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Intuitionistic generalized closed sets in generalized intuitionistic topological space

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Abstract

In this article, we analyse the μ_I g-closed set in generalized intuitionistic topological space and discuss the properties of closure and interior of μ_I g-closed. Also we have investigated some of their basic attributes.

Keywords

$\mu_I g -closed,\ \mu_I g - open,\ c^*_{\mu_I}(A),\ i^*_{\mu_I}(A).$

AMS Subject Classification

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Contents

1 Introduction 1142
2 Preliminaries 1142
3 Intuitionistic generalized closed sets in generalized intuitionistic topological space 1143
4 Closure of GITS 1144
5 Interior of GITS 1145
6 Some operators in intuitionistic topological space 1146
7 Conclusion 1146
8 Acknowledgement 1146
References 1146

1. Introduction

The documentation of intuitionistic set was introduced by Coker[5] in 1996. It has membership and non-membership degrees, so this notion gives us more flexible approaches to representing vagueness in mathematical objects and also in some of the engineering fields with classical set logic. In 1996 Csaszar [4] introduced generalized topological space. Thereafter in 2007 P. Sivagami and D. Sivaraj [11] developed generalized topological space and reveled many more results of it.

Now, G. Hari Siva Annam and G. Mathan Kumar [8] introduced a new concept of generalized topological space by using intuitionistic sets. This is called as generalized intuitionistic topological spaces (GITS). They discussed intuitionistic closed sets and intuitionistic open sets in generalized intuitionistic topological space. In this article, we establish intuitionistic generalised closed sets in GITS and discussed their properties. We also demonstrate the correlation between intuitionistic closed sets and intuitionistic generalised closed sets in GITS. Moreover, we explore closure and interior of intuitionistic generalised closed sets.

2. Preliminaries

In this segment, we list some definition and fundamental results which are to be used further.

Definition 2.1. *[5] Let X be a non-empty set. An intuitionistic set A is an object having the form A=$\langle X, A_1, A_2\rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non member of A.*

Definition 2.2. *[5] Let X be a non-empty set and let A, B be an intuitionistic sets in the form A=$\langle X, A_1, A_2\rangle$ and B=$\langle X, B_1, B_2\rangle$ respectively. Then*

1) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$.

2) A=B if and only if $A \subseteq B$ and $B \subseteq A$.

3) $\bar{A} = \langle X, A_2, A_1 \rangle$ *, (In intuitionistic, A =* A^c*)*

4) $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle$.

5) $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle$.

6) A-B = $A \cap \bar{B}$.

7) $\underset{\sim}{\phi} = \langle X, \phi, X \rangle; \underset{\sim}{X} = \langle X, X, \phi \rangle$.

Definition 2.3. *[6] An intuitionistic topology on a non-empty set X is a family* τ *of intuitionistic sets in X containing* $\underset{\sim}{\phi}, \underset{\sim}{X}$ *and closed under finite union and intersection. The pair* (X, τ) *is called an intuitionistic topological space. Any intuitionistic set in* τ *is known as an intuitionistic open set (IOS) in X and the complement of IOS is called an intuitionistic closed set (ICS).*

Definition 2.4. *[4] Let X be a non-empty set and* μ *be a collection of subsets of X. Then (X,*μ*) is called a generalized topology if* $\phi \in \mu$ *and arbitrary union of elements of* μ *is in* μ*. The elements of* μ *is called* μ *- open set and the complement of* μ *- open set is called* μ *- closed set.*

Definition 2.5. *[8] Let X be a non-empty set and* μ_I *be the collection of intuitionistic subset of X. Then* μ_I *is called the generalized intuitionistic topology on X if* $\phi \in \mu_I$ *and* μ_I *is closed under arbitrary unions. The elements of* μ_I *are called* μ_I*-open sets and their complements are called* μ_I *- closed sets.*

Definition 2.6. *[8] The* μ_I *- closure of A is the intersection of all* μ_I*- closed superset of A, and the* μ_I *- interior of A (its denoted by* $i_{\mu I}(A)$*) is the union of all* μ_I*- open sets contained in A.*

Definition 2.7. *[8]*

1) $If A \subseteq B$ *then* $c_{\mu I}(A) \subseteq c_{\mu I}(B)$.

2) $c_{\mu I}(A \cup B) \supseteq c_{\mu I}(A) \cup c_{\mu I}(B)$.

3) $c_{\mu I}(A) \cap c_{\mu I}(B) \supseteq c_{\mu I}(A \cap B)$.

4) $If\ A \subseteq B$ *then* $i_{\mu I}(A) \subseteq i_{\mu I}(B)$.

5) $i_{\mu I}(A \cup B) \supseteq i_{\mu I}(A) \cup i_{\mu I}(B)$.

6) $i_{\mu I}(A \cap B) \subseteq i_{\mu I}(A) \cap i_{\mu I}(B)$.

7) $If A$ *is* $\mu_I - closed$ *then* $c_{\mu I}(A) = A$.

8) $c_{\mu I}[c_{\mu I}(A)] = c_{\mu I}(A)$.

9) $c_{\mu I}(\underset{\sim}{\phi}) \neq \underset{\sim}{\phi}$.

10) $i_{\mu I}(\underset{\sim}{X}) \neq \underset{\sim}{X}$.

11) $\bar{\bar{A}} = A$

12) $\underset{\sim}{\phi} = \langle X, \phi, X \rangle; \underset{\sim}{X} = \langle X, X, \phi \rangle$.

