Malaya Journal of Matematik, Vol. 8, No. 3, 1070-1073, 2020

https://doi.org/10.26637/MJM0803/0058

On slightly b-$\mathscr{I}$-continuous multifunctions

M. Sebasti Jeya Pushpam[1] and N. Rajesh[2]*

Abstract

In this paper, we introduce and study the concept of slightly b-$\mathscr{I}$-continuous multifunctions on ideal topological space.

Keywords

Ideal topological spaces, b-$\mathscr{I}$-open sets, b-$\mathscr{I}$-closed sets, slightly b-$\mathscr{I}$-continuous multifunctions.

AMS Subject Classification

54C05, 54C08, 54C10, 54C60.

[1] *Department of Mathematics, Auxilium college of Arts and Science for Women (Affiliated to Bharathidasan University), Karambakudi-622302, Tamil Nadu, India.*

[2] *Department of Mathematics, Rajah Serfoji Government College (Affiliated to Bharathidasan University), Thanjavur-613005, Tamil Nadu, India.*

***Corresponding author**: [1]pushjeya@gmail.com; [2]nrajesh_topology@yahoo.co.in

Article History: Received **05** March **2020**; Accepted **26** May **2020**

Contents

1. Introduction

It is well known that various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good number of them have been extended to the setting of multifunctions [6, 9, 10, 12, 13]. This implies that both, functions and multifunctions are important tools for studying other properties of spaces and for constructing new spaces from previously existing ones. The concept of ideals in topological spaces has been introduced and studied by Kuratowski [8] and Vaidyanathaswamy, [15]. An ideal $\mathscr{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A\in\mathscr{I}$ and $B\subset A$ implies $B\in\mathscr{I}$ and (ii) $A\in\mathscr{I}$ and $B\in\mathscr{I}$ implies $A\cup B\in\mathscr{I}$. Given a topological space (X,τ) with an ideal $\mathscr{I}$ on X and if $\mathscr{P}(X)$ is the set of all subsets of X, a set operator $(\cdot)^\star$: $\mathscr{P}(X)\to\mathscr{P}(X)$, called the local function [15] of A with respect to τ and $\mathscr{I}$, is defined as follows: for $A\subset X$, $A^\star(\tau,\mathscr{I})=\{x\in X|U\cap A\notin\mathscr{I}$ for every $U\in\tau(x)\}$, where $\tau(x)=\{U\in\tau|x\in U\}$. A Kuratowski closure operator $\mathrm{Cl}^\star(\cdot)$ for a topology $\tau^\star(\tau,\mathscr{I})$ called the $\star$-topology, finer than τ is defined by $\mathrm{Cl}^\star(A)=A\cup A^\star(\tau,\mathscr{I})$ when there is no chance of confusion, $A^\star(\mathscr{I})$ is denoted by $A^\star$. If $\mathscr{I}$ is an ideal on X, then $(X,\tau,\mathscr{I})$ is called an ideal topological space. In 2007, Akadag [1] introduced the notion of b-$\mathscr{I}$-open sets in ideal topological space. In this paper, to introduce a new generalization of b-$\mathscr{I}$-continuous multifunction called slightly b-$\mathscr{I}$-continuous multifunctions in ideal topological spaces.

2. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed unless explicitly stated. For a subset A of (X,τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset S of an ideal topological space $(X,\tau,\mathscr{I})$ is b-$\mathscr{I}$-open [1] if $S\subset\mathrm{Int}(\mathrm{Cl}^\star(S))\cup\mathrm{Cl}^\star(\mathrm{Int}(S))$. The complement of a b-$\mathscr{I}$-closed set is said to be a b-$\mathscr{I}$-open set. The b-$\mathscr{I}$-closure and the b-$\mathscr{I}$-interior, that can be defined in the same way as $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively, will be denoted by $b\mathscr{I}\,\mathrm{Cl}(A)$ and $b\mathscr{I}\,\mathrm{Int}(A)$, respectively. The family of all b-$\mathscr{I}$-open (resp. b-$\mathscr{I}$-closed) sets of $(X,\tau,\mathscr{I})$ is denoted by $B\mathscr{I}O(X)$ (resp. $B\mathscr{I}C(X)$). The family of all b-$\mathscr{I}$-open (resp. b-$\mathscr{I}$-closed) sets of $(X,\tau,\mathscr{I})$ containing a point $x\in X$ is denoted by $B\mathscr{I}O(X,x)$ (resp. $B\mathscr{I}C(X,x)$). A subset U of X is called a b-$\mathscr{I}$-neighborhood of a point $x\in X$ if there exists $V\in B\mathscr{I}O(X,x)$ such that $V\subset U$. By a multifunction $F:(X,\tau)\to(Y,\sigma)$, following [3], we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B)=\{x\in X:F(x)\subset B\}$ and $F^-(B)=\{x\in X:F(x)\cap B\neq\emptyset\}$. In particular, $F^-(Y)=\{x\in X:y\in F(x)\}$

for each point $y \in Y$ and for each $A \subset X$, $F(A) = \cup_{x\in A} F(x)$. Then F is said to be surjection if $F(x) = y$. A multifunction $F : (X,\tau) \to (Y,\sigma)$ is said to be surjective if $F(X) = Y$.

Definition 2.1. *An ideal topological space $(X,\tau,\mathscr{I})$ is said to be b-$\mathscr{I}$-T_2 [2] if for each pair of distinct points x and y in X, there exist disjoint b-$\mathscr{I}$-open sets U and V in X such that $x \in U$ and $y \in V$.*

Definition 2.2. *A multifunction $F : X \to Y$ is said to be [6]:*

1. *upper b-$\mathscr{I}$-continuous if for each point $x \in X$ and each open set V containing $F(x)$, there exists $U \in B\mathscr{I}O(X,x)$ such that $F(U) \subset V$;*
2. *lower b-$\mathscr{I}$-continuous if for each point $x \in X$ and each open set V such that $F(x) \cap V \neq \emptyset$, there exists $U \in B\mathscr{I}O(X,x)$ such that $U \subset F^-(V)$.*

3. Slightly b-$\mathscr{I}$-continuous multifunctions

Definition 3.1. *A multifunction $F : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ is said to be :*

1. *upper slightly b-$\mathscr{I}$-continuous at $x \in X$ if for each clopen set V of Y containing $F(x)$, there exists $U \in B\mathscr{I}O(X)$ containing x such that $F(U) \subset V$;*
2. *lower slightly b-$\mathscr{I}$-continuous at $x \in X$ if for each clopen set V of Y such that $F(x) \cap V \neq \varnothing$, there exists $U \in B\mathscr{I}O(X)$ containing x such that $F(u) \cap V \neq \varnothing$ for every $u \in U$;*
3. *upper (lower) slightly b-$\mathscr{I}$-continuous if it has this property at each point of X.*

Remark 3.2. *It is clear that every upper b-$\mathscr{I}$-continuous multifunction is upper slightly b-$\mathscr{I}$-continuous. But the converse is not true in general, as the following example shows.*

Example 3.3. *Let $X = \{a,b,c\}$, $\tau = \{\emptyset,\{a\},X\}$, $\sigma = \{\emptyset,\{a\},\{c\},\{a,c\},X\}$ and $\mathscr{I} = \{\emptyset\}$. The multifunction $F : (X,\tau,\mathscr{I}) \to (X,\sigma)$ defined by $F(x) = \{x\}$ for all $x \in X$ is upper slightly b-$\mathscr{I}$-continuous but is not upper b-$\mathscr{I}$-continuous.*

Theorem 3.4. *For a multifunction $F : (X,\tau,\mathscr{I}) \to (Y,\sigma)$, the following statements are equivalent :*

