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Numerical solution of time fractional Kuramoto-Sivashinsky equation by Adomian decomposition method and applications

Sharvari Kulkarni[1]*, Kalyanrao Takale[2], Shrikisan Gaikwad[3]

Abstract

In the paper, we develop the Adomian Decomposition Method for fractional order nonlinear Kuramoto-Sivashinsky (KS) equation. Caputo fractional derivatives are used to define fractional derivatives. We know that KS equation has many applications in physical phenomenon such as reaction diffusion system, long waves on the boundary of two viscous fluids and hydrodynamics. In this paper, we will solve time fractional KS equation which may help to researchers for their work. We solve some examples numerically, which will show the efficiency and convenience of Adomian Decomposition Method.

Keywords

Kurmoto-Sivashinsky equation, Fractional derivative, Adomian Decomposition Method, Convergence, Mathematica.

AMS Subject Classification

26A33, 30E25, 34A12, 34A34, 34A37, 37C25, 45J05.

[1] *Department of Mathematics, Model College , Dombivali, Thane-421201, Maharashtra, India.*
[2] *Department of Mathematics, RNC Arts, JDB Commerce and NSC Science College, Nashik-422101, Maharashtra, India.*
[3] *Department of Mathematics, New Atrs, Commerce and Science College, Ahmadnagar-414001, Maharashtra, India.*
***Corresponding author**: [1] sharvari12july@gmail.com; [2]kalyanraotakale@rediffmail.com;[3]sbgmathsnagar@gmail.com
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Contents

1. Introduction

In the present scenario fractional calculus is useful in the various fields of science. In past few years, the increase of interest in the subject is witnessed by series of conferences, research papers and several monographs. The dynamic models of a large number of phenomena can be modeled by fractional order partial differential equations which are characterized by fractional space and/or time derivatives [2]. Fractional calculus is pragmatic to archetypal occurrences dependent damping behavior of many viscoelastic materials, continuum mechanics, statistical mechanics, economics etc [1]. But, many times it is difficult to obtain exact solutions, hence numerical methods must be used. Now a days, Adomian Decomposition Method (ADM) is used to obtain the solution of fractional differential equations . This method gives rapidly convergent series solutions by using a few iterations for both linear and nonlinear equations. This method is very useful to avoid linearization, perturbation, massive computation and transformations [3, 4]. Various instabilities and spatio - temporal chaotic behavior are exhibited in many thermodynamical systems. Pattern formation, travelling wave problems, reaction-diffusion systems, long waves on thin films, unstable drift waves in plasmas etc. are some of the physical phenomenon which arise from chaotic instabilities. In this context Kuramoto-Sivashinsky (KS) equation has a wide range of applicability in science. It is used to model fluctuations of the position of a flame front, the motion of a fluid going down a vertical wall, spatially uniform oscillating chemical reaction in a homogeneous medium, solitary pulses in a falling thin film [5] etc. It is also useful to physical problems such as viscous flow problems, hydrodynamics in thin films, Belousov-Zhabotinsky reactions and instabilities of solidification fronts of dilute binary alloys [6]. Kuramoto Sivashinsky equation was developed by Kuramoto and Sivashinsky is written as follow

$$w_t + \lambda_1 w w_x + \lambda_2 w_{xx} + \lambda_3 w_{xxxx} = 0 \tag{1.1}$$

where λ_1, λ_2, λ_3 are unknown parameters. The second order and fourth order spatial derivatives are making this equation's behaviour complicated and interesting. The nonlinear term transforms energy from low to high wave numbers. Also, Maziar Raissi and George have developed methodology applied to the problem of learning, system identification or data-driven discovery of partial differential equation and provides new direction to design learning machines without requiring large quantities of data. They gave following observations for values of parameters λ_1, λ_2, λ_3 with clean and noisy data [7]:
(i) The correct KS partial differential equation is

