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Edge irregular neutrosophic soft graphs

G. Upender Reddy*

Abstract

This present article, author deduce an explanation of the neutrosophic soft graphs (NSG) w.r.t. a neighborly edge irregular as well as neighborly edge totally irregular NSG. The results based on the neutrosophic soft graphs with a constant function to evaluate a neighborly edge irregular as well as totally irregular on edge neighborly NSG.

Abbreviation

1. NS: Neutrosophic set, 2. SVN: Single valued neutrosophic, 3. IFS: intuitionistic fuzzy sets, 4. NSS: neutrosophic soft set, 5. NSG: neutrosophic soft graph

Keywords

Neutrosophic Soft graph, irregular on neighborly edge and totally irregular on neighborly edge.

AMS Subject Classification

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Department of Mathematics, Nizam College(A), Osmania University, Basheerbagh, Hyderabad–500001, Telangana, India.

***Corresponding author**: yuviganga@gmail.com

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Contents

1. Introduction

The neutrosophic sets launch by Smarandache [10, 11] is a great exact implement for the situation uncertainty in the real world. This uncertainty idea comes from the theories of fuzzy Theory [5], IFS [2, 4] and interval valued IFS [3]. The representation of the neutrosophic values are truth, indeterminacy and falsity value. These T, I, F values belongs to standard or nonstandard unit interval denoted by $]-0,1+[$ [6, 9].

The idea of subclass in the NS and SVNS derived by Wang et al. [12]. The idea of SVNS initiation by IFS [1, 7], in this the functions Truth value, Indeterminacy value, Falsity values are independent and these values are present within $[0,1]$ [8]. Neutrosophic theory is widely expands in all fields especially authors discoursed about topology with respect to neutrosophic [13].

Graph theory has at this time turn into a most important branch of mathematics. It is the division of combinatory. The Graph is a extensively important to analyze combinatorial complication in dissimilar areas in mathematics, optimization and computer science. Mainly significant object is well-known. The uncertainty on the subject of vertices and edges or both representations to become a neutrosophic concept.

2. Preliminaries

Definition 2.1 (SVN set)**.** *A SVN set is explained as the membership functions represented as a triplet set in W is denoted by $\{< w,T,I,F >: w \in W\}$, these functions are mapping from W to $[0,1]$. Where T denote truth membership, I denote indeterminate value and F denote false value of W.*

Example 2.2. *Let $W=\{w_1,w_2,w_3\}$ and $A=\{< w_1,0.3,0.2,0.7 >,< w_2,0.5,0.3,0.1 >,< w_3,0.8,0.05,0.4 >\}$ is a SVN set in W.*

Definition 2.3 (SVN relation on W)**.** *Let W be a non-empty set. Then we call mapping $Z=(W,T,I,F)$, $F(w): W\times W \to [0,1]\times[0,1]$, is a SVN relation on W such that $T_z(w_1,W_2)\in[0,1]$, $I_z(w_1,w_2)\in[0,1]$, $F_z(w_1,W_2)\in[0,1]$.*

Definition 2.4. *Let $Z_1=(T_{z_1},I_{z_1},F_{z_1})$ and $Z_2=(T_{z_2},I_{z_2},F_{z_2})$ be a SVN graphs on a set W. If Z_2 is a SVN relation on Z_1, then $T_{z_2}(w_1,w_2)\le \min(T_{z_1}(w_1),T_{z_1}(w_2)$, $I_{z_2}(w_1,w_2)\ge \max(I_{z_1}(w_1),I_{z_1}(w_2))$, $F_{z_2}(w_1,w_2)\ge \max(F_{z_1}(w_1),F_{z_1}(w_2))$, for all $w_1,w_2\in W$.*

Definition 2.5. *The symmetric property defined on SVN relation Z on W is explained by $T_z(w_1,w_2)=T_z(w_2,w_1)$, $I_z(w_1,w_2)=I_z(w_2,w_1)$, $F_z(w_1,w_2)=F_z(w_2,w_1)$.*

Definition 2.6 (SVN Graph)**.** *The new graph in SVN is denoted by $G^* = (V,E)$ is a pair $G = (Z_1, Z_2)$, where $Z_1 = (T_{z_1}, I_{z_1}, F_{z_1})$ is a BSVNS in V and $Z_2 = (T_{z_2}, I_{z_2}, F_{z_2})$ is SVNS in V^2 defined as $T_{z_2}(w_1,w_2) \leq \min(T_{z_1}(w_1), T_{z_1}(w_2)$, $I_{z_2}(w_1,w_2) \geq \max(I_{z_1}(w_1), I_{z_1}(w_2))$, $F_{z_2}(w_1,w_2) \geq \max(F_{z_1}(w_1), F_{z_1}(w_2))$, for all $w_1, w_2 \in V$. SVNSG of an edge denoted by $w_1w_2 \in V^2$.*

