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Measures of (i, j)-semi-connectedness of L-fuzzy bitopological spaces

P. Gomathi Sundari[1] and R. Menaga[2]*

Abstract
In this paper, we use L-fuzzy (i, j)-semi-open operator to introduce the degree of (i, j)-s-separatedness and the degree of (i, j)-s-connectedness in L-fuzzy bitopological spaces. Many characterizations of the degree of (i, j)-s-connectedness are presented in L-fuzzy bitopological spaces.

Keywords
L-topological spaces, fuzzy (i, j)-s-connected, fuzzy (i, j)-s-connectedness degree.

AMS Subject Classification
54A40, 54D30, 03E72.

[1] *Department of Mathematics, Rajah Serfoji Government College (Affiliated to Bharathidasan University), Thanjavur-613005, Tamil Nadu, India.*
[2] *Department of Mathematics, Sengamala Thayaar Educational Trust Women's College (Affiliated to Bharathidasan University), Mannargudi-614016, Tamil Nadu, India.*
***Corresponding author**: [1]rsgcgomathi18@gmail.com; [2]menagamohan2008@gmail.com
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Contents

1. Introduction

It is well known that after the introduction of the L-fuzzy topological space by Kubiak [6] and Sostak [9] in 1985, a large number of Mathematicians have taken great interests in generalizing and extending different concepts of set topology and Chang's fuzzy topology [1] into L-fuzzy topology. The concept of connectedness along with some of its allied forms is one of the directions that have been ventured with meticulous attention. In [4], the authors introduced the notion of L-fuzzy (i, j)-semi-open operator $\tau_{(i,j)s}$ in L-fuzzy bitopological spaces as a generalization of (i, j)-semi-open L-subsets, where L completely distributive DeMorgan algebra. $\tau_{(i,j)s}(A)$ can be regarded as the degree to which A is (i, j)-semi-open. So that, actually $\tau_{(i,j)s}$ reflects the essence of L-fuzzy bitopology. In this paper, we use L-fuzzy (i, j)-semi-open operator to introduce the degree of (i, j)-s-separatedness and the degree of (i, j)-s-connectedness in L-fuzzy bitopological spaces. Many characterizations of the degree of (i, j)-s-connectedness are presented in L-fuzzy bitopological spaces.

2. Preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, X a nonempty set and L^X the set of all L-fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L are denoted by 0 and 1. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$. An element a in L is called a prime element if $b \wedge c \leq a$ implies that $b \leq a$ or $c \leq a$. a in L is called a co-prime element if a' is a prime element [2]. The set of nonunit prime elements in L is denoted by $P(L)$ and the set of nonzero co-prime elements in L by $M(L)$. The binary relation $\prec$ in L is defined as follows: for $a, b \in L, a \prec b$ if and only if for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [2]. In a completely distributive De Morgan algebra L, each element b is a sup of $\{a \in L | a \prec b\}$. The set $s(b) = \{a \in L | a \prec b\}$ is called the greatest minimal family of b in the sense of [7, 10].

Definition 2.1. *[6, 9] Let X be a universe of discourse, $\tau \in \Im(P(X))$, satisfying the following conditions:*

1. *$\tau(\underline{0}) = \tau(\underline{1}) = 1$;*
2. *for any $A, B \in L^X$, $\tau(A \wedge B) \geq \tau(A) \wedge \tau(B)$;*
3. *for any $A_\lambda \in L^X, \lambda \in \Delta$, $\tau(\bigvee_{\lambda \in \Delta} A_\lambda) \geq \bigwedge_{\lambda \in \Delta} \tau(A_\lambda)$.*

The pair (X, τ) is called an L-fuzzy topological space. $\tau(U)$ is called the degree of openness of U, $\tau^\star(U) = \tau(U')$

is called the degree of closedness of U, where U' is the L-complement of U.

