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Characterization of j-open sets in minimal structure spaces

M. Deepa[1]* and S.S. Surekha[2]

Abstract

The aim of this paper is to introduce the notion of m-j open sets in minimal structure spaces together with its corresponding interior and closure operators. Furthermore, we investigated the basic properties of the sets and studied their relationship with other existing sets. Moreover, the minimal structure properties of j-border, j-frontier and j-exterior of the sets have been discussed.

Keywords

m-j open;m-j closed;m-j border;m-j frontier and m-j exterior.

AMS Subject Classification

54BXX, 54FXX.

[1,2] *Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore-641004, Tamil Nadu, India.*

***Corresponding author**: [1] mdeepa@psgrkcw.ac.in [2]surekhasuresh97@gmail.com

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1. Introduction

Popa and Noiri [12] introduced the concepts of minimal structure (briefly m-structure) in 2000. Also they studied the notion of m_x-open and m_x-closed sets and distinguish the sets using m_x-closure and m_x-interior operators respectively. The space which contains the mother set X and the null set $\emptyset$ is said to be minimal structure. In 2009, W.K.Min and Y.K. Kim introduced Semiopen[7], Preopen[8] and α-open[9] in minimal structure. In 2012, I.Arockiarani and D.Sasikala[2] introduced the notion of j-open sets. In [6], Caldas studied the properties of α-border, α-frontier and α-exterior of α-open sets in topological space. In this paper, we introduced and studied the minimal structure properties of m-j border, m-j frontier and m-j exterior of the j-open sets. Also, we discussed some of the basic properties of m-j open sets in minimal structure.

2. Preliminaries

Definition 2.1. [12] *Let X be a non-empty set and let $m_x \subseteq P(X)$ denotes the power set of X. We say that m_x is an minimal structure on X if $\emptyset$ and X belongs to the minimal structure m_x. The sets which are in minimal structure are called m-open sets and the complement of the m-open sets are called m-closed sets.*

Definition 2.2. [12] *Let A be the subset of X. The m_x-interior and m_x-closure of the subset A is defined by*
i)m_x-$Int(A) = \cup\{U : U \in m_x; U \subseteq A\}$;where U is an open set.
ii)m_x-$Cl(A) = \cap\{F - A \subseteq F; X - F \in m_x\}$,where F is a closed set.

Definition 2.3. *Let X be the non-empty set and A be the subset of X. Then A is called*
i)m-semi open [7] *if $A \subseteq m_x$-$Cl(m_x$-$Int(A))$*
ii)m-preopen [8] *if $A \subseteq m_x$-$Int(m_x$-$Cl(A))$*
iii)m-b open [10] *if $A \subseteq m_x$-$Int(m_x$-$Cl(A)) \cup m_x$-$Cl(m_x$-$Int(A))$*
iv)m-α open [9] *if $A \subseteq m_x$-$Int(m_x$-$Cl(m_x$-$Int(A)))$*

Lemma 2.4. *Consider a minimal structure (X, m_x). For the subsets A and B the following conditions are true:*
i)m_x-$Cl(X - A) = X - m_x$-$Int(A)$ and m_x-$Int(X - A) = m_x$-$Cl(A)$
ii)If $(X - A) \in m_x$,then m_x-$Cl(A) = A$ and if $A \in m_x$,then m_x-$Int(A) = A$
iii)m_x-$Cl(\emptyset) = \emptyset, m_x$-$Cl(X) = X, m_x$-$Int(\emptyset) = \emptyset, m_x$-$Int(X) = X$

iv)If $A \subset B$, then m_x-$Cl(A) \subset m_x$-$Cl(B)$ and m_x-$Int(A) \subset m_x$-$Int(B)$
v)$A \subset m_x - Cl(A)$ and $m_x - Int(A) \subset A$
vi)m_x-$Cl(m_x$-$Cl(A)) = m_x$-$Cl(A)$

Definition 2.5. [12] *Let (X, m_x) be the minimal structure and A be the subset of X. A point $x \in X$ is called a limit point of A if and only if every neighborhood of A contains a point of A other than X and it is denoted by $D(A)$.*

3. m-j open sets

Definition 3.1. *Let (X, m_x) be the minimal structure. Let A be the subset of X. The subset A is called the m-j open set if $A \subseteq m_x$-$Int(m_x$-$pcl(A))$.*

Example 3.2. *Consider $X = \{p, q, r\}$ with minimal structure $m_x = \{\emptyset, X, \{p\}, \{r\}\}$.Then its corresponding m-j open sets are $\emptyset, X, \{p\}, \{q\}, \{r\}, \{p, r\}$.*

Definition 3.3. *Let (X, m_x) be the minimal structure with subset A of X. Then,*
i)The m_x-$Int_j(A)$ is the union of all the m-j open sets contained in A.
ii)The m_x-$Cl_j(A)$ is the intersection of all the m-j closed sets containing A.

