness if and only if for every family $\mathscr{U}$ of $(i,j)$-semi-open $L$-sets, it follows that $\bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{U}} A(x)) \leq \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} \bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{V}} A(x))$. So for every family $\mathscr{U}$ of $(i,j)$-semi-open $L$-sets, $[[G\tilde{\subset} \vee \mathscr{U}] \leq \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} [G\tilde{\subset} \vee \mathscr{V}]] = \underline{1}$. We know that an $L$-topology $\tau$ can be looked as a special $L$-fuzzy topology. Therefore, $A \in L^X$ is an $(i,j)$-semi-open set if and only if $\tau_{(i,j)s}(A) = 1$. Thus $G$ is fuzzy $(i,j)$-$s$-compactness if and only if for every family $\mathscr{V} \subset L^X$, it follows that $\tau_{(i,j)s}(U) \leq [[G\tilde{\subset} \vee \mathscr{U}] \leq \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} [G\tilde{\subset} \vee \mathscr{U}]]$. Therefore, we can naturally generalize the notion of fuzzy $(i,j)$-$s$-compactness degrees to L-fuzzy bitopological spaces as follows:

**Definition 3.1.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G \in L^X$. The fuzzy $(i,j)$-s-compactness degree $cd_{(i,j)s}$ of G is defined as $cd_{(i,j)s}(G) = \bigwedge_{\mathscr{U}\subset L^X} (\tau_{(i,j)s}(\mathscr{U}) \to ([G\tilde{\subset} \vee \mathscr{U}] \to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} [G\tilde{\subset} \vee \mathscr{V}])) = \bigwedge_{\mathscr{U}\subset L^X} (\tau_{(i,j)s}(\mathscr{U}) \to (\bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{U}} A(x)) \to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}} \bigwedge_{x\in X} (G'(x) \vee \bigvee_{A\in\mathscr{V}} A(x))))$*

**Theorem 3.2.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G \in L^X$. Then $cd_{(i,j)s}(G) \leq cd_{\tau_i}(G)$.*

*Proof.* Straightforward. □

**Theorem 3.3.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G \in L^X$. Then G is fuzzy $(i,j)$-s-compactness in $(X,\tau_1,\tau_2)$ if and only if $scd_{\chi_{\tau_1,\tau_2}}(G) = 1$.*

*Proof.* Let $(X,\tau_1,\tau_2)$ be an $L$-fuzzy bitopological space. The mapping $\chi_{\tau_1,\tau_2}: L^X \to L$ defined by

$$\chi_{\tau_1,\tau_2}(A)=\begin{cases}1 & \text{if } A\in\tau_1 \text{ or } \tau_2\\ 0 & \text{if } A\notin\tau_1 \text{ and } \tau_2\end{cases}$$

is a special $L$-fuzzy bitopology. Then $A\in L^X$ is an $(i,j)$-semi-open set in $L$-bitopology $\tau$ if and only if $(\chi_{\tau_1,\tau_2})_{(i,j)s}(A)=1$. Thus by the definition of fuzzy $(i,j)$-$s$-compactness and the properties of $\to$, we know that $G$ is fuzzy $(i,j)$-$s$-compactness if and only if for every family $\mathscr{U}\subset L^X$, then $(\chi_{\tau_1,\tau_2})_{(i,j)s}(\mathscr{U})\le [[G\tilde{\subset}\vee\mathscr{U}]\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]]$. This implies that G is fuzzy $(i,j)$-$s$-compact if and only if for every family $\mathscr{U}\subset L^X$, it follows that $(\chi_{\tau_1,\tau_2})_{(i,j)s}(\mathscr{U})\to([[G\tilde{\subset}\vee\mathscr{U}]\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]]=1$. By the definition of $scd_{(\chi_{\tau_1,\tau_2})_{(i,j)s}}$, the conclusion is hold. □

**Theorem 3.4.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G\in L^X$. G is L-fuzzy $(i,j)$-s-compactness in $(X,\tau_1,\tau_2)$ if and only if $cd_{(i,j)s}(G)=1$.*