3. Intuitionistic generalized closed sets in generalized intuitionistic topological space

Definition 3.1. *In (X,*μ_I*), an intuitionistic set A of X is said to be an intuitionistic generalised closed sets in generalized intuitionistic topological space (GITS) if* $c_{\mu I}(A) \subseteq U$ *whenever* $A \subseteq U$*and U is* μ_I*–open and it is denoted by* $\mu_I g$*–closed.*

Theorem 3.2. *Every* μ_I *– closed set is* $\mu_I g$ *–closed set but the converse is not true.*

Proof. Let A be μ_I – closed set. Then $c_{\mu I}(A)$ = A (using 2.7,[7]). Suppose $A \subseteq U$. Then $c_{\mu I}$(A) = $A \subseteq U$. Hence A is μ_I g–closed. But the converse is not true. We can see in the succeeding illustration, Let $X = \{a,b,c\}$. Then

μ_I={ $\langle X,\phi,X\rangle$, $\langle X,\{c\},\{a\}\rangle$, $\langle X,\{b,c\},\{a\}\rangle$, $\langle X,\{b\},\{a,c\}\rangle$ }, μ_I – closed = { $\langle X,X,\phi\rangle$, $\langle X,\{a\},\{c\}\rangle$, $\langle X,\{a\},\{b,c\}\rangle$,$\langle X,\{a,c\},\{b\}\rangle$} *and* μ_I g–closed = { $\langle X,X,\phi\rangle$,$\langle X,\phi,\phi\rangle$, $\langle X,\phi,\{b\}\rangle$, $\langle X,\phi,\{c\}\rangle$, $\langle X,\{a\},\phi\rangle$, $\langle X,\{a\},\{b\}\rangle$, $\langle X,\{c\},\phi\rangle$, $\langle X,\{a\},\{c\}\rangle$, $\langle X,\{a\},\{b,c\}\rangle$, $\langle X,\{c,a\},\{b\}\rangle$, $\langle X,\{c,a\},\phi\rangle$, $\langle X,\{b\},\{c\}\rangle$, $\langle X,\{b\},\phi\rangle$, $\langle X,\phi,\{b,c\}\rangle$, $\langle X,\{c\},\{b\}\rangle$ $\langle X,\{a,b\},\phi\rangle$ $\langle X,\{a,b\},\{c\}\rangle$ $\langle X,\{b,c\},\phi\rangle$} .
Here $\langle X,\{c\},\{b\}\rangle$, $\langle X,\{b\},\{c\}\rangle$, $\langle X,\{b,c\},\phi\rangle$ are $\mu_I g$ – closed sets. But these are not
μ_I – closed sets. □

Remark 3.3. *Union of two* $\mu_I g$ *– closed sets need not be* $\mu_I g$ *– closed. Now we can see the successive illustration. Let* $X = \{a,b,c\}$*. In the topological space (X,*μ_I*). Then*
μ_I={ $\langle X,\phi,X\rangle$, $\langle X,\{b\},\phi\rangle$, $\langle X,\{a,b\},\phi\rangle$,$\langle X,\{a\},\{b\}\rangle$ } , μ_I *– closed =* { $\langle X,X,\phi\rangle$, $\langle X,\phi,\{b\}\rangle$,$\langle X,\phi,\{a,b\}\rangle$, $\langle X,\{b\},\{a\}\rangle$} *and* μ_I *g–closed=* { $\langle X,X,\phi\rangle$,$\langle X,\phi,X\rangle$, $\langle X,\phi,\{a\}\rangle$, $\langle X,\phi,\{a,b\}\rangle$, $\langle X,\{b\},\{a\}\rangle$, $\langle X,\{b\},\{a,c\}\rangle$, $\langle X,\{c\},\phi\rangle$, $\langle X,\{c\},\{a\}\rangle$, $\langle X,\{c\},\{b\}\rangle$, $\langle X,\{c\},\{a,b\}\rangle$, $\langle X,\{b,c\},\phi\rangle$, $\langle X,\{b,c\},\{a\}\rangle$, $\langle X,\{c,a\},\phi\rangle$, $\langle X,\{c,a\},\{b\}\rangle$, $\langle X,\phi,\{b\}\rangle$, $\langle X,\phi,\{b,c\}\rangle$, $\langle X,\phi,\{c,a\}\rangle$ } *. Now A =*$\langle X,\phi,\{a\}\rangle$*,*$B = \langle X,\phi,\{b\}\rangle$*,*$A \cup B = \langle X,\phi,\phi\rangle$*. Here A and B are* $\mu_I g$ *– closed. But* $A \cup B$ *is not* $\mu_I g$ *– closed.*