Theorem 3.4. *Every m-open sets are m-j open sets.*

Proof. Let (X, m_x) be the minimal structure. Let P be an m-open set, then we have $P = m_x$-$Int(P)$. Also we know that $P \subseteq m_x$-$pcl(P)$. Hence we get $P \subseteq m_x$-$Int(m_x$-$pcl(P)$. Therefore, P is m-j open set.

The converse of the above theorem is not true by the following example: Consider $X = \{p, q, r\}$ with minimal structure $m_x = \{\emptyset, X, \{p\}, \{q\}, \{r\}, \{p, r\}\}$. The set $A = \{p, r\}$ is m-j open but not m-open set. □

Theorem 3.5. *For any subsets A and B with minimal structure the following conditions are true. i)The m_x-$Int_j(A)$ is the largest m-j open set contained in A.*
ii)A is an m-j open set if and only if $A = m_x$-$Int_j(A)$.
iii)$A \subseteq B$, then m_x-$Int_j(A) \subseteq m_x$-$Int_j(B)$.
iv)m_x-$Int_j(\emptyset) = \emptyset$.
v)m_x-$Int_j(X) = X$.
vi)m_x-$Int_j(A \cap B) \subseteq m_x$-$Int_j(A) \cap m_x$-$Int_j(B)$.
vii)m_x-$Int_j(A) \cup m_x$-$Int_j(B) \subseteq m_x$-$Int_j(A \cup B)$.

Proof. i)Since, m_x-$Int_j(A) = \cup\{G : G$ is m-j open, $G \subset A\}$. Hence it contains every m-j open subset G of A. Therefore, m_x-$Int_j(A)$ is the largest m-j open set contained in A.
ii)Let $A = m_x$-$Int_j(A)$. Since m_x-$Int_j(A)$ is an m-j open set, A is also an m-j open set. Conversely, let A be any m-j open set and also it is the largest m-j open set. Hence we have $A = m_x$-$Int_j(A)$.
iii)Let $A \subseteq B$. We have to prove that m_x-$Int_j(A) \subseteq m_x$-$Int_j(B)$. Let $x \in m_x$-$Int_j(A)$. Then there exists an open set G such that $x \in G \subset A$. Since $A \subseteq B$, G is also contained in B. Hence $x \in G \subset B$ which implies that $x \in m_x$-$Int_j(B)$. Hence m_x-$Int_j(A) \subseteq m_x$-$Int_j(B)$.
vi)Let $x \in m_x$-$Int_j(A \cap B)$. Hence there exist a open set G in X such that $x \in A \cap B$ which implies that $x \in A$ and $x \in B$. Therefore, $x \in G \subset A$ and $x \in G \subset B$. Hence $x \in m_x$-$Int_j(A)$ and $x \in m_x$-$Int_j(B)$. Hence m_x-$Int_j(A \cap B) \subseteq m_x$-$Int_j(A) \cap m_x$-$Int_j(B)$.
vii) Let $x \in m_x$-$Int_j(A) \cup m_x$-$Int_j(B)$. Hence $x \in m_x$-$Int_j(A)$ or $x \in m_x$-$Int_j(B)$. If $x \in m_x - Int_j(A)$, then there exists an open set G such that $x \in G \subset A$. If $x \in m_x - Int_j(B)$, then there exists an open set G such that $x \in G \subset B$. Hence $x \in G \subset A \cup B$. Therefore, m_x-$Int_j(A) \cup m_x$-$Int_j(B)$
$\subseteq m_x$-$Int_j(A \cup B)$. □

Theorem 3.6. *In minimal structure, the arbitrary union of m-j open sets is always an m-j open set.*

Proof. Let $A = \bigcup_i A_i$ be the arbitrary union of m-j open sets. Hence $A \subseteq m_x$-$Int(m_x$-$pcl(A))$ which implies $\bigcup_i A_i \subseteq m_x$-$Int(m_x$-$pcl(\bigcup_i A_i))$. Therefore, each A_i is m-j open. Hence the arbitrary union of m-j open is always m-j open.

