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$\mathscr{A}\mathscr{L}$ and $\mathscr{A}\mathscr{L}_2$-Paracompact spaces

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Abstract

This paper deals with the new concepts of $\mathscr{A}\mathscr{L}$-Paracompact spaces and $\mathscr{A}\mathscr{L}_2$-Paracompact spaces. Also we have proved that every $\mathscr{A}\mathscr{L}$-Paracompactness and $\mathscr{A}\mathscr{L}_2$-Paracompactness has a topological property.

Keywords

Angelic spaces, $\mathscr{L}$-Paracompact, $\mathscr{L}_2$–Paracompact, $\mathscr{L}$-Normal, $\mathscr{A}\mathscr{L}$-Paracompact, $\mathscr{A}\mathscr{L}_2$–Paracompact, $\mathscr{A}\mathscr{L}$-normal.

AMS Subject Classification

46A50, 54D10, 54D20.

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Contents

1. Introduction

Paracompact space was introduced by Dieudonne.J [7]. Paracompactness [7] is the most useful generalization of compactness. Many researchers have studied certain stronger as well as weaker forms of paracompactness. $\mathscr{C}$-Paracompact and $\mathscr{C}_2$ –Paracompact were defined by Arhangel'skii. $\mathscr{C}$-Paracompact and $\mathscr{L}_2$-Paracompact were studied in [17]. L.Kalantan defined $\mathscr{L}$-Paracompact and $\mathscr{L}_2$-Paracompact.L. Kalantan and M. Saeed defined the $\mathscr{L}$-normality. Fermlin's concept of angelic space [6] and some of its emanation bring us with he mandatory tools for presenting those results in a mordant idea.

2. Preliminaries

Definition 2.1. *[2] A topological space (shortly, TS) T is termed as angelic, if for each relatively countably compact subset $\mathscr{S}$ of T the ensuing hold: (a) $\mathscr{S}$ is relatively compact (b) If $s \in \overline{\mathscr{S}}$, then $\exists$ a sequence in $\mathscr{S}$ that converges to s.*

Definition 2.2. *[12] A TS X is paracompact, if each open cover has a locally finite open refinement.*

Definition 2.3. *[15] A TS M termed as $\mathscr{C}$-paracompact if $\exists$ a paracompact space N and a bijective function $p : M \to N$ $\ni$ the restriction $p|_S : S \to p(S)$ is homeomorphism for every compact subspace $S \subseteq M$.*

Definition 2.4. *[15] A TS M is termed as $\mathscr{C}_2$-Paracompact if $\exists$ a Hausdorff paracompact space N and a bijective mapping $p : M \to N \ni$ the restriction $p|_S : S \to p(S)$ is homeomorphism for every compact subspace $S \subseteq M$.*

Definition 2.5. *[12] A space for which open covering includes a countable subcovering is called a Lindelöf.*

Definition 2.6. *[11] A TS X is temed as $\mathscr{L}$-Paracompact if $\exists$ a paracompact space Y and a bijective mapping $f : X \to Y$ $\ni$ the restriction $f|_A : A \to f(A)$ is homeomorphism for every Lindelof subspace $A \subseteq X$.*

Definition 2.7. *[11] A TS X is termed as $\mathscr{L}_2$-paracompact if $\exists$ a Hausdorff paracompact space Y and a bijective mapping $f : X \to Y \ni$ the restriction $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$.*

Definition 2.8. *[8] A space M is termed as $\mathscr{L}$-Normal if $\exists$ a normal space N and a function $p : M \to N \ni$ the restriction $p|S : S \to p(S)$ is homeomorphism for each Lindelöf subspace $S \subseteq M$.*

Definition 2.9. *[17] A space M termed as mildly normal, if for any two disjoint closed domains U and V of M $\exists$ two disjoint open sets U and V of $M \ni U \subseteq U$ and $V \subseteq V$.*

Definition 2.10. *[12] A TS X is normal if for each A, B of disjoint closed sets of X, $\exists$ disjoint open sets including A and B, respectively.*

Definition 2.11. *[12] A TS X termed as regular if for each pair consisting of a point x and a closed set B disjoint from x, $\exists$ disjoint open sets including x and B, respectively.*

Definition 2.12. *[20] A TS M termed as completely Hausdorff, if for each distinct element $a, b \in M$ $\exists$ two open sets U and $V \ni a \in U$, $b \in V$ and $\overline{U} \cup \overline{V} = \emptyset$.*