Theorem 2.2. *[8] Let $\tau : L^X \to L$ be a function. Then the following conditions are equivalent:*

1. *τ is an L-fuzzy topology on X,*
2. *$\tau_{[a]} = \{A \in L^X : \tau(A) \geq a\}$ is an L-topology on X for each $a \in M(L)$.*

Definition 2.3. *An L-fuzzy bitopological space (or L-fbts for short) is an ordered triple (X, τ_1, τ_2), where τ_1 and τ_2 are subfamilies of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima.*

Definition 2.4. *Let (X, τ_1, τ_2) be an L-fbts. For $A \in L^X$, define $\tau_{(i,j)s} : L^X \to L$ by*

$$\tau_{(i,j)s}(A) = \bigwedge_{x_\lambda \prec A} \bigvee_{x_\lambda \prec B} \{\tau(B) \wedge \bigwedge_{y_\lambda \prec A} \bigwedge_{y_\mu \not\leq D \geq A} (\tau(D'))'\}.$$

Then $\tau_{(i,j)s}$ is called L-fuzzy (i,j)-semi-open operator induced by τ_1 and τ_2, where $\tau_{(i,j)s}(A)$ can be regarded as the degree to which A is (i,j)-semi-open and $\tau^\star_{(i,j)s}(A) = \tau_{(i,j)s}(A')$ can be regarded as the degree to which A is (i,j)-semi-closed.

Theorem 2.5. *Let (X, τ_1, τ_2) be an L-fbts and $A \in L^X$. Then $A \in (\tau_{(i,j)s})_{[a]}$ if and only if A is (i,j)-semi-open in $\tau_{[a]}$, where $a \in M(L)$ and $(\tau_{(i,j)s})_{[a]} = \{A \in L^X : \tau_{(i,j)s}(A) \geq a\}$.*

Lemma 2.6. *Let $\tau_{(i,j)s} : L^X \to L$ be an L-fuzzy (i,j)-semi-open operator induced by τ. Then $\tau_{(i,j)s}$ satisfies the following conditions:*

1. *$\tau_{(i,j)s}(\underline{0}) = \tau_{(i,j)s}(\underline{1}) = 1$;*
2. *for any $A_\lambda, \lambda \in \Delta$, $\tau_{(i,j)s}(\bigvee_{\lambda \in \Delta} A_\lambda) \geq \bigwedge_{\lambda \in \Delta} \tau_{(i,j)s}(A_\lambda)$.*

Definition 2.7. *An L-fuzzy (i,j)-s-closure operator on X is a mapping (i,j)-$s\mathrm{Cl} : L^X \to L^{M(L^X)}$ satisfying the following conditions:*

1. *(i,j)-$s\mathrm{Cl}(A)(x_\lambda) = \bigwedge_{\mu \prec \lambda} (i,j)$-$s\mathrm{Cl}(A)(x_\mu)$ for all $x_\lambda \in M(L^X)$,*
2. *(i,j)-$s\mathrm{Cl}(\underline{0})(x_\lambda) = 0$ for all $x_\lambda \in M(L^X)$,*
3. *(i,j)-$s\mathrm{Cl}(A)(x_\lambda) = 1$ for all $x_\lambda \leq A$,*
4. *for all $a \in L_0$, $((i,j)$-$s\mathrm{Cl}(\vee(i,j)$-$s\mathrm{Cl}(A))_{[a]}))_{[a]} \subset ((i,j)$-$s\mathrm{Cl}(A))_{[a]}$,*

(i,j)-$s\mathrm{Cl}(A)_{(x_\lambda)}$ is called the degree to which x_λ belongs to the (i,j)-s-closure of A.

Theorem 2.8. *Let $\tau_{(i,j)s} : L^X \to L$ be the L-fuzzy (i,j)-semi-open operator on X and (i,j)-$s\mathrm{Cl}^{\tau_{(i,j)s}}$ be the L-fuzzy (i,j)-s-closure operator induced by $\tau_{(i,j)s}$. Then for each $x_\lambda \in M(L^X)$ and $A \in L^X$, (i,j)-$s\mathrm{Cl}^{\tau_{(i,j)s}}(A)(x_\lambda) = \bigwedge_{x \not\leq D \geq A} (\tau_{(i,j)s}(D'))'$.*

3. On (i,j)-s-separatedness degree

Definition 3.1. *Let (X, τ_1, τ_2) be an L-fuzzy bitopological space and $A, B \in L^X$. The (i,j)-s-separatedness degree of A and B defined by $\mathscr{P}(A,B) = (\bigwedge_{x_\lambda \leq A} ((i,j)$-$s\mathrm{Cl}(B)(x_\lambda))') \wedge (\bigwedge_{y_\mu \leq B} ((i,j)$-$s\mathrm{Cl}(A)(y_\mu))')$.*

Proposition 3.2. *Let (X, τ_1, τ_2) be an L-fuzzy bitopological space and $A, B \in L^X$. Then $\mathscr{P}(A,B) = 1$ if and only if A and B are (i,j)-s-separated in (X, τ_1, τ_2).*