1. *F is upper slightly b-$\mathscr{I}$-continuous;*
2. *For each $x \in X$ and for each clopen set V such that $x \in F^+(V)$, there exists a b-$\mathscr{I}$-open set U containing x such that $U \subset F^+(V)$;*
3. *For each $x \in X$ and for each clopen set V such that $x \in F^+(Y\backslash V)$, there exists a b-$\mathscr{I}$-closed set H such that $x \in X\backslash H$ and $F^-(V) \subset H$;*
4. *$F^+(V)$ is a b-$\mathscr{I}$-open set for any clopen set V of Y;*
5. *$F^-(V)$ is a b-$\mathscr{I}$-closed set for any clopen set V of Y;*
6. *$F^-(Y\backslash V)$ is a b-$\mathscr{I}$-closed set for any clopen set V of Y;*
7. *$F^+(Y\backslash V)$ is a b-$\mathscr{I}$-open set for any clopen set V of Y.*

Proof. (1)⇔(2): Clear. (2)⇔(3): Let $x \in X$ and V be a clopen set of Y such that $x \in F^+(Y\backslash V)$. By (2), there exists a b-$\mathscr{I}$-open set U containing x such that $U \subset F^+(Y\backslash V)$. Then $F^-(V) \subset X\backslash U$. Take $H = X\backslash U$. We have $x \in X\backslash H$ and H is b-$\mathscr{I}$-open. The converse is similar. (1)⇔(4): Let $x \in F^+(V)$ and V be a clopen set of Y. By (1), there exists a b-$\mathscr{I}$-open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x\in F^+(V)} U_x$. Since any union of b-$\mathscr{I}$-open sets is b-$\mathscr{I}$-open, $F^+(V)$ is b-$\mathscr{I}$-open. The converse can be shown similarly. (4)⇔(5)⇔(6)⇔(7) : Clear. □

Theorem 3.5. *For a multifunction $F : (X,\tau,\mathscr{I}) \to (Y,\sigma)$, the following statements are equivalent :*

1. *F is lower slightly b-$\mathscr{I}$-continuous;*
2. *For each $x \in X$ and for each clopen set V such that $X \in F^-(V)$, there exists a b-$\mathscr{I}$-open set U containing x such that $U \subset F^-(V)$;*
3. *For each $x \in X$ and for each clopen set V such that $x \in F^-(Y\backslash V)$, there exists a b-$\mathscr{I}$-closed set H such that x $\in X\backslash H$ and $F^+(V) \subset H$;*
4. *$F^-(V)$ is a b-$\mathscr{I}$-open set for any clopen set V of Y;*
5. *$F^+(V)$ is a b-$\mathscr{I}$-closed set for any clopen set V of Y;*
6. *$F^+(Y\backslash V)$ is a b-$\mathscr{I}$-closed set for any clopen set V of Y;*
7. *$F^-(Y\backslash V)$ is a b-$\mathscr{I}$-open set for any clopen set V of Y.*

Proof. The proof is similar to that of Theorem 3.4. □

Lemma 3.6. *[1] Let A and B be subsets of a topological space (X,τ). If $A \in B\mathscr{I}O(X)$ and $B \in \tau$, then $A \cap B \in B\mathscr{I}O(B)$;*

Theorem 3.7. *Let $F : (X,\tau,\mathscr{I}) \to (Y,\sigma)$ be a multifunction and U be an open subset of X. If F is a lower (upper) slightly b-$\mathscr{I}$-continuous multifunction, then multifunction $F_{|_U} : (U,\tau|U,\mathscr{I}|U) \to (Y,\sigma)$ is a lower (upper) slightly b-$\mathscr{I}|U$-continuous multifunction.*

Proof. Let V be any clopen subset of Y, $x \in U$ and $x \in F^-_{|_U}(V)$. Since F is lower slightly b-$\mathscr{I}$-continuous multifunction, it follows that there exists a b-$\mathscr{I}$-open set G containing x such that $G \subset F^-(V)$. By Lemma 3.6, we have $x \in G \cap U \in B\mathscr{I}O(U)$ and $G \cap U \subset F^-_{|_U}(V)$. This shows that the restriction multifunction $F_{|_U}$ is a lower slightly b-$\mathscr{I}$-continuous. The proof of the upper slightly b-$\mathscr{I}|U$-continuity of $F_{|_U}$ can be done by a similar manner. □

Lemma 3.8. *[11] For a multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$, the following holds:*

1. *$G_F^+(A\times B)=A\cap F^+(B)$;*
2. *$G_F^-(A\times B)=A\cap F^-(B)$*

for any subset A of X and B of Y.