Definition 2.13. *[4] A space X termed as countable tightness if for every subset A and each $x \in X$ with $x \in \overline{A}$ $\exists$ a countable subset $B \subseteq A \ni x \in \overline{B}$.*

Definition 2.14. *[3] Let X be TS. If $\exists X' = X \times \{1\}$ and $X \cap X' = \emptyset \ni A(X) = X \cup X'$, then $A(X)$ with the unique topology τ is termed as Alexandroff duplicate of X.*

3. $\mathscr{AL}$ and $\mathscr{AL}_2$-Paracompact spaces

Definition 3.1. *Let $\mathscr{P}$ be an angelic space and $\mathscr{A}$ be an angelic Lindelöf subspace of $\mathscr{P}$. If there is a bijection mapping $g : \mathscr{P} \to \mathscr{Q}$, $\mathscr{Q}$ is an angelic paracompact space and the restriction $g|_{\mathscr{A}} : \mathscr{A} \to g(\mathscr{A})$ is a homeomorphism, then $\mathscr{P}$ is said to be an $\mathscr{AL}$-paracompact space.*

Definition 3.2. *Let $\mathscr{P}$ be an angelic space and $\mathscr{A}$ be an angelic Lindelöf subspace of $\mathscr{P}$. If there is a bijection mapping $g : \mathscr{P} \to \mathscr{Q}$, $\mathscr{Q}$ is a Hausdorff paracompact space and the restriction $g|_{\mathscr{A}} : \mathscr{A} \to g(\mathscr{A})$ is a homeomorphism, then $\mathscr{P}$ is said to be an $\mathscr{AL}$-paracompact space.*

Theorem 3.3. *If $\mathscr{P}$ is an $\mathscr{AL}$-paracompact ($\mathscr{AL}_2$-paracompact) and of countable tightness and $g : \mathscr{P} \to \mathscr{Q}$ is a witness function of the $\mathscr{AL}$-paracompact ($\mathscr{AL}_2$-paracompact) space of $\mathscr{P}$, then g is continuous.*

Proof. Since $\mathscr{A}$ is a non-empty subset of $\mathscr{P}$. Consider $q \in g(\overline{\mathscr{A}})$ is arbitrary and $p \in \mathscr{P}$ is a unique element $\ni g(P) = q$. Then $p \in \mathscr{A}$. choose a countable subset $\mathscr{A}_l \subseteq \mathscr{A} \ni p \in \mathscr{A}_l$. Take $B = \{p\} \cup \mathscr{A}_0$; then $\mathscr{B}$ is a Lindelöf subspace of $\mathscr{P}$, and $g|_{\mathscr{B}} : \mathscr{B} \to g(\mathscr{B})$ is homeomorphism. Suppose $V \subseteq \mathscr{Q}$ is a open neighborhood of q; next $V \cap g(\mathscr{B})$ is open in subspace $g(\mathscr{B})$ including q. hence, $g^{-1}(V) \cap \mathscr{B}$ is open in $\mathscr{B}$ including q. Hence, $g^{-1}(V) \cap \mathscr{B} \cap \mathscr{A}_0 \neq \emptyset$. So $g^{-1}(V) \cap \mathscr{B} \cap \mathscr{A}_0 \neq \emptyset$. Thus, $\emptyset \neq g(g^{-1}(V) \cap \mathscr{B}) \cap \mathscr{B} \subseteq g(g^{-1}(V) \cap \mathscr{A}) = V \cap \mathscr{A})$. Hence $q \in g(\mathscr{A})$ and g is continuous. □

Corollary 3.4. *Any $\mathscr{AL}_2$-paracompact space which is of countable tightness must be at least Hausdorff.*

Theorem 3.5. *Let $\mathscr{P}$ be the T_3 separable $\mathscr{AL}$-paracompact and of countable tightness. Then $\mathscr{P}$ is an angelic paracompact T_4.*