Lemma 3.3. *Let (X, τ_1, τ_2) be an L-fbts and $A, B \in L^X$. If $A \wedge B \neq \underline{0}$, then $\mathscr{P}(A,B) = 0$.*

Proof. Let $z_\mu \in M(L^X)$ such that $x_\mu \leq A \wedge B$. Then $\mathscr{P}(A,B) = (\bigwedge_{x_\lambda \leq A} ((i,j)$-$s\mathrm{Cl}(B)(x_\lambda))') \wedge (\bigwedge_{x_\lambda \leq B} ((i,j)$-$s\mathrm{Cl}(A)(x_\lambda))') \leq ((i,j)$-$s\mathrm{Cl}(B)(z_\mu))' \wedge ((i,j)$-$s\mathrm{Cl}(A)(z_\mu))' = 1' \wedge 1' = 0$. □

Lemma 3.4. *Let (X, τ_1, τ_2) be an L-fbts and $A, B, C, D \in L^X$. If $C \leq A$ and $D \leq B$, then $\mathscr{P}(A,B) \leq \mathscr{P}(C,D)$.*

Proof. If $C \leq A$ and $D \leq B$, then (i,j)-$s\mathrm{Cl}(C) \leq (i,j)$-$s\mathrm{Cl}(A)$ and (i,j)-$s\mathrm{Cl}(D) \leq (i,j)$-$s\mathrm{Cl}(B)$. Hence we have $\mathscr{P}(A,B) = (\bigwedge_{x_\lambda \leq A} ((i,j)$-$s\mathrm{Cl}(B)(x_\lambda))') \wedge (\bigwedge_{y_s \leq B} ((i,j)$-$s\mathrm{Cl}(A)(y_s))') \leq (\bigwedge_{x_\lambda \leq A} ((i,j)$-$s\mathrm{Cl}(D)(x_\lambda))') \wedge (\bigwedge_{y_s \leq B} ((i,j)$-$s\mathrm{Cl}(C)(y_s))') \leq (\bigwedge_{x_\lambda \leq C} ((i,j)$-$s\mathrm{Cl}(D)(x_\lambda))') \wedge (\bigwedge_{y_s \leq D} ((i,j)$-$s\mathrm{Cl}(C)(y_s))') = \mathscr{P}(C,D)$. □

Lemma 3.5. *Let (X, τ_1, τ_2) be an L-fuzzy bitopological space, $A, B \in L^X$ and $a \in M(L)$. Then $(\mathscr{P}(A,B))' \geq a$ if and only if there exist $D, E \in L^X$ such that $D \geq A$, $E \geq B$, $D \wedge B = E \wedge A = \underline{0}$ and $(\tau_{(i,j)s}(D'))' \vee (\tau_{(i,j)s}(E'))' \not\geq a$.*

Proof. Suppose that $(\mathscr{P}(A,B))' \geq a$. Then $(\mathscr{P}(A,B))' \geq b$ for some $b \in s^\star(a)$. Then $\bigvee_{x_\lambda \leq A} (i,j)$-$s\mathrm{Cl}(B)(x_\lambda) \vee \bigvee_{y_s \leq B} (i,j)$-$s\mathrm{Cl}(A)(y_s) \not\geq b$. Moreover, we have $\bigvee_{x_\lambda \leq A} \bigwedge_{x_\lambda \not\leq E \geq B} (\tau_{(i,j)s}(E'))' \vee \bigvee_{y_s \leq B} \bigwedge_{y_s \not\leq D \geq A} (\tau_{(i,j)s}(D'))' \not\geq b$. Hence for any $x_\lambda \leq A$ and for any $y_s \leq B$, there exist $D_{y_s}, E_{x_\lambda} \in L^X$ such that $x_\lambda \not\leq E_{x_\lambda} \geq B$, $y_s \not\leq D_{y_s} \geq A$ and $(\tau_{(i,j)s}(D'_{y_s}))' \vee (\tau_{(i,j)s}(E'_{x_\lambda}))' \not\geq b$. Let $E = \bigwedge_{x_\lambda \leq A} E_{x_\lambda}$ and $D = \bigwedge_{y_s \leq B} D_{y_s}$. Then we have $D \geq A$, $E \geq B$, $D \wedge B = E \wedge A = \underline{0}$ and $(\tau_{(i,j)s}(D'_{y_s}))' \vee (\tau_{(i,j)s}(E'_{x_\lambda}))' = (\tau_{(i,j)s}(\bigvee_{y_s \leq B} D'_{y_s}))' \vee (\tau_{(i,j)s}(\bigvee_{x_\lambda \leq B} E'_{x_\lambda}))' \leq \bigvee_{y_s \leq B} (\tau_{(i,j)s}(D'_{y_s}))' \vee \bigvee_{x_\lambda \leq A} (\tau_{(i,j)s}(E'_{x_\lambda}))' \not\geq a$. Conversely, there exist $D, E \in L^X$ with $D \geq A$, $E \geq B$, $D \wedge B = E \wedge A = \underline{0}$ and $(\tau_{(i,j)s}(D'))' \vee (\tau_{(i,j)s}(E'))' \not\geq a$. So $(\mathscr{P}(A,B))' = \bigvee_{x_\lambda \leq A} (i,j)$-$s\mathrm{Cl}(B)(x_\lambda) \vee \bigvee_{y_s \leq B} (i,j)$-$s\mathrm{Cl}(A)(y_s) = \bigvee_{x_\lambda \leq A} \bigwedge_{x_\lambda \not\leq G \geq B} (\tau_{(i,j)s}(G'_{y_s}))' \vee \bigvee_{y_s \leq B} \bigwedge_{y_s \not\leq H \geq A} (\tau_{(i,j)s}(H'_{y_s}))' \leq (\tau_{(i,j)s}(D'_{y_s}))' \vee (\tau_{(i,j)s}(E'_{x_\lambda}))'$. Then $(\mathscr{P}(A,B))' \not\geq a$. □