Theorem 3.9. *Let $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ be a multifunction. If the graph multifunction of F is an upper slightly b-$\mathscr{I}$-continuous, then F is an upper slightly b-$\mathscr{I}$-continuous.*

Proof. Let $x\in X$ and V be any clopen subset of Y such that $x\in F^+(V)$. We obtain that $x\in G_F^+(X\times V)$ and that $X\times V$ is a clopen set. Since the graph multifunction G_F is upper slightly b-$\mathscr{I}$-continuous, it follows that there exists a b-$\mathscr{I}$-open set U of X containing x such that $U\subset G_F^+(X\times V)$. Since $U\subset G_F^+(X\times V)=X\cap F^+(V)=F^+(V)$. We obtain that $U\subset F^+(V)$. Thus, F is upper slightly b-$\mathscr{I}$-continuous. □

Theorem 3.10. *A multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is lower slightly b-$\mathscr{I}$-continuous if $G_F:(X,\tau,\mathscr{I})\to(X\times Y,\tau\times\sigma)$ is lower slightly b-$\mathscr{I}$-continuous.*

Proof. Suppose that G_F is lower slightly b-$\mathscr{I}$-continuous. Let $x\in X$ and V be any clopen set of Y such that $x\in F^-(V)$. Then $X\times V$ is clopen in $X\times Y$ and $G_F(x)\cap(X\times V)=(\{x\}\times F(x))\cap(X\times V)=\{x\}\times(F(x)\cap V)\neq\varnothing$. Since G_F is lower slightly b-$\mathscr{I}$-continuous, there exists a b-$\mathscr{I}$-open U containing x such that $U\subset G_F^-(X\times V)$; hence $U\subset F^-(V)$. This shows that F is lower slightly b-$\mathscr{I}$-continuous. □

Theorem 3.11. *Suppose that (X_α,τ_α) are topological spaces where $\alpha\in J$. Let $F:X\to\prod_{\alpha\in J}X_\alpha$ be a multifunction from an ideal topological space $(X,\tau,\mathscr{I})$ to the product space $\prod_{\alpha\in J}X_\alpha$ and let $P_\alpha:\prod_{\alpha\in J}X_\alpha\to X_\alpha$ be the projection multifunction for each $\alpha\in J$ which is defined by $P_\alpha((x_\alpha))=\{x_\alpha\}$. If F is an upper (lower) slightly b-$\mathscr{I}$-continuous multifunction, then $P_\alpha\circ F$ is an upper (lower) slightly b-$\mathscr{I}$-continuous multifunction for each $\alpha\in J$.*

Proof. Take any $\alpha_0\in J$. Let V_{α_0} be a clopen set in $(X_{\alpha_0},\tau_{\alpha_0})$. Then $(P_{\alpha_0}\circ F)^+(V_{\alpha_0})=F^+(P_{\alpha_0}^+(V_{\alpha_0}))=F^+(V_{\alpha_0}\times\prod_{\alpha\neq\alpha_0}X_\alpha)$ (resp. $(P_{\alpha_0}\circ F)^-(V_{\alpha_0})=F^-(P_{\alpha_0}^-(V_{\alpha_0}))=F^-(V_{\alpha_0}\times\prod_{\alpha\neq\alpha_0}X_\alpha)$). Since F is an upper (lower) slightly b-$\mathscr{I}$-continuous multifunction and since $V_{\alpha_0}\times\prod_{\alpha\neq\alpha_0}X_\alpha$ is a clopen set, it follows that $F^+(V_{\alpha_0}\times\prod_{\alpha\neq 0}X_\alpha)$ (resp. $F^-(V_{\alpha_0}\times\prod_{\alpha\neq\alpha 0}X_\alpha)$) is a b-$\mathscr{I}$-open set in $(X,\tau,\mathscr{I})$. This shows that $P_{\alpha 0}\circ F$ is an upper (lower) slightly b-$\mathscr{I}$-continuous multifunction. Hence, we obtain that $P_\alpha\circ F$ is an upper (lower) slightly b-$\mathscr{I}$-continuous multifunction for each $\alpha\in J$. □