$$w_t + w w_x + w_{xx} + w_{xxxx} = 0 \tag{1.2}$$

(ii) The identified KS partial differential equation **(clean data)** is

$$w_t + 0.952 w w_x + 1.005 w_{xx} + 0.980 w_{xxxx} = 0 \tag{1.3}$$

(iii) The identified KS partial differential equation
One percent noisy data is

$$w_t + 0.908 w w_x + 0.951 w_{xx} + 0.927 w_{xxxx} = 0 \tag{1.4}$$

Recently, some researchers have used Homotopy Perturbation Method [8], He's Variational Iteration Method [9] and Lattice Boltzmann mehod [10] to solve KS equation. Saad A. Manna, Fadhil H. Easif [11] have used ADM to solve KS equation. Weishi Yin, Fei Xu et.al. found the asymptotic expansion of solutions to time-space fractional KS equation by residual power series method [12].
Therefore, models which represent wave phenomenon needs to study travelling wave solutions. As per Abdul Wazwaz, in the study of solitary wave theory, we can obtain travelling wave solutions. These solutions are used by scientists to study various physical applications in plasma physics. In [13], the researchers used Bogning-Dijeumen Tchalo-Kofane method (BDK Method) to solve very strong nonlinear KS equation. By applying BDK method they make up modulated soliton solution of KS equation. In paper [14] KS equation was solved by truncated expansion method and compared with Ansatz method. Also researchers analyzed new solitary wave solutions of KS equation with comparison of solutions given by Chen and Zhand, Wazwaz and Wazaan. In this connection in our paper, we used ADM to solve time fractional KS equation because ADM is a powerful method to obtain the solution of linear and nonlinear fractional partial differential equations of higher order.
We organize the paper as follows: We have given some formulae and theorem in Section 2, which are useful for further developments. Section 3, is devoted for ADM to solve time fractional KS equation and prove convergence. In section 4, numerical problems are solved and presented their solutions graphically by using mathematica software.

2. Preliminaries

Some basic concepts, which we will be using are as follows:-

Definition 2.1. *The Caputo fractional derivative [?] of the function $f(x)$ is defined as*

$$\begin{aligned} D_*^{\beta} f(x) &= J^{(m-\beta)} D^m f(x) \\ &= \frac{1}{\Gamma(m-\beta)} \int_0^x \frac{1}{(x-t)^{(1-m+\beta)}} f^{(m)}(t)dt, \\ & \text{for } m-1 < \beta \le m,\ m \in N, x > 0,\ f \in C_{-1}^m. \end{aligned}$$

Properties:
For $f(x) \in C_\mu$, $\mu \ge -1$, $\alpha, \beta \ge 0$ and $\gamma > -1$ [15], we have

(i) $J^{\alpha} J^{\beta} f(x) = J^{\alpha+\beta} f(x)$,

(ii) $J^{\alpha} J^{\beta} f(x) = J^{\beta} J^{\alpha} f(x)$,

(iii) $J^{\alpha} x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{(\alpha+\gamma)}$.

Lemma 2.2. *If $m-1 < \alpha \le m$, $m \in N$ and $f \in C_\mu^m$, $\mu \ge -1$, then*

$$D_*^{\alpha} J^{\alpha} f(x) = f(x)$$

$$J^{\alpha} D_*^{\alpha} f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!}, x > 0.$$

3. Fractional Adomian Decomposition Method

We consider following time fractional KS equation to develop the time Fractional ADM [15] for solving KS equation,

$$w_t^{\alpha} + \lambda_1 w w_x + \lambda_2 w_{xx} + \lambda_3 w_{xxxx} = 0,\ \ 0 < \alpha \le 1, t > 0 \tag{3.1}$$

$$\text{initial condition} : w(x,0) = f(x) \tag{3.2}$$

We will operate J^{α} on R.H.S. and L.H.S. of equation,

$$J^{\alpha}\left[w_t^{\alpha} + \lambda_1 w w_x + \lambda_2 w_{xx} + \lambda_3 w_{xxxx} = 0\right] = 0,\ 0 < \alpha \le 1, t > 0$$

Now, consider following decomposition series:-

$$w(x,t) = \sum_{n=0}^{\infty} w_n(x,t) \tag{3.3}$$

The decomposed series of nonlinear terms $Nw(x,t)$ are:

$$Nw(x,t) = \sum_{n=0}^{\infty} A_n \tag{3.4}$$

where the formula for Adomian polynomial is as follows:

$$A_n = \frac{1}{n!}\left[\frac{d^nN}{d\lambda^n}\left(\sum_{k=0}^{n}\lambda^k u_k\right)\right]_{\lambda=0} \tag{3.5}$$