*Proof.* By the definition of $L$-fuzzy $(i,j)$-$s$-compactness, we know that $G$ is L-fuzzy $(i,j)$-$s$-compactness in $(X,\tau_1,\tau_2)$ if and only if for every family $G\in L^X$, it follows that

$$\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{V}]\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}].$$

By the properties of $\to$, we obtain that $G$ is $L$-fuzzy $(i,j)$-$s$-compactness in $(X,\tau_1,\tau_2)$ if and only if for every family $\mathscr{U}\subset L^X$, it follows that

$$\tau_{(i,j)s}(\mathscr{U})\to([G\tilde{\subset}\vee\mathscr{U}]\to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]]=1.$$

By the definition of $cd_{(i,j)s}$, the conclusion is hold. □

**Lemma 3.5.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G\in L^X$. Then $cd_{(i,j)s}(G)\ge a$ if and only if for any $\mathscr{U}\subset L^X$, $\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge a\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]$.*

*Proof.* For any $a\in L$, $cd_{(i,j)s}(G)\ge a$, that is,

$$\bigwedge_{\mathscr{U}\subset L^X}(\tau_{(i,j)s}(\mathscr{U})\to([G\tilde{\subset}\vee\mathscr{U}]\to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}])]\ge a$$

if and only if for any $\mathscr{U}\subset L^X$,

$$\tau_{(i,j)s}(\mathscr{U})\to([G\tilde{\subset}\vee\mathscr{U}]\to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}])\ge a$$

if and only if (by the property (6) of $\to$) for any $\mathscr{U}\subset L^X$,

$$\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}])\to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]\ge a.$$

if and only if (by the property (1) of $\to$) for any $\mathscr{U}\subset L^X$,

$$\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge a\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}].$$

□

**Theorem 3.6.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G\in L^X$. Then $cd_{(i,j)s}(G)\ge a$ if and only if for any $\mathscr{P}\subset L^X$, $\bigvee_{F\in\mathscr{P}}\tau^\star_{(i,j)s}(F)'\vee(\bigvee_{x\in X}(G(x)\wedge\bigwedge_{F\in\mathscr{P}}F(x)))\vee a'\ge \bigwedge_{\mathscr{H}\in 2^{(\mathscr{P})}}\bigvee_{x\in X}(G(x)\wedge\bigwedge_{F\in\mathscr{H}}F(x))$.*

*Proof.* It can be easily obtained by Lemma 3.5 and the definition of $\tau^\star_{(i,j)s}$. □

**Theorem 3.7.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G\in L^X$. Then $cd_{(i,j)s}(G)=\vee\{a\in L:\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge a\le \bigwedge_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{U}],\forall\mathscr{U}\subset L^X\}$.*

*Proof.* By Lemma 3.5, $cd_{(i,j)s}(G)$ is an upper bound of $\{a\in L:\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge a\le \bigwedge_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{U}],\forall\mathscr{U}\subset L^X\}$. Since $cd_{(i,j)s}(G)=\bigwedge_{\mathscr{U})\subset L^X}(\tau_{(i,j)s}(\mathscr{U})\to ([G\tilde{\subset}\vee\mathscr{U}]\to \bigwedge_{\mathscr{U}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]))$, then for every family $\mathscr{U}\subset L^X$, we have

$$cd_{(i,j)s}(G)\le\tau_{(i,j)s}(\mathscr{U})\to([G\tilde{\subset}\vee\mathscr{U}]\to \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}])$$

$$=(\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}])\to \bigwedge_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}].$$

By the property (1) of $\to$, we obtain that for every family $\mathscr{U}\subset L^X$, $\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge cd_{(i,j)s}(G)\le \bigvee_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}]$, thus $cd_{(i,j)s}(G)=\vee\{a\in L:\tau_{(i,j)s}(\mathscr{U})\wedge[G\tilde{\subset}\vee\mathscr{U}]\wedge a\le \bigwedge_{\mathscr{V}\in 2^{(\mathscr{U})}}[G\tilde{\subset}\vee\mathscr{V}],\forall\mathscr{U}\subset L^X\}$. Therefore, the conclusion is hold. □

In order to write simply, for any mapping $\tau: L^X\to L$, denote $\tau_b=\{A\in L^X:\tau(A)\ge b\}$.