Theorem 3.12. *Suppose that $(X,\tau,\mathscr{I}),(Y,\sigma\,\mathscr{J})$ are ideal topological spaces and (Z,η) is a topological space and $F_1:(X,\tau,\mathscr{I})\to(Y,\sigma)$, $F_2:(Y,\sigma\,\mathscr{J})\to(Z,\eta)$ are multifunctions. Let $F_1\times F_2:X\to Y\times Z$ be a multifunction which is defined by $(F_1\times F_2)(x)=F_1(x)\times F_2(x)$ for each $x\in X$. If $F_1\times F_2$ is upper (lower) slightly b-$\mathscr{I}$-continuous multifunction, then F_1 and F_2 are upper (lower) slightly b-$\mathscr{I}$-continuous multifunctions.*

Proof. Let $x\in X$, $K\subset Y$ and $H\subset Z$ be clopen sets such that $x\in F_1^+(K)$ and $x\in F_2^+(H)$. Then we obtain that $F_1(x)\subset K$ and $F_2(x)\subset H$ and thus, $F_1(x)\times F_2(x)=(F_1\times F_2)(x)\subset K\times H$. We have $x\in(F_1\times F_2)^+(K\times H)$. Since $F_1\times F_2$ is upper slightly b-$\mathscr{I}$-continuous multifunction, it follows that there exists a b-$\mathscr{I}$-open set U containing x such that $U\subset(F_1\times F_2)^+(K\times H)$. We obtain that $U\subset F_1^+(K)$ and $U\subset F_2^+(H)$. Thus, F_1 and F_2 are upper slightly b-$\mathscr{I}$-continuous multifunction. The proof of the lower slightly b-$\mathscr{I}$-continuity of F_1 and F_2 is similar to the above. □

Definition 3.13. *[4] Let (X,τ) be a topological space. X is said to be a strongly normal space if for every disjoint closed subsets K and F of X, there exist two clopen sets U and V such that $K\subset U$, $F\subset V$ and $U\cap V=\emptyset$.*

Recall that a multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is said to be punctually closed if for each $x\in X$, $F(x)$ is closed.

Theorem 3.14. *Let $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ be an upper slightly b-$\mathscr{I}$-continuous multifunction and punctually closed from a topological space X to a strongly normal space Y and let $F(x)\cap F(y)=\varnothing$ for each pair of distinct points x and y of X. Then X is a b-$\mathscr{I}$-T_2 space.*

Proof. Let x and y be any two distinct points in X. Then we have $F(x)\cap F(y)=\varnothing$. Since Y is strongly normal, it follows that there exist disjoint clopen sets U and V containing $F(x)$ and $F(y)$, respectively. Thus $F^+(U)$ and $F^+(V)$ are disjoint b-$\mathscr{I}$-open sets containing x and y, respectively and hence (X,τ) is b-$\mathscr{I}$-T_2. □

Definition 3.15. *Let $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ be a multifunction. The multigraph $G(F)$ is said to be $\mathscr{I}$-co-closed if for each $(x,y)\notin G(F)$, there exist b-$\mathscr{I}$-open set U and clopen set V containing x and y, respectively, such that $(U\times V)\cap G(F)=\varnothing$.*

Definition 3.16. *[5] A topological space (X,τ) is said to be clopen T_2 (clopen Hausdorff) if for each pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X such that $x\in U$ and $y\in V$.*

Theorem 3.17. *If a multifunction $F:(X,\tau,\mathscr{I})\to(Y,\sigma)$ is an upper slightly b-$\mathscr{I}$-continuous such that $F(x)$ is mildly compact relative to Y for each $x\in X$ and Y is a clopen Hausdorff space, then the multigraph $G(F)$ of F is b-$\mathscr{I}$-co-closed in $X\times Y$.*