From (3.3) and using lemma (2.1), we get

$$\sum_{n=0}^{\infty} w_n(x,t) = \sum_{k=0}^{m-1}\frac{\partial^k w(x,0)}{\partial t^k}\frac{t^k}{k!} - J^\alpha\left[\lambda_2\sum_{n=0}^{\infty}D_x^2 w_n(x,t)\right.$$

$$\left.+\lambda_3\sum_{n=0}^{\infty}D_x^4 w_n(x,t) + \lambda_1\sum_{n=0}^{\infty}A_n\right],\ x>0$$

The value of $w_n(x,t)$, $n \geq 0$ can be obtained as follows:

$$w_0(x,t) = w(x,0) = f(x) \tag{3.6}$$

$$w_{n+1}(x,t) = -J^\alpha\left[\lambda_2 D_x^2 w_n(x,t) + \lambda_3 D_x^4 w_n(x,t) + \lambda_1 A_n\right] \tag{3.7}$$

For , $x > 0$, now we can obtain solution by calculating value of each component.

$$\phi_N(x,t) = \sum_{n=0}^{N-1} w_n(x,t) \tag{3.8}$$

$$\lim_{N\to\infty}\phi_N = w(x,t) \tag{3.9}$$

Theorem 3.1. ***Uniqueness Theorem*** *[16]*
Consider time fractional KS equation for $\lambda_1 = 1$, $\lambda_2 = 1$, and $\lambda_3 = 1$, as follows

$$w_t^\alpha + ww_x + w_{xx} + w_{xxxx} = 0,\ \ 0 < \alpha \leq 1, t > 0 \tag{3.10}$$

$$\textit{initial condition}: w(x,0) = f(x) \tag{3.11}$$

The equation has a unique solution whenever $0 < \gamma < 1$ where $\gamma = \frac{(C_1+C_2+C_3)t^\alpha}{\Gamma\alpha+1}$.

Proof:- Let $X = (C(I), \|.\|)$ be the Banach space of all continuous functions on $I = [0,T]$ with norm

$$\|w(t)\| = \max_{t\in I} \mid w(t) \mid .$$

We define a mapping $M : X \to X$, such that

$$M(w(t)) = f(x) - J^\alpha N(w(t)) - J^\alpha S(w(t)) - J^\alpha F(w(t)).$$

Now, $N(w(t))$ denotes nonlinear term and $S(w(t))$ denotes second order spatial term and $F(w(t))$ denotes fourth order spatial term. Also nonlinear term $N(w(t))$ is Lipschitzian that is

$$\mid N(w) - N(p) \mid \leq C_1 \mid w - p \mid$$

where C_1 is Lipschitz constant. Let $w, w' \in X$, we have

$$||M(w) - M(w')|| = \max_{t\in I} | -J^\alpha N(w(t)) - J^\alpha S(w(t) - J^\alpha F(w(t)$$

$$+J^\alpha N(w'(t)) + J^\alpha S(w'(t) + J^\alpha F(w'(t)|$$

$$= \max_{t\in I} \mid -J^\alpha(Nw - Nw') - J^\alpha(Sw - Sw') - J^\alpha(Fw - Fw') \mid$$

$$= \max_{t\in I} \mid J^\alpha(Nw - Nw') + J^\alpha(Sw - Sw') + J^\alpha(Fw - Fw') \mid$$

$$\leq \max_{t\in I} \mid J^\alpha(Nw - Nw') \mid + \mid J^\alpha(Sw - Sw') \mid + \mid J^\alpha(Fw - Fw') \mid$$

Now suppose $S(w(t))$ and $F(w(t))$ are also Lipschitzian that is

$$\mid S(w) - S(p) \mid \leq C_2 \mid w - p \mid$$

and

$$\mid F(w) - F(p) \mid \leq C_3 \mid w - p \mid,$$

where C_2 and $C_3$are Lipschitz constants.
Therefore

$$\| M(w) - M(w') \| \leq \max_{t\in I}(C_1 J^\alpha \mid w - w' \mid + C_2 J^\alpha \mid w - w' \mid$$

$$+C_3 J^\alpha \mid w - w' \mid) \leq (C_1 + C_2 + C_3) \| w - w' \| \frac{t^\alpha}{\Gamma\alpha + 1}$$

$$\| M(w) - M(w') \| \leq \gamma \| w - w' \|, where\ \gamma = \frac{(C_1 + C_2 + C_3)t^\alpha}{\Gamma\alpha + 1}$$

Therefore, whenever $0 < \gamma < 1$, the mapping is contraction. Hence with the reference of Banach fixed point theorem for contraction, we proved that equation has unique solution.