Remark 3.4. *Intersection of any two* μ_I *g – closed sets need not be* μ_I *g – closed set. Now we can see in the following example. Let* $X = \{a,b,c\}$ *in (X,*μ_I*), we found that,*
μ_I= { $\langle X,\phi,X\rangle$, $\langle X,\phi,\{b\}\rangle$, $\langle X,\{a\},\phi\rangle$, $\langle X,\{a,b\},\phi\rangle$, $\langle X,\{a\},\{c\}\rangle$ } , μ_I *– closed =* { $\langle X,X,\phi\rangle$, $\langle X,\{b\},\phi\rangle$, $\langle X,\phi,\{a\}\rangle$, $\langle X,\phi,\{a,b\}\rangle$, $\langle X,\{c\},\{a\}\rangle$} *. Then*
$\mu_I g$*–closed =* { $\langle X,X,\phi\rangle$, $\langle X,\phi,\{a\}\rangle$, $\langle X,\phi,\{a,b\}\rangle$, $\langle X,\{b\},\phi\rangle$, $\langle X,\{b\},\{a\}\rangle$, $\langle X,\{b\},\{c\}\rangle$, $\langle X,\{b\},\{a,c\}\rangle$, $\langle X,\{c\},\phi\rangle$, $\langle X,\{c\},\{a\}\rangle$, $\langle X,\{c\},\{b\}\rangle$, $\langle X,\{c\},\{a,b\}\rangle$, $\langle X,\{b,c\},\phi\rangle$, $\langle X,\{b,c\},\{a\}\rangle$, $\langle X,\{c,a\},\phi\rangle$, $\langle X,\{c,a\},\{b\}\rangle$} .

Suppose $A = \langle X,\phi,\{a\}\rangle; B = \langle X,\{b\},\{c\}\rangle$, *then* $A\cap B = \langle X,\phi,\{a,c\}\rangle$. *Here A and B are* $\mu_I g$*–closed sets. But* $A\cap B$ *is not a* $\mu_I g$*–closed sets.*

4. Closure of GITS

Definition 4.1. *The* μ_I *g – closure of A, denoted by* $c^*_{\mu I}(A)$*, is the intersection of all* $\mu_I g$ *– closed superset of A.*

Note 4.2.

(i) $c^*_{\mu I}(\underset{\sim}{\phi}) \neq \underset{\sim}{\phi}$;

(ii) $c^*_{\mu I}(\underset{\sim}{X}) = \underset{\sim}{X}$

Proof. The result is obvious. □

Theorem 4.3. *Let* (X,μ_I) *be a GITS. A and B are the subset of X*

(i) $c^*_{\mu_I}$ *is enhancing:* $A \subseteq c^*_{\mu I}(A)$,

(ii) *If* $A\subseteq B$ *then* $c^*_{\mu I}(A) \subseteq c^*_{\mu I}(B)$.

(iii) $c^*_{\mu I}$ *is idempotent* : $c^*_{\mu I}[c^*_{\mu I}(A)] = c^*_{\mu I}(A)$.

Proof. i) Since $c^*_{\mu I}(A)$ is the intersection of all $\mu_I g$- closed superset of A, then $A \subseteq c^*_{\mu I}(A)$.

ii) Let $A\subseteq B$, Suppose $x \notin c^*_{\mu I}(B)$. Then $x \notin \cap F, F$ *is* $\mu_I g$ – closed set and $B \subseteq F$. Which impiles $x \notin F$, for some $\mu_I g$-closed superset of $B \subseteq F$. Since $A \subseteq B$, $A\subseteq F$. Hence $x \notin F$, for some $\mu_I g$-closed superset of A. So $x \notin c^*_{\mu I}(A)$. Therefore we get a result.

iii) From (i), $A\subseteq c^*_{\mu I}(A)$. Then $c^*_{\mu I}(A) \subseteq c^*_{\mu I}c^*_{\mu I}(A)$. Take B = $c^*_{\mu I}(A)$. Let $x \notin c^*_{\mu I}(B)$. Then $x \notin \cap F$, F is μ_I g – closed and $B\subseteq F$, which implies $x \notin F$, for some μ_I g – closed set F such that $B \subseteq F$. This gives $x \notin B = c^*_{\mu I}(A)$, we get $c^*_{\mu I}c^*_{\mu I}(A) \subseteq c^*_{\mu I}(A)$. Therefore $c^*_{\mu I}[c^*_{\mu I}(A)] = c^*_{\mu I}(A)$. □

Theorem 4.4. $c^*_{\mu I}(A)\cup c^*_{\mu I}(B) \subseteq c^*_{\mu I}(A\cup B)$.

Proof. We know that $A\subseteq A\cup B$ *and* $B\subseteq A\cup B$. Then by theorem 4.3(ii)
$c^*_{\mu I}(A) \subseteq c^*_{\mu I}(A\cup B)$ and $c^*_{\mu I}(B) \subseteq c^*_{\mu I}(A\cup B)$. Therefore $c^*_{\mu I}(A)\cup c^*_{\mu I}(B) \subseteq c^*_{\mu I}(A\cup B)$. □

Example 4.5. *The inclusion may be strict or equal, we can see in the succeeding illustration. Let* $X = \{a,b,c\}$ *in the topological space* (X,μ_I) *then,*
μ_I={ $\langle X,\phi,X\rangle, \langle X,\{a\},\{c\}\rangle, \langle X,\{c\},\{a\}\rangle, \langle X,\{a,c\},\phi\rangle, \langle X,a,\phi\rangle$} μ_I *– closed* = { $\langle X,X,\phi\rangle$, $\langle X,\{c\},\{a\}\rangle, \langle X,\{a\},\{c\}\rangle, \langle X,\phi,\{a,c\}\rangle, \langle X,\phi,\{a\}\rangle$}
$\mu_I g$*– closed* = { $\langle X,\phi,X\rangle, \langle X,X,\phi\rangle, \langle X,\phi,\{a\}\rangle, \langle X,\phi,\{c\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\phi,\{b,c\}\rangle, \langle X,\phi,\{c,a\}\rangle, \langle X,\{a\},\{b,c\}\rangle, \langle X,\{c\},\{a\}\rangle, \langle X,\{a,b\}\phi\rangle, \langle X,\{b\},\{a,c\}\rangle, \langle X,\{c\},\{a,b\}\rangle, \langle X,\{a,b\},\{c\}\rangle, \langle X,\{b,c\},\phi\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{a\},\{c\}\rangle, \langle X,\{b\},\{c\}\rangle, \langle X,\{b\},\{a\}\rangle$ } .