Proof. Let $(x,y)\in(X\times Y)\backslash G(f)$. That is $y\notin F(x)$. Since Y is clopen Hausdorff, for each $z\in F(x)$, there exist disjoint clopen sets $V(z)$ and $U(z)$ of Y such that $z\in U(z)$ and $y\in$

$V(z)$. Then $\{U(z) : z \in F(x)\}$ is a clopen cover of $F(x)$ and since $F(x)$ is mildly compact, there exists a finite number of points, say, $z_1, z_2, ..., z_n$ in $F(x)$ such that $F(x) \subset \cup\{U(z_i) : i = 1,2,...,n\}$. Put $U = \cup\{U(z_i) : i = 1,2,...,n\}$ and $V = \cap\{V(y_i) : i = 1,2,...,n\}$. Then U and V are clopen sets in Y such that $F(x) \subset U$, $y \in V$ and $U \cap V = \varnothing$. Since F is upper slightly b-$\mathscr{I}$-continuous multifunction, there exists a b-$\mathscr{I}$-open set W of X containing x such that $F(W) \subset U$. We have $(x,y) \in W \times V \subset (X \times Y) \backslash G(F)$. We obtain that $(W \times V) \cap G(F) = \varnothing$; hence $G(F)$ is b-$\mathscr{I}$-co-closed in $X \times Y$. □

References

[1] M. Akadag, On b-$\mathscr{I}$-open sets and b-$\mathscr{I}$-continuous functions, *Internat. J. Math. Math. Sci.*, (2007), 1–13.

[2] R. Balaji and N. Rajesh, Some new separation axioms via b-$\mathscr{I}$-open sets, *Inter. J. Pure Appl. Math.*, 94 (2)(2014), 223–232.

[3] T. Banzaru, Multifunctions and M-product spaces, *Bull. Stin. Tech. Inst. Politech. Timisoara, Ser. Mat. Fiz. Mer. Teor. Apl.*, 17(31)(1972), 17–23.

[4] E. Ekici, Slightly continuous multifunctions, *International J. Math. Sci.*, 4(1)(2005), 69–78.

[5] E. Ekici, Generalization of perfectly continuous, Regular set-connected and clopen functions, *Acta. Math. Hungarica*, 107(3)(2005), 193–206.

[6] P. Gomathi Sundari, N. Rajesh and R. Muthu Vijayalakshmi, On upper and lower b-$\mathscr{I}$-continuous multifunctions, *Aryabhatta Journal of Mathematics & Informatics*, 11(1)(2019), 87–90.

[7] D. Jankovic and T. R. Hamlett, New Toplogies From Old Via Ideals, *Amer. Math. Monthly*, 97 (4) (1990), 295–310.

[8] K. Kuratowski, *Topology*, Academic Press, New York, 1966.

[9] T. Noiri and V. Popa, Almost weakly continuous multifunctions, *Demonstratio Math.*, 26 (1993), 363–380.

[10] T. Noiri and V. Popa, A unified theory of almost continuity for multifunctions, *Sci. Stud. Res. Ser. Math. Inform.*, 20(1) (2010), 185–214.

[11] T. Noiri and V. Popa, Almost weakly continuous multifunctions, *Demonstraio Math.*, 26(1993), 363–380.

[12] V. Popa, A note on weakly and almost continuous multifunctions, *Univ, u Novom Sadu, Zb. Rad. Prirod-Mat. Fak. Ser. Mat.*, 21(1991), 31–38.

[13] V. Popa, Weakly continuous multifunction, *Boll. Un. Mat. Ital.*, (5) 15-A(1978), 379–388.

[14] R. Staum, The algebra of bounded continuous fuctions into a nonarchimedean field, *Pacific J. Math.*, 50(1974), 169–185.

[15] R. Vaidyanathaswamy, The localization theory in set topology, *Proc. Indian Acad. Sci.*, 20(1945), 51–61.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