Definition 2.7. *Let $G = (Z_1, Z_2)$ be a SVNSG and $a, b \in V$. A path $P : a = w_0, w_1, w_2 \ldots, w_{k-1}, w_k = b$ in G is sequence of distinct vertices such that $(T_s(w_{m-1}, w_m) > 0)$, $(I_s(w_{m-1}, w_m) > 0)$, $(F_s(w_{m-1}, w_m) > 0)$, $m = 1, 2, \ldots, k$ and length of the path is k, here a is said to be initial vertex and b is terminal vertex in the path.*

Definition 2.8. *Let μ be the universal and $N(\mu)$ be the neutrosophic Universal. X be the variables that indicate the members of μ and $A \subseteq X$. A two of a kind (T, A) is the NSS over μ, here T is a function $T : A \to N(\mu)$. In the NSS (T, A) is varies given by $\{S(e_k), k = 1, 2, 3, e \in A\}$.*

Definition 2.9. *Let X_1, $X_2 \in X$, (F_1, X_1), (F_2, X_2) are two NSS over μ then (F_1, X_1) is to be a neutrosophic soft sub set of (F_2, X_2) if*

(i) $X_1 \subseteq X_2$

(ii) $T_{F_1(e)}(x) \leq T_{F_2(e)}(x)$, $I_{F_1(e)}(x) \leq I_{F_2(e)}(x)$, $F_{F_1(e)}(x) \leq F_{F_2(e)}(x)$, *for all* $e \in X_1$, $x \in \mu$.

Thus, $(F_1, X_1) \subseteq (F_2, X_2)$

Definition 2.10. *Suppose (F_1, X_1) and (F_2, X_2) are two NSS to be equal if (F_1, X_1) is a NS contained in (F_2, X_2) and (F_2, X_2) is a NS contained in (F_1, X_1) vise versa then $(F_1, X_1) = (F_2, X_2)$.*

Definition 2.11. *Let μ be an universe, K be the set of variables.*

(a) (F_1, K) *is to be a relative complete NSS (with respect to the variable set K), represented by f_K, if $T_{F_1(e)} = 1$, $I_{F_1(e)} = 1$, $F_{F_1(e)} = 0$ for all $e \in K$, $x \in \mu$.*

(b) (F_2, K) *is to be a relative void NSS (with respect to the variable set K), represented by f_K, if $T_{F_2(e)} = 0$, $I_{F_2(e)} = 0$, $F_{F_2(e)} = 1$ for all $e \in K$, $x \in \mu$.*

The relative complete NSS with respect to the set of variables K is known as the complete NSS over μ and notated by μ_A. For comparable method the relative null NSS with respect to K is the null NSS over μ and is notated by f_K.

Definition 2.12. *Let G^* be a graph and K be the set of variables. Consider $N(V)$ be the set of all NS in V. The NSG, means 4-tuple $G_N = (G^*, A, F_1, F_2)$, here $F_1 : K \to N_s(V)$, $F_2 : K \to N_s(V \times V)$ it gives $F_1(e) = F_{1e} = \{< w, T_{F_{1e}}(w), I_{F_{1e}}(w), F_{F_{1e}}(w) >: w \in V\}$ and $F_2(e) = F_{2e} = \{< (w_1,w_2), T_{F_{2e}}(w_1,w_2), I_{F_{2e}}(w_1,w_2), F_{F_{2e}}(w_1,w_2) >: (w_1,w_2) \in V \times V\}$ are NS over V and $V \times V$ correspondingly, such that $T_{F_{2e}}(w_1,w_2) \leq \min\{T_{F_{1e}}(w_1), T_{F_{1e}}(w_2)\}$, $I_{F_{2e}}(w_1,w_2) \leq \min\{I_{fe}(w_1), T_{fe}(w_2)\}$, $F_{F_{2e}}(w_1,w_2) \geq \max\{F_{F_{1e}}(w_1), F_{F_{1e}}(w_2)\}$ for all $(w_1,w_2) \in V \times V$ and $e \in K$. Also represented a NSG by $G_N = (G^*, A, F_1, F_2) = \{N(e) : e \in K\}$ which is a varied class of graphs $N(e)$, we call it as NSG.*