Definition 3.6. *Let (X,τ_1,τ_2) be an L-fuzzy bitopological space and $G\in L^X$. Define (i,j)-$s\mathscr{C}(G)=\wedge\{\mathscr{P}(A,B)':A,B\in L^X\setminus\{\underline{0}\},G=A\vee B\}$. Then (i,j)-$s\mathscr{C}(G)$ is said to be the (i,j)-s-connectedness degree of G. That is, (i,j)-$s\mathscr{C}(G)=\bigwedge_{A,B\in L^X\setminus\{\underline{0}\},G=A\vee B}\{\bigvee_{x_\lambda\leq A}(i,j)\text{-}s\mathrm{Cl}(B)(x_\lambda)\vee\bigvee_{y_s\leq B}(\mathrm{Cl}(A)(y_s)\}$.*

Theorem 3.7. *Let (X,τ_1,τ_2) be an L-fuzzy bitopological space and $G\in L^X$. Then (i,j)-$s\mathscr{C}(G)=\bigwedge_{G\wedge A\neq 0,G\wedge B\neq 0,G\wedge A\wedge B\neq 0,G\leq A\vee B}\{(\tau_{(i,j)s}(A'_{y_s}))'\vee(\tau_{(i,j)s}(B'_{x_\lambda}))'\}$.*

Proof. We have (i,j)-$s\mathscr{C}(G)=\bigwedge_{A,B\in L^X\setminus\{\underline{0}\},G=A\vee B}\{\bigvee_{x_\lambda\leq A}(i,j)\text{-}s\mathrm{Cl}(B)(x_\lambda)\vee\bigvee_{y_s\leq B}(i,j)\text{-}s\mathrm{Cl}(A)(y_s)\}=\bigwedge_{A,B\in L^X\setminus\{0\},G=A\vee B}\{\bigvee_{x_\lambda\leq A}\bigwedge_{x_\lambda\not\leq D\geq B}(\tau_{(i,j)s}(D'))'\vee\bigvee_{y_s\leq B}\bigwedge_{y_s\not\leq E\geq A}(\tau_{(i,j)s}(E'))'\}=\bigwedge_{G\wedge A\neq 0,G\wedge B\neq 0,G\wedge A\wedge B\neq 0,G=A\vee B}\{\bigvee_{x_\lambda\leq G\wedge A}\bigwedge_{x_\lambda\not\leq D\geq G\wedge B}(\tau_{(i,j)s}(D'))'\vee\bigvee_{y_s\leq G\wedge B}\bigwedge_{y_s\not\leq E\geq G\wedge A}(\tau_{(i,j)s}(E'))'\}\leq\bigwedge_{G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B\neq\underline{0},G=A\vee B}\{\bigvee_{x_\lambda\leq G\wedge A}(\tau_{(i,j)s}(D'))'\vee\bigvee_{y_s\leq G\wedge B}(\tau_{(i,j)s}(E'))'\}=\bigwedge_{G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B\neq\underline{0},G=A\vee B}\{(\tau_{(i,j)s}(D'))'\vee(\tau_{(i,j)s}(E'))'\}$.
Conversely, suppose (i,j)-$s\mathscr{C}(G)\not\geq a$ where $a\in M(L)$. Then there exist $A,B\in L^X\setminus\{\underline{0}\}$ with $G=A\vee B$ and $(\mathscr{P}(A,B))\not\geq a$. By Lemma 3.5, there exist $D,E\in L^X$ such that $D\geq A$, $E\geq B$, $D\wedge B=E\wedge A=\underline{0}$ and $(\tau_{(i,j)s}(D'))'\vee(\tau_{(i,j)s}(E'))'\not\geq a$. Hence $\bigwedge_{G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B=\underline{0},G\leq A\vee B}\{(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(E'))'\}\not\geq a$. Then (i,j)-$s\mathscr{C}(G)\geq\bigwedge_{G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B=\underline{0},G\leq A\vee B}\{(\tau_{(i,j)s}(B'))'\vee\tau_{(i,j)s}(E'))'\}$ and this completes the proof. □