□

Remark 3.7. *The intersection of any two m-j open sets need not be m-j open by the following example.*
Consider $X = \{p, q, r\}$ with minimal structure m_x given by $m_x = \{\emptyset, X, \{p\}, \{s\}, \{q, r\}, \{p, q\}, \{p, r, s\}\}$. The corresponding m-j open set is given by m-$jO(X) = \{\emptyset, X, \{p\}, \{s\}, \{p, q\}, \{q, r\}, \{p, s\}, \{p, q, r\}, \{p, r, s\}, \{p, q, s\}, \{q, r, s\}\}$. Here $\{p, q\} \cap \{q, r\} = \{q\}$ which is not in the m-j open set.

Theorem 3.8. *Every m-j open sets are m-preopen sets.*

Proof. Let A be the m-j open set. Hence $A \subseteq m_x$-$Int(m_x$-$pcl(A))$. Since $pcl(A) \subseteq Cl(A)$ we have $A \subseteq m_x$-$Int(m_x$-$Cl(A))$. Hence A is m-preopen. □

The converse of the above theorem is not true by the above example. m-$PO(X) = \{\emptyset, X, \{p\}, \{s\}, \{p, q\}, \{q, r\}, \{r, s\}, \{p, s\}\{p, r\}, \{q, s\}, \{p, q, r\}, \{p, r, s\}, \{p, q, s\}, \{q, r, s\}\}$. Here $A = \{r, s\}$ is m-preopen but not m-j open.

Theorem 3.9. *Every m-j open sets are m-b open sets.*

Proof. We have to show that A is m-b open. Since A is m-j open, we have A is m-preopen. That is $A \subseteq m_x$-$Int(m_x$-$Cl(A))$. Hence we have $A \subseteq m_x$-$Int(m_x$-$Cl(A)) \cup m_x$-$Cl(m_x$-$Int(A))$. Therefore, A is m-b open. □

The converse of the above theorem is not true by the following example. Consider $X = \{p, q, r\}$ with minimal structure m_x given by $m_x = \{\emptyset, X, \{p\}, \{q\}, \{p, r\}\}$ with m-$bO(X) = \{\emptyset, X, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, r\}\}$ and m-$jO(X) = \{\emptyset, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$. Here $A = \{r\}$ is m-b open but not m-j open.

4. m-j border,m-j frontier,m-j exterior of the sets

Definition 4.1. *In a minimal structure* (X,m_x), m_x-$b_j(A) = A\backslash m_x$-$Int_j(A)$ *is called the m-j border of A, where A is the subset of X.*

Example 4.2. *Consider the minimal structure* (X,m_x) *with* $X=\{p,q,r,s\}$ *and* $m_x=\{\emptyset,X,\{p\},\{s\},\{q,r\},\{p,q\},\{p,r,s\}\}$. *And the corresponding m-j open set is given by* m-$jO(X) = \{\emptyset,X,\{p\},\{s\},\{p,q\},\{q,r\},\{p,s\},\{p,q,r\},\{p,r,s\},\{p,q,s\},\{q,r,s\}\}$. *Let* $A=\{r,s\}$ *then* m_x-$b_j(A)=\{r\}$.

Theorem 4.3. *Consider the minimal structures* (X,m_x) *with the subset A of X, the following conditions are true:*