Theorem 3.2. ***Convergence Theorem****[16]*
Let Q_n be the n^{th} partial sum, that is

$$Q_n = \sum_{i=0}^{n} w_i(x,t) \tag{3.12}$$

Then we shall prove that $\{Q_n\}$ is a Cauchy sequence in Banach space X.

Proof: For proving this theorem, we consider

$$\| Q_{n+p} - Q_n \| = \max_{t\in I} \mid Q_{n+p} - Q_n \mid$$

$$= \max_{t\in I} \left| \sum_{i=n+1}^{n+p} w_i(x,t) \right|$$

$$= \max_{t\in I} \left| -J^\alpha \sum_{i=n+1}^{n+p} Sw_{i-1}(x,t) - J^\alpha \sum_{i=n+1}^{n+p} Fw_{i-1}(x,t)\right.$$

$$\left. -J^\alpha \sum_{i=n+1}^{n+p} Nw_{i-1}(x,t) \right|$$

$$= \max_{t\in I} |J^\alpha SQ_{n+p-1} - SQ_{n-1} + J^\alpha FQ_{n+p-1} - FQ_{n-1}$$

$$+J^\alpha NQ_{n+p-1} - NQ_{n-1}|$$

$$\leq \max_{t\in I} J^\alpha(|(SQ_{n+p-1} - SQ_{n-1}|) + \max_{t\in I} J^\alpha(|(FQ_{n+p-1} - FQ_{n-1}|)$$

$$+ \max_{t\in I} J^\alpha |NQ_{n+p-1} - NQ_{n-1}|$$

$$\leq C_2 \max_{t\in I} J^\alpha(|(Q_{n+p-1} - Q_{n-1}|) + C_3 \max_{t\in I} J^\alpha(|(Q_{n+p-1} - Q_{n-1}|)$$

$$+C_1 \max_{t\in I} J^{\alpha}|Q_{n+p-1}-Q_{n-1}|$$
$$\leq (C_1+C_2+C_3)\frac{t^{\alpha}}{\Gamma\alpha+1} \parallel Q_{n+p-1}-Q_{n-1} \parallel$$
$$\parallel Q_{n+p}-Q_n \parallel \leq \gamma \parallel Q_{n+p-1}-Q_{n-1} \parallel,$$

where $\gamma=(C_1+C_2+C_3)\frac{t^{\alpha}}{\Gamma\alpha+1}$

$$\parallel Q_{n+p}-Q_n \parallel \leq \gamma \parallel Q_{n+p-1}-Q_{n-1} \parallel$$

Similarly, we have

$$\begin{aligned}\parallel Q_{n+p}-Q_n \parallel &\leq \gamma^2 \parallel Q_{n+p-2}-Q_{n-2} \parallel\\ &\vdots\\ &\leq \gamma^n \parallel Q_p - Q_0 \parallel\\ &\leq \gamma^n \parallel Q_1 - Q_0 \parallel, \ forp=1\\ &\leq \gamma^n \parallel w_1 \parallel\end{aligned}$$

Now, for $n>m$, where $n,m\in N$,

$$\begin{aligned}\parallel Q_n-Q_m \parallel &\leq \parallel Q_{m+1}-Q_m \parallel + \parallel Q_{m+2}-Q_{m+1} \parallel\\ &+\cdots+\parallel Q_n-Q_{n-1} \parallel\\ &\leq (\gamma^m+\gamma^{m+1}+\cdots+\gamma^{n-1}) \parallel w_1 \parallel\\ &\leq \gamma^m\left[\frac{1-\gamma^{n-m}}{1-\gamma}\right] \parallel w_1 \parallel\end{aligned}$$

Since, $0<\gamma<1$, then $1-\gamma^{n-m}<1$, so we have,

$$\parallel Q_n-Q_m \parallel \leq \frac{\gamma^m}{1-\gamma} \parallel w_1 \parallel$$

Since, $w(t)$ is bounded, therefore $\parallel w_1 \parallel < \infty$

$$\lim_{n\to\infty} \parallel Q_n-Q_m \parallel \to 0$$

Hence, we proved that solution is convergent because $\{Q_n\}$ is a Cauchy sequence in X.