Corollary 3.8. *Let (X,τ_1,τ_2) be an L-fuzzy bitopological space. Then (i,j)-$s\mathscr{C}(\underline{1})=\bigwedge_{A\neq\underline{0},A\wedge B=\underline{0},A\vee B=\underline{0}}\{(\tau_{(i,j)s}(A))'\vee(\tau_{(i,j)s}(B))'\}$.*

Theorem 3.9. *For any $x_\lambda\in M(L^X)$, it follows that (i,j)-$s\mathscr{C}(x_\lambda)=1$.*

Proof. Straightforward. □

Theorem 3.10. *For any $G\in L^X$, we have $\bigvee_{b\in M(L^X)}((i,j)\text{-}s\mathscr{C}(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[b]})\geq(i,j)\text{-}s\mathscr{C}(G)$.*

Proof. Let $a\leq(i,j)$-$s\mathscr{C}(G)$ where $a\in M(L)$ and suppose that $\bigvee_{b\in M(L^X)}(i,j)\text{-}s\mathscr{C}(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[b]}))\not\geq a$. Then (i,j)-$s\mathscr{C}(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\not\geq a$. By using Theorem 3.7, there exist $A,B\in L^X$ with $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\wedge A\neq 0$, $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\wedge B\neq\underline{0}$, $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\wedge A\wedge B\neq\underline{0}$, $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\leq A\wedge B$ and $(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(A'))'\not\geq a$. Since $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\wedge A\neq\underline{0}$, there exist $x_\lambda\leq A$ with (i,j)-$s\mathrm{Cl}(G)(x_\lambda)\geq a$. Since $(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]})\wedge A\wedge B=\underline{0}$, we have $x_\lambda\not\leq B$. If $G\wedge A\neq\underline{0}$, then $G\leq\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[a]}\leq A\vee B$ we have $G\leq B$, hence it follows that $a\leq(i,j)$-$s\mathrm{Cl}(G)(x_\lambda)=\bigwedge_{x_\lambda\not\leq E\geq G}(\tau_{(i,j)s}(E'))'\leq(\tau_{(i,j)s}(B'))'$, which is a contradiction. Analogously we can prove $G\wedge B\neq\underline{0}$. Thus by $G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B\neq\underline{0},G\leq A\vee B$, $(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(A'))'\not\geq a$ and Theorem 3.7, we know that (i,j)-$s\mathscr{C}(G)\not\geq a$, contradicting (i,j)-$s\mathscr{C}(G)\geq a$. It is proved that $\bigvee_{b\in M(L^X)}(i,j)$-$s\mathscr{C}(\vee((i,j)\text{-}s\mathrm{Cl}(G))_{[b]})\geq(i,j)$-$s\mathscr{C}(G)$. □

Theorem 3.11. *For any $G,H\in L^X$, we have (i,j)-$s\mathscr{C}(G\vee H)\geq(\mathscr{P}(G,H))'\wedge(i,j)$-$s\mathscr{C}(G)\wedge(i,j)$-$s\mathscr{C}(H)$.*