Proof. Suppose $\mathscr{Q}$ be an angelic paracompact space and $g : \mathscr{P} \to \mathscr{Q}$ be a bijective witness to an $\mathscr{AL}$-paracompactness of $\mathscr{P}$. Then g is continuous because $\mathscr{P}$ is of countable tightness, by Theorem 3.1. Consider D is a countable dense subset of $\mathscr{P}$. We produce g is closed. Assume that H is a non-empty closed proper subset of $\mathscr{P}$. Suppose $g(p)\inf(H)$; next $p \notin H$. Using regularity, let U and V be disjoint open subsets of P including p and H, respectively. Then $U \cap (D \cup \{p\})$ is open in the angelic Lindelöf subspace $D \cup \{p\}$ including p, so $g(U \cap (D \cup \{p\}))$ is open in the subspace $g(D \cup \{p\})$ of $\mathscr{Q}$ including q. Thus $g(U \cap (D \cup \{p\})) = g(U) \cap g(D \cup p) = W \cap g(D \cap \{p\})$ for some open subset W in $\mathscr{Q}$ with $q \in W$. We claim that $W \cap g(H) = \emptyset$. Suppose otherwise, and take $q \in W \cap g(H)$. Let $p \in H \ni g(p) = q$. Note that $p \in V$. By reason of D is dense in $\mathscr{P}$, D is also dense in the open set V. Thus $p \in V \cap D$. Presently since W is open in $\mathscr{Q}$ and g is continuous, $g^{-1}(W)$ is an open set in $\mathscr{P}$; it also includes p. Thus we can choose $d \in g^{-1}(W) \cap V \cap D$. Next $p(d) \in W \cap g(V \cap D) \subseteq W \cap g(D \cup \{p\}) = g(U \cap (D \cup \{p\}))$. So $g(d) \in g(u) \cap g(V)$. Hence $W \cap g(H) = \emptyset$. Note that $q \in W$. As $q \in \mathscr{Q}/g(H)$ was arbitrary, $g(H)$ is closed. So g is a homeomorphism and $\mathscr{P}$ is an angelic paracompact. Since $\mathscr{P}$ is also T_2, $\mathscr{P}$ is normal. Note that $\mathscr{P}$ is also an angelic Lindelöf being separable and an angelic paracompact. □

Theorem 3.6. *Every $\mathscr{AL}$-paracompactness ($\mathscr{AL}_2$-paracompactness) has a topological property.*

Proof. Suppose $\mathscr{P}$ is an $\mathscr{AL}$-paracompact space and $\mathscr{P} \cong Z$. Since $\mathscr{Q}$ be an angelic paracompact space and $g : \mathscr{P} \to \mathscr{Q}$ be a bijection $\ni g|_{\mathscr{A}} : \mathscr{A} \to g(\mathscr{A})$ is homeomorphism for every angelic Lindelöf subspace $\mathscr{A}$ of $\mathscr{P}$. Take $h : \mathscr{P} \to Z$ is homeomorphism. Then $g \circ h : Z \to \mathscr{P}$ fulfills topological property. □

Theorem 3.7. *Every $\mathscr{AL}$-paracompactness ($\mathscr{AL}_2$-paracompactness) has an additive property.*

Proof. Since $\mathscr{P}_\alpha$ be an $\mathscr{AL}$-paracompact space for every $\alpha \in \Lambda$. Consider their sum $\oplus_{alpha \in \Lambda} \mathscr{P}_\alpha$ is an $\mathscr{AL}$-paracompact. For every $\alpha \in \Lambda$, take an angelic paracompact space $\mathscr{P}_\alpha$ and a bijective function $p_\alpha : \mathscr{P}_\alpha \to \mathscr{Q}_\alpha \ni g_{\alpha/C_\alpha} : C_\alpha \to g_\alpha(C_\alpha)$ is homeomorphism for every angelic Lindelöf subspace $C_(\alpha)$ of $\mathscr{P}_\alpha$. Suppose $\mathscr{Q}_\alpha$ is an angelic paracompact for every $\alpha \in \Lambda$, afterwords the sum $\oplus_{\alpha \in A} \mathscr{Q}_\alpha$ is an angelic paracompact. . Consider the function sum, $\oplus_{\alpha \in \Lambda} p_\alpha : \oplus_{\alpha \in \Lambda} \mathscr{P}_\alpha \to \oplus_{\alpha \in \Lambda} \mathscr{Q}_\alpha$ defined by $\oplus_{\alpha \in \Lambda} p_\alpha(m) = p(m)$ if $m \in M_\beta$, $\beta \in \Lambda$. Presently, a subspace $C \subseteq \oplus_{\alpha \in \Lambda} \mathscr{P}_\alpha$ is an angelic Lindelöf iff the $\Lambda_0 = \{\alpha \in \Lambda : S \cap \mathscr{P}_\alpha \notin \emptyset\}$ is finite and $C \cap \mathscr{P}_\alpha$ is an angelic compact in $\mathscr{P}_\alpha$ for each $\alpha \in \Lambda_0$. If $S \subseteq \oplus_{alpaha \in \Lambda} \mathscr{P}_\alpha$ is an angelic compact, hence $(\oplus_{\alpha \in \Lambda} \mathscr{P}_\alpha)|_C$ is homeomorphism as $p_{\alpha/C \cap \mathscr{P}_\alpha}$ is homeomorphism for every $\alpha \in \Lambda_0$. □