4. Numerical Examples

Example 4.1: We will consider the following time fractional KS equation

$$w_t^{\alpha}+ww_x+w_{xx}+w_{xxxx}=0, \ \ 0<\alpha\leq 1, t>0 \quad (4.1)$$

$$\text{initial condition}: w(x,0)=\sec h^2\left(\frac{x}{2}\right) \quad (4.2)$$

Now, using equations (3.6) and (3.7), we have

$$w_0(x,t)=w(x,0)=f(x)$$
$$w_{k+1}(x,t)=-J^{\alpha}\left[A_k+D_x^2w_k(x,t)+D_x^4w_k(x,t)\right], \ x>0$$
$$w_0(x,t)=w(x,0)=\sec h^2\left(\frac{x}{2}\right)$$
$$w_1(x,t)=-J^{\alpha}\left[A_0+D_x^2w_0(x,t)+D_x^4w_0(x,t)\right]$$

$$A_0=w_0(w_0)_x=\left[-\frac{1}{2}\tan h\left(\frac{x}{4}\right)+\tan h^3\left(\frac{x}{4}\right)-\frac{1}{2}\tan h^5\left(\frac{x}{4}\right)\right]$$

$$D_x^2w_0(x,t)=\left[-\frac{1}{8}\sec h^2\left(\frac{x}{4}\right)+\frac{3}{8}\tan h^2\left(\frac{x}{4}\right)-\frac{3}{8}\tan h^4\left(\frac{x}{4}\right)\right]$$

$$D_x^2w_0(x,t)=\left[-\frac{1}{8}\left(1-\tan h^2\left(\frac{x}{4}\right)\right)+\frac{3}{8}\tan h^2\left(\frac{x}{4}\right)-\frac{3}{8}\tan h^4\left(\frac{x}{4}\right)\right]$$

$$D_x^2w_0(x,t)=\left[-\frac{1}{8}+\frac{1}{2}\tan h^2\left(\frac{x}{4}\right)-\frac{3}{8}\tan h^4\left(\frac{x}{4}\right)\right]$$

$$D_x^4w_0(x,t)=\left[\frac{1}{16}\sec h^2\left(\frac{x}{4}\right)-\frac{15}{32}\tan h^2\left(\frac{x}{4}\right)\sec h^2\left(\frac{x}{4}\right)+\frac{15}{32}\tan h^4\left(\frac{x}{4}\right)\sec h^2\left(\frac{x}{4}\right)\right]$$

$$D_x^4w_0(x,t)=\left[\frac{1}{16}\left(1-\tan h^2\left(\frac{x}{4}\right)\right)-\frac{15}{32}\tan h^2\left(\frac{x}{4}\right)+\frac{15}{32}\tan h^4\left(\frac{x}{4}\right)\left(1-\tan h^2\left(\frac{x}{4}\right)\right)\right]$$

$$D_x^4w_0(x,t)=\left[\frac{1}{16}-\frac{17}{32}\tan h^2\left(\frac{x}{4}\right)+\frac{15}{16}\tan h^4\left(\frac{x}{4}\right)\right]$$

$$w_1(x,t)=-J^{\alpha}\left[A_0+D_x^2w_0(x,t)+D_x^4w_0(x,t)\right]$$

$$w_1(x,t)=-J^{\alpha}\left[-\frac{1}{2}\tan h\left(\frac{x}{4}\right)+\tan h^3\left(\frac{x}{4}\right)-\frac{1}{2}\tan h^5\left(\frac{x}{4}\right)-\frac{3}{8}\tan h^4\left(\frac{x}{4}\right)+\frac{1}{16}-\frac{17}{32}\tan h^2\left(\frac{x}{4}\right)\right]$$