Let $A = \langle X,\{c\},\{a,b\}\rangle, B = \langle X,\{b,c\},\phi\rangle, A\cup B = \langle X,\{b,c\},\phi\rangle$
$c^*_{\mu I}(A) = \langle X,\{c\},\{a,b\}\rangle; c^*_{\mu I}(B) = \langle X,\{b,c\},\phi\rangle; c^*_{\mu I}(A)\cup c^*_{\mu I}(B) = \langle X,\{b,c\},\phi\rangle$
$c^*_{\mu I}(A\cup B) = \langle X,\{b,c\},\phi\rangle$. *Hence* $c^*_{\mu I}(A\cup B) = c^*_{\mu I}(A)\cup c^*_{\mu I}(B)$.

Let A=$\langle X,\phi,\{a\}\rangle, B = \langle X,\phi,\{c\}\rangle$ $c^*_{\mu I}(A) = \langle X,\phi,\{a\}\rangle; c^*_{\mu I}(B) = \langle X,\phi,\{c\}\rangle; c^*_{\mu I}(A)\cup c^*_{\mu I}(B) = \langle X,\phi,\phi\rangle; A\cup B = \langle X,\phi,\phi\rangle$;
$c^*_{\mu I}(A\cup B) = \langle X,\{b\},\phi\rangle$. *Hence* $c^*_{\mu I}(A)\cup c^*_{\mu I}(B) \subset c^*_{\mu I}(A\cup B)$

Theorem 4.6. $c^*_{\mu I}(A\cap B) \subseteq c^*_{\mu I}(A)\cap c^*_{\mu I}(B)$.

Proof. We know that $A\cap B\subseteq A$ *and* $A\cap B\subseteq B$. Then $c^*_{\mu I}(A\cap B) \subseteq c^*_{\mu I}(A)$ and $c^*_{\mu I}(A\cap B) \subseteq c^*_{\mu I}(B)$. Therefore $c^*_{\mu I}(A\cap B) \subseteq c^*_{\mu I}(A)\cap c^*_{\mu I}(B)$. □

Example 4.7. *The inclusion may be strict or equal, we can see in the ensuing illustration. Let* $X = \{a,b,c\}$,
μ_I={ $\langle X,\phi,X\rangle$, $\langle X,\{b\},\phi\rangle$, $\langle X,\{a,b\},\phi\rangle, \langle X,\{a\},\{b\}\rangle$ } , μ_I *– closed* = { $\langle X,X,\phi\rangle$, $\langle X,\phi,\{b\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\{b\},\{a\}\rangle$} *and* μ_I *g–closed*= { $\langle X,X,\phi\rangle, \langle X,\phi,X\rangle, \langle X,\phi,\{a\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\{b\},\{a\}\rangle, \langle X,\{b\},\{a,c\}\rangle, \langle X,\{c\},\phi\rangle, \langle X,\{c\},\{a\}\rangle, \langle X,\{c\},\{b\}\rangle, \langle X,\{c\},\{a,b\}\rangle, \langle X,\{b,c\},\phi\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{c,a\},\phi\rangle, \langle X,\{c,a\},\{b\}\rangle$, $\langle X,\phi,\{b\}\rangle$, $\langle X,\phi,\{b,c\}\rangle$, $\langle X,\phi,\{c,a\}\rangle$ } .

Take $A = \langle X,\phi,\{a\}\rangle, B = \langle X,\phi,\{c\}\rangle$, $c^*_{\mu I}(A) = \langle X,\phi,\{a\}\rangle, c^*_{\mu I}(B) = \langle X,\{c\},\phi\rangle$, $c^*_{\mu I}(A)\cap c^*_{\mu I}(B) = \langle X,\phi,\{a\}\rangle, A\cap B = \langle X,\phi,\{a,c\}\rangle, c^*_{\mu I}(A\cap B) = \langle X,\phi,\{a,c\}\rangle$. *Hence* $c^*_{\mu I}(A\cap B) \subset c^*_{\mu I}(A)\cap c^*_{\mu I}(B)$.

Take $A = \langle X,\phi,\phi\rangle, B = \langle X,\phi,\{a\}\rangle, A\cap B = \langle X,\phi,\{a\}\rangle$, $c^*_{\mu I}(A\cap B) = \langle X,\phi,\{a\}\rangle; c^*_{\mu I}(A) = \langle X,\{c\},\phi\rangle; c^*_{\mu I}(B) = \langle X,\phi,\{a\}\rangle, c^*_{\mu I}(A)\cap c^*_{\mu I}(B)$=$\langle X,\phi,\{a\}\rangle$. *Hence* $c^*_{\mu I}(A\cap B) = c^*_{\mu I}(A)\cap c^*_{\mu I}(B)$.

Theorem 4.8. $A \subseteq c^*_{\mu I}(A) \subseteq c_{\mu I}(A)$.