Proof. Let $a\leq(\mathscr{P}(G,H))'\wedge(i,j)$-$s\mathscr{C}(G)\wedge(i,j)$-$s\mathscr{C}(H)$, where $a\in M(L)$ and suppose that (i,j)-$s\mathscr{C}(G\vee H)\not\geq a$. Then by using Theorem 3.7, there exist $A,B\in L^X$ such that $(G\vee H)\wedge A\neq 0,(G\vee H)\wedge B\neq\underline{0},(G\vee H)\wedge A\wedge B=\underline{0},G\vee H\leq A\vee B$ and $(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(A'))'\not\geq a$. Since $(G\vee H)\wedge A\neq\underline{0}$, we have $G\wedge A\neq\underline{0}$ and $H\wedge A\neq\underline{0}$. Suppose that $G\wedge A\neq\underline{0}$ (The case of $H\wedge A\neq\underline{0}$ is analogous). Then we have $G\wedge B=\underline{0}$, otherwise if $G\wedge B\neq\underline{0}$, then by $G\wedge A\neq\underline{0},G\wedge B\neq\underline{0},G\wedge A\wedge B=\underline{0},G\leq A\vee B$ and $(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(E'))'\not\geq a$, we know that (i,j)-$s\mathscr{C}(G)\not\geq a$, which is a contradiction. In this case by $(G\vee H)\wedge B\neq\underline{0}$, we know that $H\wedge B\neq\underline{0}$. Analogously we can prove $H\wedge A=\underline{0}$. Thus by $G\vee H\leq A\vee B$ we can obtain that $G\leq A$ and $H\leq B$. Hence by $G\leq A$, $H\leq B$, $G\wedge B=H\wedge A=\underline{0}$, $(\tau_{(i,j)s}(B'))'\vee(\tau_{(i,j)s}(A'))'\not\geq a$ and by Lemma 3.5, we have $(\mathscr{P}(G,H))'\not\geq a$, which is a contradiction. This shows that (i,j)-$s\mathscr{C}(G\vee H)\geq a$ and this completes the proof. □

Corollary 3.12. *Let (X,τ_1,τ_2) be an L-fuzzy bitopological space and $G,H\in L^X$. If $A\wedge B\neq\underline{0}$, then (i,j)-$s\mathscr{C}(G\vee H)\geq(i,j)$-$s\mathscr{C}(G)\wedge(i,j)$-$s\mathscr{C}(H)$.*

Theorem 3.13. *Let (X,τ_1,τ_2) be an L-fuzzy bitopological space and $G\in L^X$. Then (i,j)-$s\mathscr{C}(G)=\bigwedge_{x_\lambda,y_s\leq G}\vee\{(i,j)\text{-}s\mathscr{C}(D_{x_\lambda,y_s}):x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G\}$.*

Proof. $\bigwedge_{x_\lambda,y_s\leq G}\vee\{(i,j)\text{-}s\mathscr{C}(D_{x_\lambda,y_s}):x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G\}\geq a$, where $a\in M(L)$. Take $x_\lambda\leq G$ fixed. Then for any $y_s\leq G$, there exist $D_{x_\lambda,y_s}\in L^X$ such that $x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G$ and (i,j)-$s\mathscr{C}(D_{x_\lambda,y_s})\geq a$. Let $D_{x_\lambda}=\bigvee_{y_s\leq G}D_{x_\lambda,y_s}$. Then $D_{x_\lambda}=G$ and $\bigvee_{y_s\leq G}D_{x_\lambda,y_s}\neq 0$. By using Corollary 3.12, we have (i,j)-$s\mathscr{C}(G)=(i,j)$-$s\mathscr{C}(D_{x_\lambda})\geq\bigvee_{y_s\leq G}(i,j)$-$s\mathscr{C}(D_{x_\lambda,y_s})\geq a$. This shows that (i,j)-$s\mathscr{C}(G)\geq\bigwedge_{x_\lambda,y_s\leq G}\vee\{(i,j)\text{-}s\mathscr{C}(D_{x_\lambda,y_s}):x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G\}$. Since (i,j)-$s\mathscr{C}(G)\leq\bigwedge_{x_\lambda,y_s\leq G}\vee\{(i,j)\text{-}s\mathscr{C}(D_{x_\lambda,y_s}):x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G\}$ is clear. Then we have (i,j)-$s\mathscr{C}(G)=\bigwedge_{x_\lambda,y_s\leq G}\vee\{(i,j)\text{-}s\mathscr{C}(D_{x_\lambda,y_s}):x_\lambda,y_s\leq D_{x_\lambda,y_s}\leq G\}$. □

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