$$w_1(x,t)=-J^{\alpha}\left[-\frac{1}{16}-\frac{1}{2}\tan h\left(\frac{x}{4}\right)-\frac{1}{32}tanh^2\left(\frac{x}{4}\right)+\frac{9}{16}\tan h^4\left(\frac{x}{4}\right)-\frac{1}{2}\tan h^5\left(\frac{x}{4}\right)-\frac{15}{32}\tan h^6\left(\frac{x}{4}\right)\right]$$

$$w_1(x,t)=\left[\frac{1}{16}+\frac{1}{2}\tan h\left(\frac{x}{4}\right)+\frac{1}{32}\tan h^2\left(\frac{x}{4}\right)-\tan h^3\left(\frac{x}{4}\right)-\frac{9}{16}\tan h^4\left(\frac{x}{4}\right)+\frac{1}{2}\tan h^5\left(\frac{x}{4}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$\vdots$$

After calculating and substituting values of various components, we have

$$w(x,t)=w_0(x,t)+w_1(x,t)+\cdots$$

$$w(x,t)=\sec h^2\left(\frac{x}{2}\right)+\left[\frac{1}{16}+\frac{1}{2}\tan h\left(\frac{x}{4}\right)+\frac{1}{32}\tan h^2\left(\frac{x}{4}\right)-\tan h^3\left(\frac{x}{4}\right)-\frac{9}{16}\tan h^4\left(\frac{x}{4}\right)+\frac{1}{2}\tan h^5\left(\frac{x}{4}\right)+\frac{15}{32}\tan h^6\left(\frac{x}{4}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}\cdots$$

Example 4.2: We will solve the following time fractional KS equation

$$w_t^{\alpha}+ww_x+w_{xx}+w_{xxxx}=0,\ \ 0<\alpha\leq 1, t>0 \quad (4.3)$$

$$\text{initial condition}: w(x,0)=cos\left(\frac{x}{2}\right) \quad (4.4)$$

Now, using equation (3.3) and (3.6), we have

$$w_0(x,t)=w(x,0)=f(x)$$

$$w_{k+1}(x,t)=-J^{\alpha}\left[A_k+D_x^2w_k(x,t)+D_x^4w_k(x,t)\right],\ x>0$$

$$w_0(x,t)=w(x,0)=cos\left(\frac{x}{2}\right)$$

$$w_1(x,t)=-J^{\alpha}\left[D_x^2w_0(x,t)+D_x^4w_0(x,t)+A_0\right]$$

$$A_0=w_0(w_0)_x=-\frac{1}{4}\sin(x)$$

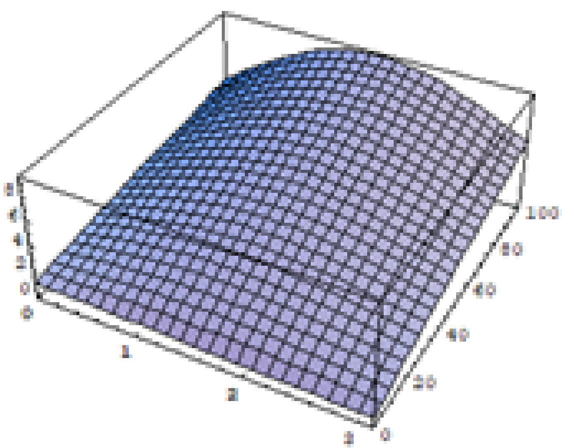

Figure 1. 3DPlot of time fractional KS eqn for $\alpha=1$

$$D_x^2w_0(x,t)=-\frac{1}{4}cos\left(\frac{x}{2}\right),\ \ D_x^4w_0(x,t)=\frac{1}{16}\cos\left(\frac{x}{2}\right)$$

$$w_1(x,t)=\left[\frac{1}{4}\sin x+\frac{1}{4}\cos\left(\frac{x}{2}\right)-\frac{1}{16}\cos\left(\frac{x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$w_1(x,t)=\left[\frac{1}{4}\sin x+\frac{3}{16}\cos\left(\frac{x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$w_2(x,t)=-J^{\alpha}\left[D_x^2w_1(x,t)+D_x^4w_1(x,t)+A_1\right]$$

$$A_1=w_1(w_0)_x+w_0(w_1)_x$$

$$A_1=\left[-\frac{9}{64}\sin(x)+\frac{1}{4}\cos\left(\frac{3x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$D_x^2w_1(x,t)=\left[-\frac{1}{4}\sin(x)-\frac{3}{64}\cos\left(\frac{x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$D_x^4w_1(x,t)=\left[\frac{1}{4}\sin(x)+\frac{3}{256}\cos\left(\frac{x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$w_2(x,t)=\left[\frac{9}{64}\sin(x)+\frac{9}{256}\cos\left(\frac{x}{2}\right)\right]\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$\vdots$$