**Theorem 3.8.** *Let $(X,\tau_1,\tau_2)$ be an L-fuzzy bitopological space and $G\in L^X$, $a\in L\backslash\{0\}$. Then the following conditions are equivalent:*

1. *$cd_{(i,j)s}(G)\ge a$.*
2. *For any $b\in P(L)$, $b\ngeq a$, each strong b-shading $\mathscr{U}$ of G with $\tau_{(i,j)s}(\mathscr{U})\nleq b$ has a finite subfamily $\mathscr{V}$ which is a strong b-shading of G.*
3. *For any $b\in P(L)$, $b\ngeq a$, each strong b- shading$\mathscr{U}$ of G with $\tau_{(i,j)s}(\mathscr{U})\nleq b$, there exists a finite subfamily $\mathscr{V}$ of $\mathscr{U}$ and $r\in\alpha^\star(b)$ such that $\mathscr{V}$ is an r-shading of G.*
4. *For any $b\in P(L)$, $b\ngeq a$, each strong b-shading $\mathscr{U}$ of G with $\tau_{(i,j)s}(\mathscr{U})\nleq b$, there exists a finite subfamily $\mathscr{V}$ of $\mathscr{U}$ and $r\in\alpha^\star(b)$ such that $\mathscr{V}$ is a strong r-shading of G.*
5. *For any $b\in M(L)$, $b\nleq a'$, each strong b-remote family $\mathscr{P}$ of G with $\tau^\star_{(i,j)s}(\mathscr{P})\nleq b'$ has a finite subfamily $\mathscr{H}$ which is a strong b-remote family of G.*

6. *For any $b \in M(L)$, $b \nleq a'$, each strong $b$-remote family $\mathscr{P}$ of $G$ with $\tau^{\star}_{(i,j)s}(\mathscr{P}) \nleq b'$, there exists a finite subfamily $\mathscr{H}$ of $\mathscr{P}$ and $r \in s^{\star}(b)$ such that $\mathscr{H}$ is an $r$-remote family of $G$.*

7. *For any $b \in M(L)$, $b \nleq a'$, each strong $b$-remote family $\mathscr{P}$ of $G$ with $\tau^{\star}_{(i,j)s}(\mathscr{P}) \nleq b'$, there exists a finite subfamily $\mathscr{H}$ of $\mathscr{P}$ and $r \in s^{\star}(b)$ such that $\mathscr{H}$ is a strong $r$-remote family of $G$.*

8. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each $Q_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a $Q_r$-cover of $G$.*

9. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each $Q_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a strong $s_r$-cover of $G$.*

10. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each $Q_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a $s_r$-cover of $G$.*

11. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each strong $s_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a $Q_r$-cover of $G$.*

12. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each strong $s_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a strong $s_r$-cover of $G$.*

13. *For any $b \leq a$, $r \in s(b)$, $b, r \neq 0$, each strong $s_b$-cover $\mathscr{U} \subset (\tau_{(i,j)s})_b$ of $G$ has a finite subfamily $\mathscr{V}$ which is a $s_r$-cover of $G$.*

In Theorem 3.8 (8)-(13), if we replace $b, r \neq 0$ and $r \in s(b)$ with $b \in M(L)$ and $r \in s^{\star}(b)$, then the conclusions are still right.