Theorem 3.8. *Let $\mathscr{P}$ be the second countable $\mathscr{AL}_2$-paracompact space. Then $\mathscr{P}$ is metrizable.*

Proof. Since $\mathscr{P}$ is a second countable space, then $\mathscr{P}$ is an angelic Lindelöf. If $\mathscr{P}$ is also $\mathscr{AL}_2$-paracompact, then $\mathscr{P}$ will be homeomorphic to a T_2 angelic paracompact space $\mathscr{Q}$

and, in particular, $\mathscr{Q}$ is T_4. Thus $\mathscr{P}$ is second countable and regular, hence metrizable. □

Corollary 3.9. *Let $\mathscr{P}$ T_2 second countable $\mathscr{AL}$-paracompact space. Then $\mathscr{P}$ is metrizable.*

Example 3.10. *An $\mathscr{AL}_2$-paracompact space whose a discrete extension of it is Tychonoff but not $\mathscr{AL}$-paracompact.*

Proof. Since $\mathbb{R}$ with the rational sequence topology is a first countable Tychonoff locally angelic compact separable space which is neither normal nor angelic paracompact. Thus $\mathbb{R}$ with the rational sequence topology has a one-point compactification. Let $\mathscr{P} = \mathbb{R} \cup \{p\}$, where $p \notin \mathbb{R}$, be a one-point compactification of $\mathbb{R}$. Since $\mathscr{P}$ is T_2 angelic compact, then it is an $\mathscr{AL}_2$-paracompact. We prove that the discrete extension $\mathscr{P}_\mathbb{R}$ is not $\mathscr{AL}_2$-paracompact. Observe that in $\mathscr{P}_\mathbb{R} = \mathbb{R} \cup \{p\}$, the singleton $\{p\}$ is closed-and-open. $\mathscr{P}_\mathbb{R}$ is first countable and T_3 because R with the rational sequence topology is, thus $\mathscr{P}_\mathbb{R}$ is of countable tightness. $\mathscr{P}_\mathbb{R}$ is also separable because $Q \cup \{p\}$ is a countable dense subset of $\mathscr{P}_\mathbb{R}$. Presently, $\mathbb{R}$ with the rational sequence topology is not normal. Pick any two closed disjoint subsets $\mathscr{A}$ and $\mathscr{B}$ of $\mathbb{R}$ cannot be separated by disjoint open sets in $\mathbb{R}$. Then $\mathscr{A}$ and $\mathscr{B}$ will be also closed and disjoint in $\mathscr{P}_\mathbb{R}$ cannot be separated by disjoint open sets in $\mathscr{P}_\mathbb{R}$. We conclude that $\mathscr{P}_\mathbb{R}$ is not normal. By Theorem 3.2, Hence $\mathscr{P}_\mathbb{R}$ cannot be an $\mathscr{AL}$-paracompact. □

4. $\mathscr{AL}$-normal and its Properties

Definition 4.1. *A TS $\mathscr{P}$ is termed as an $\mathscr{AL}$-Normal if $\exists$ a normal space $\mathscr{Q}$ and a bijective function $g : \mathscr{P} \to \mathscr{Q}$, $\mathscr{Q}$ is an angelic paracompact space and the restriction $g|_\mathscr{A} : \mathscr{A} \to g(\mathscr{A})$ for each angelic Lindelöf subspace $\mathscr{A} \subseteq \mathscr{P}$.*

Example 4.2. *$\mathscr{AL}$-normal does not imply $\mathscr{AL}$-paracompactness*