Example 2.13. *Let G^* be a graph with vertex set $V = \{a, b, c\}$ selection of edges $A = \{e_1, e_2, e_3\}$. A NSG is followed by in **Table.1** and $T_{F_2e}(w_i, w_j) = I_{F_2e}(w_i, w_j) = 0$ and $F_{F_2e}(w_i, w_j) = 1$ for all $(w_i, w_j) \in V \times V$ $\{(w_1,w_2), (w_2,w_3), (w_3,w_1)\}$ and every $e \in A$.*

Table 1

f	a	B	c
e_1	(0.3, 0.5, 0.7)	(0.4, 0.6, 0.8)	(0, 0, 1)
e_2	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.6)	(0.1, 0.3, 0.4)
e_3	(0.1, 0.2, 0.4)	(0.2, 0.4, 0.6)	(0.3, 0.5, 0.7)
g	(x_1, x_2)	(x_2, x_3)	(x_3, x_1)
e_1	(0.2, 0.4, 0.9)	(0, 0, 1)	(0, 0, 1)
e_2	(0.1, 0.1, 0.7)	(0, 0, 1)	(0.1, 0.1, 0.6)
e_3	(0.1, 0.2, 0.7)	(0.1, 0.2, 0.8)	(0.2, 0.3, 0.8)

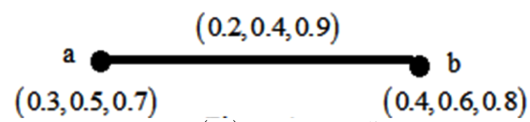

Figure 1. $N(e_1)$ Corresponding to e_1

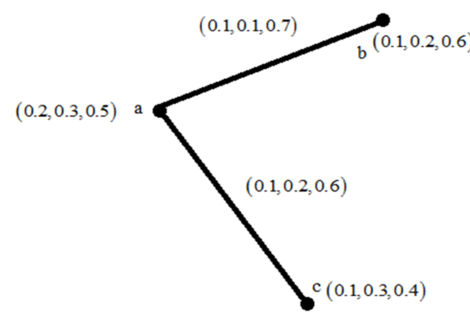

Figure 2. $N(e_2)$ Corresponding to e_2

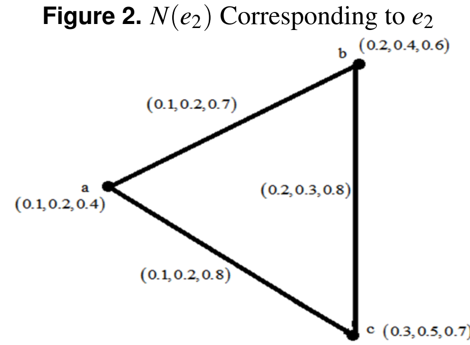

Figure 3. $N(e_3)$ Corresponding to e_3

Definition 2.14. *A NSG $G = (G^*, K^1, F_1^1, F_2^2)$ is a neutrosophic soft sub graph of $G = (G^*, K, F_1, F_2)$ if*

(i) $K^1 \subseteq K$

(ii) $F^1_{1e} \subseteq F_1$, *which gives* $T_{F^1_{1e}}(w) \leq T_{F_{1e}}(w)$, $I_{F^1_{1e}}(w) \leq I_{F_{1e}}(w)$, $F_{F^1_{1e}}(w) \geq F_{F_{1e}}(w)$

(iii) $F^1_{2e} \subseteq F_2$, *which gives* $T_{F^1_{2e}}(w_1, w_2) \leq T_{F_{2e}}(w_1, w_2)$, $I_{F^1_{2e}}(w_1, w_2) \leq I_{F_{2e}}(w_1, w_2)$, $F_{F^1_{2e}}(w_1, w_2) \geq F_{F_{2e}}(w_1, w_2)$ *for every* $e \in K^1$.

Definition 2.15. *Let* $G_N = (G^*, A, F_1, F_2)$ *be an NSS of* G^*. *If* $H(e)$ *is a neighborly edge irregular NSG for all* $e \in A$ *then G is the neighborly edge irregular NSG. Consistently, if any two neighboring edges have different degrees in* $H(e)$ *for arbitrary* $e \in A$ *then a NSG G is a irregular on neighborly edge.*

Definition 2.16. *Let* $G_N = (G^*, A, F_1, F_2)$ *be a NSG of* G^*. *The neighborly edge totally irregular NG* $H(e)$ *for all* $e \in A$ *then* G_N *is a totally irregular on neighborly edge NSG. Consistently, if any two neighboring edges have different total degrees in* $H(e)$ *for all* $e \in A$ *then a NSG G is the totally irregular on neighborly edge NSG.*

Theorem 2.17. *Consider* $G_N = (G^*, A, F_1, F_2)$ *be NSG of* G^* *and* F_2 *is a constant function. If G is a neighborly edge irregular (totally irregular on neighborly edge) NSG, then G is totally irregular on neighborly edge (irregular on neighborly edge) NSG.*