Proof. Assume $x \notin c_{\mu I}(A)$ which implies $x \notin \cap F$, where F is a μ_I - closed superset of A. It gives $x \notin \cap F$, F is a $\mu_I g$ - closed superset of A (By theroem 3.2) then $x \notin F$ for some $\mu_I g$ - closed superset of A. Therefore $x \notin A$ and hence we have $A \subseteq c^*_{\mu I}(A) \subseteq c_{\mu I}(A)$. □

Example 4.9. *Consider the topological space* $X = \{a,b,c\}$. *Let* μ_I = { $\langle X,X,\phi\rangle$, $\langle X,\{a\},\phi\rangle, \langle X,\phi,\{b\}\rangle, \langle X,\{c\},\phi\rangle, \langle X,\{a,c\},\phi\rangle, \langle X,\{a,\},\{b,c\}\rangle, \langle X,\{a\},\{b\}\rangle$ } μ_I *– closed* ={ $\langle X,X,\phi\rangle$, $\langle X,\phi,\{a\}\rangle, \langle X,\{b\},\phi\rangle, \langle X,\phi,\{c\}\rangle, \langle X,\phi,\{a,c\}\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{b\},\{a\}\rangle$ } $\mu_I g$ *– closed* ={ $\langle X,X,\phi\rangle, \langle X,\phi,\{a\}\rangle, \langle X,\phi,\{c\}\rangle, \langle X,\phi,\{c,a\}\rangle$,

$\langle X,\{b\},\{a\}\rangle, \langle X,\{b\},\phi\rangle, \langle X,\{a,b\},\phi\rangle, \langle X,\{b\},\{a,c\}\rangle, \langle X,\{a,b\},\{c\}\rangle, \langle X,\{b,c\},\phi\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{b\},\{c\}\rangle\}$.

Let $A=\langle X,\{b\},\phi\rangle, c^*_{\mu I}(A)=\langle X,\{b\},\phi\rangle . c_{\mu I}(A)=\langle X,\{b\},\phi\rangle$. *Then* $A=c^*_{\mu I}(A)=c_{\mu I}(A)$

Let $A=\langle X,\{a\},\phi\rangle, c^*_{\mu I}(A)=\langle X,\{a,b\},\phi\rangle, c_{\mu I}(A)=\langle X,X,\phi\rangle$. *Then* $A\subset c^*_{\mu I}(A)\subset c_{\mu I}(A)$.

Theorem 4.10. *If a subset A of X is* $\mu_I g-closed$ *set and* $A\subseteq B\subseteq c_{\mu I}(A)$ *then B is* $\mu_I g-closed$ *sets in X.*

Proof. Let A be a $\mu_I g$ – closed set and $A\subseteq B\subseteq c_{\mu I}(A)$. *Let U be a* μ_I*–closed set of X such that* $B\subseteq U$. *Since A is* $\mu_I g$*–closed set, then we have* $c_{\mu I}(A)\subseteq U$. Now $c_{\mu I}(A)\subseteq c_{\mu I}(B)\subseteq c_{\mu I}[c_{\mu I}(A)]=c_{\mu I}(A)\subseteq U$. Then $c_{\mu I}(B)\subseteq U$, *U is* μ_I*–closed set and* $B\subseteq U$. *Therefore we get B is* $\mu_I g$ – closed set in X. □

Remark 4.11. *The converse of the theorem need not be true as seen from the following example. Consider the topological space* (X,μ_I) *where* $X=\{a,b,c\}$ $\mu_I=\{\langle X,\phi,X\rangle, \langle X,\{a,c\},\{b\}\rangle, \langle X,\{c\},\phi\rangle, \langle X,\{a,c\},\phi\rangle\}$ μ_I *– closed* $=\{\langle X,X,\phi\rangle, \langle X,\{b\}\{a,c\}\rangle, \langle X,\phi,\{c\}\rangle, \langle X,\phi,\{a,c\}\rangle\}$ $\mu_I g$ *– closed* $=\{\langle X,X,\phi\rangle, \langle X,\phi,\{c\}\rangle, \langle X,\phi,\{a,c\}\rangle, \langle X,\{b\},\{a\}\rangle, \langle X,\{b\},\phi\rangle, \rangle X,\{b\},\{a,c\}\rangle, \langle X,\{b\},\{c\}\rangle, \langle X,\{a,b\},\phi\rangle, \langle X,\{a,b\},\{c\}\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{b,c\},\phi\rangle\}$.

Let $A=\langle X,\{b\},\{a,c\}\rangle, c_{\mu I}(A)=\langle X,\{b\},\{a,c\}\rangle, B=\langle X,\phi,\{c\}\rangle$. *Here B is* $\mu_I g-closed$ *set but* $A\nsubseteq B\nsubseteq c_{\mu I}(A)$.

Let $A=\langle X,\phi,\{a,c\}\rangle, c_{\mu I}(A)=\langle X,\phi,\{a,c\}\rangle, B=\langle X,\{b\},\{a\}\rangle$. *Here B is* $\mu_I g-closed$ *set but* $A\subseteq B\nsubseteq c_{\mu I}(A)$.

Let $A=\langle X,\phi,\{c\}\rangle; B=\langle X,\phi,\{a,c\}\rangle, c_{\mu I}(A)=\langle X,\phi,\{c\}\rangle$. *Here B is* $\mu_I\ g-closed$ *set but* $A\nsubseteq B\subseteq c_{\mu I}(A)$.