Proof. Consider $\mathscr{P} = [0,\infty)$. Define $\tau = \{\emptyset, \mathscr{P}\} \cup \{[0,p) : p \in \mathbb{R}, 0 < p\}$. declare that $(\mathscr{P},\tau)$ is normal because there are no two non-empty closed disjoint subsets. Thus $(\mathscr{P},\tau)$ is an $\mathscr{AL}$-normal. Declare that $(\mathscr{P},\tau)$ is second countable, thus hereditary angelic Lindelöf. $(\mathscr{P},\tau)$ cannot be an angelic paracompact because τ is coarser than the particular point topology on $\mathscr{P}$, here the particular point is 0. That is because any non-empty open set includes 0. Therefore, $\mathscr{P}$ is an $\mathscr{AL}$-normal but not an $\mathscr{AL}$-paracompact. □

Theorem 4.3. *Every T_3 separable $\mathscr{AL}$-normal and of countable tightness is normal.*

Proof. If $\mathscr{Q}$ is a normal space and $g : \mathscr{P} \to \mathscr{Q}$ be a mapping witness to $\mathscr{AL}$-normality of $\mathscr{P}$. Then g is continuous because $\mathscr{P}$ is of countable tightness. Consider D is a countable dense subset of $\mathscr{P}$. We produce g is closed. Assume H is a non-empty closed proper subset of $\mathscr{P}$. Suppose $g(p) = q \in Q/g(H)$; then $p \notin H$. Using regularity, consider U and V be disjoint open subsets of $\mathscr{P}$ including p and H, respectively. Then $U \cap (D \cup \{p\})$ is open in Lindelöf subspace $D \cup \{p\}$ including p, so $g(U \cap (D \cup \{p\}))$ is open in the subspace $g((D \cap \{p\}))$ of $\mathscr{Q}$ including q. Thus, $g(U \cap (D \cup \{p\})) = g(U) \cap g(D \cap \{p\}) = W \cap g(D \cap \{p\})$ for some open subset W in $\mathscr{Q}$ with $q \in W$.

We claim that $W \cap f(H) = \emptyset$. Suppose otherwise, and take $q \in W \cap g(H)$. Let $p \in H \ni g(p) = q$. Note that $p \in V$. As D is dense in $\mathscr{P}$, D is also dense in the open set V. Thus, ap $p \in V \cap D$. Presently since W is open in $\mathscr{Q}$ and g is continuous, $g^{-1}(W)$ is an open set in $\mathscr{P}$; it also includes p. Thus, we can choose $d \in g^{-1}(W) \cap V \cap D$. Next $g(d) \in W \cap g(V \cap D) \subseteq W \cap g(D \cup \{p\}) = g(U \cap (D \cup \{p\}))$. So $g(d) \in g(U) \cap g(V)$ is contradiction. Hence, $W \cap g(H) = \emptyset$. Note that $q \in W$. As $q \in \mathscr{Q}/g(H)$ was arbitrary, $g(H)$ is closed. Thus g is a homeomorphism and $\mathscr{P}$ is normal. □

Theorem 4.4. *Let $\mathscr{P}$ be an $\mathscr{AC}$-normal space $\ni$ each angelic Lindelöf subspace is included in an angelic compact subspace. Then $\mathscr{P}$ is an $\mathscr{AL}$-normal.*

Proof. Since $\mathscr{P}$ is a $\mathscr{AC}$-Normal space $\ni$ if $\mathscr{A}$ is a angelic Lindelöf subspace of $\mathscr{P}$, $\exists$ an angelic compact subspace $\mathscr{B} \ni \mathscr{A} \subseteq \mathscr{B}$. Consider $\mathscr{Q}$ is a normal space and $g : \mathscr{P} \to \mathscr{Q}$ be a bijective mapping $\ni g|_\mathscr{C} : \mathscr{C} \to g(\mathscr{C})$ is homeomorphism for every angelic compact subspace $\mathscr{C}$ of $\mathscr{P}$. Presently, consider $\mathscr{A}$ be any angelic Lindelöf subspace of $\mathscr{P}$. Take an angelic compact subspace $\mathscr{B}$ of $\mathscr{P} \ni \mathscr{A} \subseteq \mathscr{B}$; $g|_\mathscr{B} : \mathscr{B} \to g(\mathscr{B})$ is homeomorphism; thus, $g|_\mathscr{A} : \mathscr{A} \to g(\mathscr{A})$ is a homeomorphism as $(g|_\mathscr{B})|_\mathscr{A} = g|\mathscr{B}$. □

Theorem 4.5. If $\mathscr{P}$ is an $\mathscr{AL}$-Normal, then its Alexandroff duplicate $A(\mathscr{P})$ is also an $\mathscr{AL}$-normal.