Proof. Let us F_2 is a constant function, $F_{2e_i}(w_1w_2) = (k_i, k_i^1)$ for all, $w_1w_2 \in V \times V$, $e_i \in A$, where k_i and k_i^1 are constants where $i = 1, 2, \ldots, k$. Let w_1w_2 and w_2w_3 be pair of adjacent edges in A. Assume G is a neighborly edge irregular NSG. Then $deg_G(w_1w_2)(e_i) \neq deg_G(w_2w_3)(e_i)$ for every $e_i \in A$, this gives

$$
\begin{aligned}
&(deg_\mu(w_1w_2)(e_i), deg_\nu(w_1w_2)(e_i)) \\
&\qquad \neq (deg_\mu(w_2w_3)(e_i), deg_\nu(w_2w_3)(e_i)) \\
&(deg_\mu(w_1w_2)(e_i), deg_\nu(w_1w_2)(e_i)) + (c_i, c_i') \\
&\qquad \neq (deg_\mu(w_2w_3)(e_i), deg_\nu(w_2w_3)(e_i)) + (k_i, k_i') \\
&deg_G(w_1w_2)(e_i) + F_2(w_1w_2)(e_i) \\
&\qquad \neq deg_G(w_2w_3)(e_i) + F_2(w_2w_3)(e_i) \\
&tdeg_G(w_1w_2)(e_i) \neq tdeg_G(w_2w_3)(e_i)
\end{aligned}
$$

where w_1w_2 and w_2w_3 are adjacent edges in A. Hence, G is a neighborly edge totally irregular NSG. □

Theorem 2.18. *Let* $G_N = (G^*, A, F_1, F_2)$ *be connected NSG on* G^* *and* F_2 *is a constant function. If G is a neighborly edge totally irregular NSG, then G is neighborly edge irregular NSG.*

Remark 2.19. *Let* $G_N = (G^*, A, F_1, F_2)$ *be connected NSG on* G^* *and* F_2 *is a constant function. If G is both a neighborly edge irregular NSG and neighborly edge totally irregular NSG Then* F_2 *not required be a constant function.*

Theorem 2.20. *Let* $G_N = (G^*, A, F_1, F_2)$ *be connected NSG of* G^* *and* F_2 *is a constant function. If G is a neighborly edge irregular NSG then G is an irregular NSG.*

Proof. Let G be connected NSG of G^* and F_2 is a constant function. $F_{2e_i}(w_1w_2) = (k_i, k_i^1)$, where k_i and k_i^1 are constants. Assume that G is a neighborly edge irregular NSG. Consider w_1w_2 and w_2w_3 are two adjacent edges in G with different degrees,

$$
\begin{aligned}
&(deg_\mu(w_1w_2)(e_i), deg_\nu(w_1w_2)(e_i)) \\
&\qquad \neq (deg_\mu(w_2w_3)(e_i), deg_\nu(w_2w_3)(e_i)) \\
&deg_\mu(w_1w_2)(e_i) \neq deg_\mu(w_2w_3)(e_i) \text{ or} \\
&\qquad deg_\nu(w_1w_2)(e_i) \neq deg_\nu(w_2w_3)(e_i) \\
&deg_\mu(w_1)(e_i) + deg_\mu(w_2)(e_i) - 2k_i \\
&\qquad \neq deg_\mu(w_2)(e_i) + deg_\mu(w_3)(e_i) - 2k_i \text{ or} \\
&deg_\nu(w_1)(e_i) + deg_\nu(w_2)(e_i) - 2k_i \\
&\qquad \neq deg_\nu(w_2)(e_i) + deg_\nu(w_3)(e_i) - 2k_i \\
&deg_\mu(w_1)(e_i) \neq deg_\mu(w_3)(e_i) \text{ or} \\
&\qquad deg_\nu(w_1)(e_i) \neq deg_\nu(w_3)(e_i) \\
&(deg_\mu(w_1)(e_i), deg_\nu(w_1)(e_i)) \\
&\qquad \neq (deg_\mu(w_3)(e_i), deg_\nu(w_3)(e_i)) \\
&deg_G(w_1)(e_i) \neq deg_G(w_3)(e_i)
\end{aligned}
$$

Hence, there exist w_2 a vertex which is adjacent to the vertices w_2 and w_3 have different degree. Hence, G is an irregular NSG. □

Theorem 2.21. *Let* $G_N = (G^*, A, F_1, F_2)$ *be connected NSG on* G^* *and* F_2 *is a constant function. If G is a neighborly edge totally irregular NSG, then G is an irregular NSG.*