Theorem 4.12. *Let* (X,μ_I) *be a GITS. If U and V are* μ_I *g-closed set and* $U\cap V=\underset{\sim}{\phi}$ *then* $(c^*_{\mu I}U)\cap V=\underset{\sim}{\phi}$ *and* $U\cap(c^*_{\mu I}V)=\underset{\sim}{\phi}$.

Proof. Given U and V are μ_I g-closed sets and $U\cap V=\underset{\sim}{\phi}$. Then $U\nsubseteq V$ and $V\nsubseteq U$. Hence $U\nsubseteq c^*_{\mu I}(V)$ *and* $V\nsubseteq c^*_{\mu I}(U)$. Therefore $(c^*_{\mu I}U)\cap V=\underset{\sim}{\phi}$ *and* $U\cap(c^*_{\mu I}V)=\underset{\sim}{\phi}$. □

5. Interior of GITS

Definition 5.1. *The complement of* $\mu_I g$*-closed set is* $\mu_I g$*-open.*

Definition 5.2. *For any* $A\subseteq X$*, the space union of all* $\mu_I g$*-open set contained in A is said to be* $\mu_I g$*- interior of A and its denoted by* $i^*_{\mu I}(A)$.

Note 5.3. *(i)* $i^*_{\mu I}(\underset{\sim}{X})\neq\underset{\sim}{X}$

(ii) $i^*_{\mu I}(\underset{\sim}{\phi})=\underset{\sim}{\phi}$.

Proof. The result is obvious. □

Theorem 5.4. *If* $A\subseteq B$*, then* $i^*_{\mu I}(A)\subseteq i^*_{\mu I}(B)$.

Proof. Suppose $A\subseteq B$, *let* $x\notin i^*_{\mu I}(B)$, then $x\notin\cup G$, G is μ_I g– open set contained in B. Hence
$x\notin G$, for all μ_I g– open set G contained in B. Therfore $x\notin\cup G$, G is μ_I g– open set contained in A.
Therefore $x\notin i^*_{\mu I}(A)$. Then we have $i^*_{\mu I}(A)\subseteq i^*_{\mu I}(B)$. □

Theorem 5.5. $i^*_{\mu I}(A)\cup i^*_{\mu I}(B)\subseteq i^*_{\mu I}(A\cup B)$.

Proof. We know that $A\subseteq A\cup B$ *and* $B\subseteq A\cup B$. Then $i^*_{\mu I}(A)\subseteq i^*_{\mu I}(A\cup B)$ and
$i^*_{\mu I}(B)\subseteq i^*_{\mu I}(A\cup B)$ which implies $i^*_{\mu I}(A)\cup i^*_{\mu I}(B)\subseteq i^*_{\mu I}(A\cup B)$. □

Example 5.6. *The inclusion may be strict or equal we can see in the following example. Let* $X=\{a,b,c\}$ *in the topological space* (X,μ_I), $\mu_I=\{\langle X,\phi,X\rangle, \langle X,\{a\},\{b\}\rangle, \langle X,\{b\},\{c\}\rangle, \langle X,\{a,b\},\phi\rangle\}$ μ_I*– closed*$=\{\langle X,X,\phi\rangle, \langle X,\{b\},\{a\}\rangle, \langle X,\{c\},\{b\}\rangle, \langle X,\phi,\{a,b\}\rangle\}$ $\mu_I g-closed=\{\langle X,X,\phi\rangle, \langle X,\phi,\{a\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\{b\},\{a\}\rangle, \langle X,\{c\},\phi\rangle, \langle X,\{c\},\{a\}\rangle, \langle X,\{b,c\},\phi\rangle, \langle X,\{c\},\{a,b\}\rangle, \langle X,\{c\},\{b\}\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{a,c\},\phi\rangle, \langle X,\{a,c\},\{b\}\rangle\}$.

Let $A=\langle X,\phi,\{c\}\rangle, i^*_{\mu I}(A)=\langle X,\phi,\{c\}\rangle, B=\langle X,\phi,\{a,b\}\rangle, i^*_{\mu I}(B)=\langle X,\phi,X\rangle, i^*_{\mu I}(A)\cup i^*_{\mu I}(B)=\langle X,\phi,\{c\}\rangle, A\cup B=\langle X,\phi,\phi\rangle, i^*_{\mu I}(A\cup B)=\langle X,\phi,\{c\}\rangle$. *Hence* $i^*_{\mu I}(A\cup B)=i^*_{\mu I}(A)\cup i^*_{\mu I}(B)$.

Let $A=\langle X,\phi,\{a\}\rangle, B=\langle X,\{a\},\{c\}\rangle, A\cup B=\langle X,\{a\},\phi\rangle, i^*_{\mu I}(A\cup B)=\langle X,\{a\},\phi\rangle, i^*_{\mu I}(A)=\langle X,\phi,\{a,c\}\rangle, i^*_{\mu I}(B)=\langle X,\{a\},\{c\}\rangle, i^*_{\mu I}(A)\cup i^*_{\mu I}(B)=\langle X,\{a\},\{c\}\rangle$. *Hence* $i^*_{\mu I}(A)\cup i^*_{\mu I}(B)\subset i^*_{\mu I}(A\cup B)$.

Theorem 5.7. $i^*_{\mu I}(A\cap B)\subseteq i^*_{\mu I}(A)\cap i^*_{\mu I}(B)$.