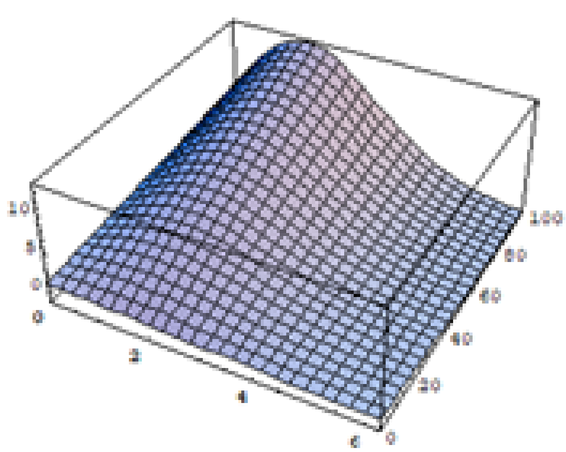

Figure 2. 3DPlot of time fractional KS eqn with $\alpha=0.9$

After calculating and substituting values of various components, we have

$$w(x,t)=w_0(x,t)+w_1(x,t)++\cdots \quad (4.5)$$

$$\begin{aligned}&w(x,t)\\&=\cos\left(\frac{x}{2}\right)+\left[\frac{1}{4}\sin x+\frac{3}{16}\cos\left(\frac{x}{2}\right)\right]\frac{t^{\alpha}}{\Gamma(\alpha+1)}\\&\quad+\left[\frac{9}{64}\sin(x)+\frac{9}{256}\cos\left(\frac{x}{2}\right)\right]\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}+\cdots\end{aligned}$$

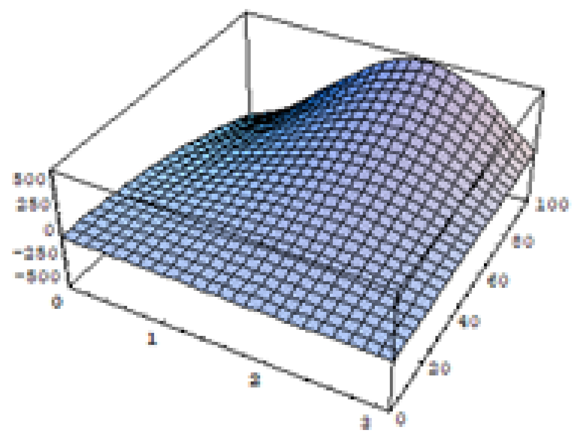

Figure 3. Plots of time fractional KS eqn $\alpha = 1$

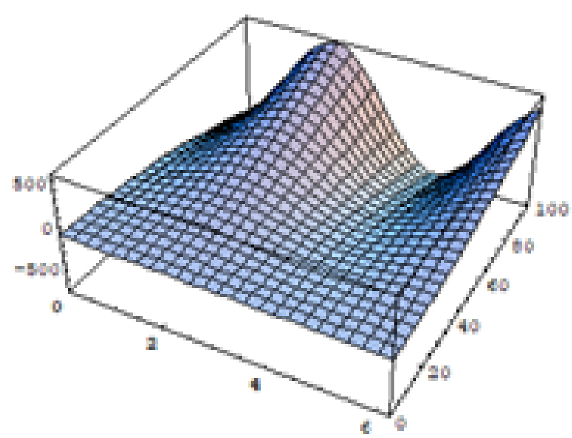

Figure 4. Plots of time fractional KS eqn with $\alpha = 0.9$

5. Conclusion

The time fractional KS equation is solved by using ADM and we can say that the formula of ADM polynomials is powerful to obtain the solution of nonlinear fractional partial differential equation. The graphical presentation of solutions of time fractional KS equation reveals the reliability of the mathematical procedure. We also prove the uniqueness and convergence theorem for time fractional KS equation.

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