**Theorem 3.9.** *Let $(X, \tau_1, \tau_2)$ be an L-fuzzy bitopological space and $G \in L^X$, $a \in L\backslash\{0\}$. If for any $c, d \in L$, $s(c \wedge d) = s(c) \wedge s(d)$. Then the following conditions are equivalent:*

1. *$cd_{(i,j)s}(G) \geq a$.*

2. *For any $b \in s(a)$, $b \neq 0$, each strong $s_b$-cover $\mathscr{U}$ of $G$ with $b \in s(\tau_{(i,j)s}(\mathscr{U}))$ has a finite subfamily $\mathscr{V}$ which is a $Q_b$-cover of $G$.*

3. *For any $b \in s(a)$, $b \neq 0$, each strong $s_b$-cover $\mathscr{U}$ of $G$ with $b \in s(\tau_{(i,j)s}(\mathscr{U}))$ has a finite subfamily $\mathscr{V}$ which is a strong $s_b$-cover of $G$.*

4. *For any $b \in s(a)$, $b \neq 0$, each strong $s_b$-cover $\mathscr{U}$ of $G$ with $b \in s(\tau_{(i,j)s}(\mathscr{U}))$ has a finite subfamily $\mathscr{V}$ which is a strong $s_b$-cover of $G$.*

**Theorem 3.10.** *Let $(X, \tau_1, \tau_2)$ be an L-fuzzy bitopological space and $G, H \in L^X$. Then $cd_{(i,j)s}(G \wedge H) = cd_{(i,j)s}(G) \wedge cd_{(i,j)s}(H)$.*

*Proof.* By Theorem 3.7, we have $cd_{(i,j)s}(G \wedge H) = \vee\{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [(G \wedge H)\tilde{\subset} \vee \mathscr{U}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [(G \wedge H)\tilde{\subset} \vee \mathscr{U}], \forall \mathscr{U} \subset L^X\} = \vee\{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [G\tilde{\subset} \vee \mathscr{U}] \wedge [H\tilde{\subset} \vee \mathscr{U}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} ([G\tilde{\subset} \vee \mathscr{V}] \wedge [H\tilde{\subset} \vee \mathscr{V}]), \forall \mathscr{U} \subset L^X\} \geq \{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [G\tilde{\subset} \vee \mathscr{V}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [G\tilde{\subset} \vee \mathscr{V}], \forall \mathscr{U} \subset L^X\} \wedge \vee\{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [H\tilde{\subset} \vee \mathscr{U}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [H\tilde{\subset} \vee \mathscr{V}], \forall \mathscr{U} \subset L^X\} = cd_{(i,j)s}(G) \wedge cd_{(i,j)s}(H)$. □

**Theorem 3.11.** *Let $(X, \tau_1, \tau_2)$ be an L-fuzzy bitopological space and $G, H \in L^X$. Then $cd_{(i,j)s}(G \wedge H) = cd_{(i,j)s}(G) \wedge \tau^{\star}_{(i,j)s}(H)$.*

*Proof.* By Theorem 3.7, $cd_{(i,j)s}(G \wedge H) = \vee\{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [(G \wedge H)\tilde{\subset} \vee \mathscr{U}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [(G \wedge H)\tilde{\subset} \vee \mathscr{U}], \forall \mathscr{U} \subset L^X\}$ $= \vee\{a \in L : \tau_{(i,j)s}(\mathscr{U}) \wedge [G\tilde{\subset}(H' \vee \vee \mathscr{U})] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [G\tilde{\subset}(H' \vee \vee \mathscr{V})], \forall \mathscr{U} \subset L^X\} \geq \{a \wedge \tau^{\star}_{(i,j)s}(H) : \tau_{(i,j)s}(\mathscr{U}) \wedge [G\tilde{\subset} \vee \mathscr{U}] \wedge a \leq \bigwedge_{\mathscr{V} \in 2^{(\mathscr{U})}} [G\tilde{\subset} \vee \mathscr{V}], \forall \mathscr{U} \subset L^X\} = cd_{(i,j)s}(G) \wedge \tau^{\star}_{(i,j)s}(H)$. □

**Corollary 3.12.** *Let $(X, \tau_1, \tau_2)$ be an L-fuzzy bitopological space and $G \in L^X$. Then $cd_{(i,j)s}(G) = cd_{(i,j)s}(\underline{1}) \wedge \tau^{\star}_{(i,j)s}(G)$.*

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