Proof. We know that $A\cap B\subseteq A$ *and* $A\cap B\subseteq B$. Then $i^*_{\mu I}(A\cap B)\subseteq i^*_{\mu I}(A)$ and $i^*_{\mu I}(A\cap B)\subseteq i^*_{\mu I}(B)$ which implies $i^*_{\mu I}(A\cap B)\subseteq i^*_{\mu I}(A)\cap i^*_{\mu I}(B)$. □

Example 5.8. *In* (X,μ_I)*, Let* $X=\{a,b,c\}$

$\mu_I=\{\langle X,\phi,X\rangle, \langle X,\{b\},\phi\rangle, \langle X,\{a,b\},\phi\rangle, \langle X,\{a\},\{b\}\rangle\}$, μ_I *– closed* $=\{\langle X,X,\phi\rangle, \langle X,\phi,\{b\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\{b\},\{a\}\rangle\}$ *and* μ_I *g–closed*$=\{\langle X,X,\phi\rangle, \langle X,\phi,X\rangle, \langle X,\phi,\{a\}\rangle, \langle X,\phi,\{a,b\}\rangle, \langle X,\{b\},\{a\}\rangle, \langle X,\{b\},\{a,c\}\rangle, \langle X,\{c\},\phi\rangle, \langle X,\{c\},\{a\}\rangle, \langle X,\{c\},\{b\}\rangle, \langle X,\{c\},\{a,b\}\rangle, \langle X,\{b,c\},\phi\rangle, \langle X,\{b,c\},\{a\}\rangle, \langle X,\{c,a\},\phi\rangle, \langle X,\{c,a\},\{b\}\rangle, \langle X,\phi,\{b\}\rangle, \langle X,\phi,\{b,c\}\rangle, \langle X,\phi,\{c,a\}\rangle\}$.

Now $A=\langle X,\phi,\{a\}\rangle$; $B=\langle X,\phi,\{b\}\rangle$; $A\cap B=\langle X,\phi,\{a,b\}\rangle$; $i^*_{\mu I}(A)=\langle X,\phi,\{a,c\}\rangle$; $i^*_{\mu I}(B)=\langle X,\phi,\{b,c\}\rangle$; $i^*_{\mu I}(A\cap B)=\langle X,\phi,X\rangle$; $i^*_{\mu I}(A)\cap$

$i^*_{\mu I}(B) = \langle X, \phi, X\rangle$. *Hence* $i^*_{\mu I}(A\cap B) = i^*_{\mu I}(A)\cap i^*_{\mu I}(B)$.

Now $A = \langle X, \{\, b,c\}\,, \phi\rangle, B = \langle X, \{\, a,c\}\,, \{\, b\}\,\rangle$; $i^*_{\mu I}(A) = \langle X, \{\, b,c\}\,, \phi\rangle, i^*_{\mu I}(B) = \langle X, \{\, a,c\}\,, \{\, b\}\,\rangle i^*_{\mu I}(A)\cap i^*_{\mu I}(B) = \langle X, \{\, c\}\,, \{\, b\}\,\rangle$, $A\cap B = \langle X, \{\, c\}\,, \{\, b\}\,\rangle, i^*_{\mu I}(A\cap B) = \langle X, \phi, \{\, b,c\}\,\rangle$. *Hence* $i^*_{\mu I}(A\cap B) \subset i^*_{\mu I}(A)\cap i^*_{\mu I}(B)$.

Theorem 5.9. $i_{\mu I}(A) \subseteq i^*_{\mu I}(A) \subseteq A$.

Proof. Suppose $x\in i_{\mu I}(A)$. Then $x\in \cup G$, where G is a μ_I - open set contained in A. Since every μ_I - open set is $\mu_I g$ - open set, $x\in\cup G$, where G is a $\mu_I g$ - open set contained in A. Thus we have $x\in i^*_{\mu I}(A)$. Since $i^*_{\mu I}(A)$, is the union of all open sets contained in A, $i^*_{\mu I}(A)\subseteq A$. Therefore $i_{\mu I}(A)\subseteq i^*_{\mu I}(A)\subseteq A$. □

Example 5.10. *In (X,* μ_I*), Let* $X = \{\, a,b,c\}$*, we have*
μ_I=$\{\ \langle X,\phi,X\rangle\ ,\langle X,\{\ c\}\ ,\{\ a,b\}\ \rangle,\langle X,\{\ b\}\ ,\{\ c\}\ \rangle, \langle X,\{\, b,c\}\,,\phi\rangle,\langle X,\{\, c\}\,,\phi\rangle,\langle X,\phi,\{\, c\}\,\rangle,\langle X,\{\, b\}\,,\{\, a\}\,\rangle, \langle X,\{\, b\}\,,\phi\rangle,\langle X,\{\, b,c\}\,,\{\, a\}\,\rangle\,\}$
$\mu_I - closed = \{\,\langle X,X,\phi\rangle\,,\langle X,\{\, a,b\}\,,\{\, c\}\,\rangle,\langle X,\{\, c\}\,,\{\, b\}\,\rangle, \langle X,\phi,\{\, b,c\}\,\rangle,\langle X,\phi,\{\, c\}\,\rangle\langle X,\{\, c\}\,,\phi\rangle, \langle X,\{\, a\}\,,\{\, b\}\,\rangle,\langle X,\phi,\{\, b\}\,\rangle,\langle X,\{\, a\}\,,\{\, b,c\}\,\rangle\,\}$.
$\mu_I\, g - closed = \{\,\langle X,\phi,X\rangle\,,\,\langle X,\phi,\phi\rangle,\langle X,\phi,\{\, b\}\,\rangle,\langle X,\phi,\{\, c\}\,\rangle, \langle X,\phi,\{\, b,c\}\,\rangle,\langle X,\{\, a\}\,,\{\, b,c\}\,\rangle,\langle X,\{\, a\}\,,\phi\rangle, \langle X,\{\, a,b\}\,,\phi\rangle,\langle X,\{\, a\}\,,\{\, b\}\,\rangle,\langle X,\{\, a\}\,,\{\, c\}\,\rangle,\langle X,\{\, c\}\,,\{\, b\}\,\rangle, \langle X,\{\, c\}\,,\phi\rangle,\langle X,\{\, a,b\}\,,\{\, c\}\,\rangle,\langle X,\{\, c,a\}\,,\phi\rangle,\langle X,\{\, c,a\}\,,\{\, b\}\,\rangle \}$.