Theorem 2.22. *Let* $G_N = (G^*, A, F_1, F_2)$ *be connected NSG on* G^* *and* F_2 *is a constant function. Then G is a neighborly edge irregular NSG iff G is extremely irregular NSG.*

Proof. Let G be connected NSG of G^* and F_2 is a constant function. $F_{2e_i}(w_1w_2)(e_i) = (k_i, k_i^1)$, for every, $w_1w_2 \in A$, where k_i and k_i^1 are constants. Let w_1 be the vertex adjacent with w_2, w_3 and t. $w_1w_2w_3$ and w_1t are adjacent edges in G. Assume G is a neighborly edge irregular NSG, gives that every pair of adjacent edges in G with different degrees, then

$$
\begin{aligned}
&deg_G(w_2w_1)(e_i) \neq deg_G(w_1w_3)(e_i) \neq deg_G(w_1t)(e_i) \\
&(deg_\mu(w_2w_1)(e_i), deg_\nu(w_2w_1)(e_i)) \\
&\qquad \neq (deg_\mu(w_1w_3)(e_i), deg_\nu(w_1w_3)(e_i)) \\
&\qquad \neq (deg_\mu(w_1t)(e_i), deg_\nu(w_1t)(e_i))
\end{aligned}
$$

Consider

$$
\begin{aligned}
&(deg_\mu(w_2w_1)(e_i), deg_\nu(w_2w_1)(e_i)) \\
&\qquad \neq (deg_\mu(w_1w_3)(e_i), deg_\nu(w_1w_3)(e_i)) \\
&deg_\mu(w_2w_1)(e_i) \neq deg_\mu(w_1w_3)(e_i) \text{ or} \\
&\qquad deg_\nu(w_2w_1)(e_i) \neq deg_\nu(w_1w_3)(e_i) \\
&deg_\mu(w_1)(e_i) + deg_\mu(w_2)(e_i) - 2k_i \\
&\qquad \neq deg_\mu(w_1)(e_i) + deg_\mu(w_3)(e_i) - 2k_i \text{ or} \\
&deg_\nu(w_1)(e_i) + deg_\nu(w_2)(e_i) - 2k_i \\
&\qquad \neq deg_\nu(w_1)(e_i) + deg_\nu(w_3)(e_i) - 2k_i
\end{aligned}
$$

$$deg_{\mu}(w_2)(e_i) \neq deg_{\mu}(w_3)(e_i) \text{ or}$$
$$\neq deg_{v}(w_2)(e_i) \neq deg_{v}(w_3)(e_i)$$
$$deg_{\mu}(w_2)(e_i), deg_{v}(w_2)(e_i)$$
$$\neq deg_{\mu}(w_3)(e_i) \neq deg_{v}(w_3)(e_i)$$
$$deg_G(w_2) \neq deg_G(w_3).$$

In the same way, $deg_G(w_3) \neq deg_G(t) \Rightarrow deg_G(w_2) \neq deg_G(w_3) \neq deg_G(t)$, obviously, any vertex w_1 is adjacent to the vertices w_2, w_3 and t with different degrees. Hence G is extremely irregular NSG.

Conversely, let w_2w_1 and w_1w_3 are arbitrarily two adjacent edges in G. Assume that G is extremely irregular NSG, then any vertex adjacent to the vertices in $H(e_i)$ for every $e_i \in A$ contains different degrees, such that $deg_G(w_2) \neq deg_G(w_3)$

$$deg_{\mu}(w_2)(e_i) + deg_{\mu}(w_1)(e_i) - 2k_i$$
$$\neq deg_{\mu}(w_3)(e_i) + deg_{\mu}(w_1)(e_i) - 2k_i \text{ or}$$
$$deg_{v}(w_2)(e_i) + deg_{v}(w_1)(e_i) - 2k_i$$
$$\neq deg_{v}(w_3)(e_i) + deg_{v}(w_1)(e_i) - 2k_i$$
$$deg_{\mu}(w_2w_1)(e_i) \neq deg_{\mu}(w_1w_3)(e_i) \text{ or}$$
$$deg_{v}(w_1w_2)(e_i) \neq deg_{v}(w_1w_3)(e_i)$$
$$(deg_{\mu}(w_1w_2)(e_i), deg_{v}(w_1w_2)(e_i))$$
$$\neq (deg_{\mu}(w_1w_3)(e_i), deg_{v}(w_1w_2)(e_i))$$
$$= deg_G(w_1w_2) \neq deg_G(w_1w_3)$$

Hence G is a neighborly edge irregular NSG. □

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