Proof. Since $\mathscr{P}$ is an $\mathscr{AL}$-Normal space. Consider a normal space $\mathscr{Q}$ and a bijective mapping $g : \mathscr{P} \to \mathscr{Q} \ni g|_\mathscr{C} : \mathscr{C} \to g(\mathscr{C})$ is homeomorphism for every angelic Lindelöf subspace $C \subseteq \mathscr{P}$. Choose the Alexandroff duplicate spaces $A(\mathscr{P})$ and $A(\mathscr{Q})$ of $\mathscr{P}$ and $\mathscr{Q}$, respectively. By reason of the Alexandroff duplicate of a normal space is normal. Hence, $A(\mathscr{P})$ is normal. Construct $g : A(\mathscr{P}) \to A(\mathscr{P})$ by $h(a) = g(a)$ if $a \in \mathscr{P}$ and b is the unique element in $\mathscr{P} \ni b' = a$, also construct $h(a) = (g(b))'$. Thus g is a bijective mapping. Presently, a subspace $C \subseteq A(\mathscr{P})$ is an angelic Lindelöf iff $C \cap \mathscr{P}$ is an angelic Lindelöf in $\mathscr{P}$, and for every open set U in $\mathscr{P}$ with $C \cap \mathscr{P} \subseteq \mathscr{A}$, we state that $(C \cap \mathscr{P}')/U'$ is countable. Choose $C \subseteq A(\mathscr{P})$ is an angelic Lindelöf subspace. We produce $h|_C : C \to h(C)$ is a homeomorphism. Let $a \in C$ be arbitrary. Take $a \in C \cap \mathscr{P}'$ and $b \in \mathscr{P}$ are the unique element $\ni b' = a$. The smallest basic open neighborhood $\{g(b))'\}$ of $h(a)$ here $\{a\}$ is open in C and $h(\{a\}) \subseteq \{(g(b))'\}$. If $a \in C \cap \mathscr{P}$. Consider W is an open set in $\mathscr{Q} \ni h(a) = g(a) \in W$ and $H = (W \cup (W'\{g(a)'\})) \cap h(C)$ is a basic open neighborhood of $g(a)$ in $g(C)$. Because $g|_{C \cap \mathscr{P})} : C \cap \mathscr{P} \to g(C \cap \mathscr{P})$ is homeomorphism, next $\exists$ an open set $U \in \mathscr{P}$, $a \in U$ and $h|_{C \cap \mathscr{P}(U \cap C))} \subseteq W \cap h(C \cap \mathscr{P})$. Presently, $(U \cup (U'/\{a'\})) \cap q(C)$ is open in $C \ni a \in G$ and $h_{c(G)} \subseteq H$. Thus, $h|_C$ is continuous. Presently, produce the $h|_C$ is open. Consider $K \cup (K'/\{k'\})$, here $k \in K$ and K is open in $\mathscr{P}$, is a basic open set in $A(\mathscr{P})$, afterwords $(K \cap C) \cup ((K' \cap C)/\{k'\})$ is a basic open set in C. As $\mathscr{P} \cap C$ is Lindelöf in $\mathscr{P}$, so $h|_{C(K\cap(\mathscr{P}\cap C))} = h|_{\mathscr{P}\cap C(K\cap(\mathscr{P}\cap C)))}$ is open in $Q \cap h(C \cap \mathscr{P})$ as $g|_{\mathscr{P}\cap C)}$ is homeomorphism. hence, $K \cap C$ is open in $Q \cap g(\mathscr{P} \cap C)$. In addition, $h(K' \cap C)/\{k'\})$ is open in $\mathscr{Q}' \cap g(C)$ being a set of isolated points. Then, $h|_C$ is an open mapping. Hence, $h|_C$ is homeomorphism. □

5. Conclusion

Our main results includes the two new concepts of an $\mathscr{AL}$-Paracompact spaces and an $\mathscr{AL}_2$-Paracompact spaces. Also proved that, every $\mathscr{AL}$-Paracompactness and an $\mathscr{AL}_2$-Parac- ompactness has a topological property. We also investigated the $\mathscr{AL}$-normal and its properties.

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