Let $A = \langle X,\phi,\{\, b\}\,\rangle, i_{\mu I}(A) = \underset{\sim}{\phi}, i^*_{\mu I}(A) = \langle X,\phi,\{\, a,b\}\,\rangle$.
Therefore $i_{\mu I}(A)\subset i^*_{\mu I}(A)\subset A$.

Let $A = \langle X,\{\, b\}\,,\{\, a\}\,\rangle, i_{\mu I}(A) = \langle X,\{\, b\}\,,\{\, a\}\,\rangle, i^*_{\mu I}(A) = \langle X,\{\, b\}\,,\{\, a\}\,\rangle$. *Therefore* $i_{\mu I}(A) = i^*_{\mu I}(A) = A$.

Axiom 5.11. *Let If* (X,μ_I) *be a GITS and A is a subset of* $\underset{\sim}{X}$*. Afterwards the subsequent statements are hold*

i) $c^*_{\mu I}(\bar{A}) = \overline{i^*_{\mu I}(A)}$

ii) $\overline{c^*_{\mu I}(A)} = i^*_{\mu I}(\bar{A})$

iii) $\overline{c^*_{\mu I}(\bar{A})} = i^*_{\mu I}(A)$

iv) $c^*_{\mu I}(A) = \overline{i^*_{\mu I}(\bar{A})}$.

Proof. i) Let $x\ \in\ c^*_{\mu I}(\bar{A})$. Then $x\ \in\ \cap F$, F is $\mu_I g -$ closed set and $\bar{A}\subseteq F$, which implies $x\ \in\ F$, for all μ_I g $-$ closed set F such that $\bar{A}\subseteq F$. Therefore $x\notin X - F$, for some μ_I g-open set X-F such that $X-F\subseteq A$. Then $x\ \notin\ i^*_{\mu I}(A)$ and hence $x\ \in\ \overline{i^*_{\mu I}(A)}$. Which implies $c^*_{\mu I}(\bar{A})\subseteq\overline{i^*_{\mu I}(A)}$. Suppose $x\notin c^*_{\mu I}(\bar{A})$, then $x\ \notin\cap F$, F is μ_I g $-$ closed set and $(\bar{A})\subseteq F$, which implies $x\ \notin\ F$, for some $\mu_I g -$ closed set contains $\bar{A}$. Therefore $x\in X-F$, for some μ_I g-open set X-F such that $X-F\subseteq A$ and hence $x\ \in i^*_{\mu I}(A)$ which implies $x\ \notin\ \overline{i^*_{\mu I}(A)}$. Then $\overline{i^*_{\mu I}(A)}\subseteq c^*_{\mu I}(\bar{A})$ and we get a result.

ii) Proof is similar to i).

iii) Following by taking complements in i)

iv) Replacing A by $\bar{A}$ in i). □

6. Some operators in intuitionistic topological space

Theorem 6.1. *The operators* $\cup$ *and* $\cap$ *satisfies associativity and De Morgan's laws*

i) $(A\cup B)\cup C = A\cup(B\cup C)$

ii) $(A\cap B)\cap C = A\cap(B\cap C)$

iii) $\overline{\bar{A}\cap\bar{B}} = A\cup B$

iv) $\overline{\bar{A}\cup\bar{B}} = A\cap B$.

Proof. Let $A = \langle X,A_1,A_2\rangle, B = \langle X,B_1,B_2\rangle, C = \langle X,C_1,C_2\rangle$

i) Now,

$$\begin{aligned}(A\cup B)\cup C &= \langle X,A_1\cup B_1,A_2\cap B_2\rangle\cup C\\ &= \langle X,(A_1\cup B_1)\cup C_1,(A_2\cap B_2)\cap C_2\rangle\\ &= \langle X,A_1\cup(B_1\cup C_1),A_2\cap(B_2\cap C_2)\rangle\\ &= A\cup(B\cup C)\end{aligned}$$

ii) Proof is similar to i).

iii) Now

$$\begin{aligned}\overline{\bar{A}\cap\bar{B}} &= \overline{\langle X,A_2\cap B_2,A_1\cup B_1\rangle}\\ &= \langle X,A_1\cup B_1,A_2\cap B_2\rangle\\ &= A\cup B.\end{aligned}$$

iv) Proof is similar to iii). □

Based on this property, some more properties on closure and interior operators will be implemented in future papers.

7. Conclusion

We conclude that the collection of intuitionistic generalized closed sets in GITS does not form the generalized intuitionistic topology and intuitionistic topology. The characterization of some more properties are in future process.

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