i) m_x-$b_j(A)\subseteq m_x$-$b(A)$

ii)m_x-$Int_j(A)\cup m_x$-$b_j(A)=A$

iii) m_x-$Int_j(A)\cap m_x$-$b_j(A)=\emptyset$

iv) A is an m-j open set if and only if m_x-$b_j(A)=\emptyset$

v) m_x-$b_j(m_x$-$Int_j(A))=\emptyset$

vi) m_x-$Int_j(m_x$-$b_j(A))=\emptyset$

vii) m_x-$b_j(m_x$-$b_j(A))=m_x$-$b_j(A)$

viii) m_x-$b_j(A)=A\cap m_x$-$Cl_j(X-A)$

ix) m_x-$b_j(A)=m_x$-$D_j(X-A)$

Proof. i) m_x-$b_j(A)=A\backslash m_x$-$Int_j(A)\subseteq A\backslash m_x$-$Int(A)\subseteq m_x$-$b(A)$.
ii)m_x-$Int_j(A)\cup m_x$-$b_j(A)=m_x$-$Int_j(A)\cup(A\backslash m_x$-$Int_j(A))=A$.
iii) m_x-$Int_j(A)\cap m_x$-$b_j(A)=m_x$-$Int_j(A)\cap(A\backslash m_x$-$Int_j(A))=\emptyset$.
iv) A is an m-j open set if and only if m_x-$Int_j(A)=A$ if and only if $A\backslash m_x$-$Int_j(A)=\emptyset$ if and only if m_x-$b_j(A)=\emptyset$.
v) By the definition of m-j border, we have m_x-$b_j(A)=A\backslash m_x$-$Int_j(A)$. When $A=m_x$-$Int_j(A)$ we have m_x-$b_j(A)=m_x$-$Int_j(A)\backslash m_x$-$Int_j(A)=\emptyset$.
vi) If $x\in m_x$-$Int_j(m_x$-$b_j(A))$ then $x\in m_x$-$b_j(A)$. On the other hand, since m_x-$b_j(A)\subset A$, we have $x\in m_x$-$Int_j(m_x$-$b_j(A))\subset m_x$-$Int_j(A)$. Hence $x\in m_x$-$Int_j(A)\cap m_x$-$b_j(A)$ which is a contradiction to (iii). Thus
m_x-$Int_j(m_x$-$b_j(A))=\emptyset$.
vii) Since m_x-$b_j(m_x$-$Int_j(A))=\emptyset$, we have m_x-$b_j(m_x$-$b_j(A))=m_x$-$b_j(A\backslash m_x$-$Int_j(A))=m_x$-$b_j(A)$.
viii)m_x-$b_j(A)=A\backslash m_x$-$Int_j(A)=A\backslash(X\backslash m_x$-$Cl_j(X-A))=A\cap m_x$-$Cl_j(X-A)$.
ix) m_x-$b_j(A)=A\backslash m_x$-$Int_j(A)=A\backslash(A\backslash m_x$-$D_j(X-A))=m_x$-$D_j(A)$. □

Consider the Example 4.2. If $A=\{p,s\}$ then m_x-$b_j(A)=\emptyset$ and m_x-$b(A)=\{s\}$. Hence m_x-$b(A)\not\subset m_x$-$b_j(A)$. In general, the converse part of the theorem 4.3(i) may not be true.

Definition 4.4. *Let* (X,m_x) *be the minimal structure and A be the subset of X, then* m_x-$Fr_j(A)=m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A)$ *is said to be m-j frontier of A.*

Example 4.5. *Let* $X=\{a,b,c\}$ *with minimal structure* $m_x=\{\emptyset,X,\{a\},\{b\},\{b,c\},\{a,b\}\}$ *and the corresponding m-j open set is given by* m-$jO(X)=\{\emptyset,X,\{a,b\},\{b,c\},\{a\},\{b\}\}$. *Let* $A=\{a,b\}$ *then* m_x-$Fr_j(A)=\{c\}$.

Theorem 4.6. *Consider the minimal structures* (X,m_x) *with the subset A of X the following conditions are true:*

i) m_x-$Fr_j(A)\subseteq m_x$-$Fr(A)$ *where* $Fr(A)$ *denotes the frontier of A.*

ii) m_x-$Cl_j(A)=m_x$-$Int_j(A)\cup m_x$-$Fr_j(A)$.

iii) m_x-$Int_j(A)\cap m_x$-$Fr_j(A)=\emptyset$.

iv) m_x-$b_j(A)\subset m_x$-$Fr_j(A)$.

v) m_x-$Fr_j(A)=m_x$-$b_j(A)\cup m_x$-$D_j(A)$.

vi) A is an m-j open set if and only if m_x-$Fr_j(A)=m_x$-$D_j(A)$.

vii) m_x-$Fr_j(A)=m_x$-$Cl_j(A)\cap m_x$-$Cl_j(X-A)$.

viii) m_x-$Fr_j(A)=m_x$-$Fr_j(X-A)$.

ix) m_x-$Fr_j(A)$ *is m-j closed.*

x)m_x-$Fr_j(m_x$-$Fr_j(A))\subseteq m_x$-$Fr_j(A)$.

xi) m_x-$Fr_j(m_x$-$Int_j(A))\subseteq m_x$-$Fr_j(A)$.

xii) m_x-$Fr_j(m_x$-$Cl_j(A))\subseteq m_x$-$Fr_j(A)$.

xiii) m_x-$Int_j(A)=A\backslash m_x$-$Fr_j(A)$.

Proof. i)m_x-$Fr_j(A)=m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A)\subseteq m_x$-$Cl(A)\backslash m_x$-$Int(A)\subseteq m_x$-$Fr(A)$
ii)m_x-$Int_j(A)\cup m_x$-$Fr_j(A)=m_x$-$Int_j(A)\cup(m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A))=m_x$-$Cl_j(A)$
iii)It is similar to (ii) and also by the definition.
iv)m_x-$b_j(A)=A\backslash m_x$-$Int_j(A)\subseteq m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A)\subseteq m_x$-$Fr_j(A)$.
v)It follows from (ii) and from the fact that m_x-$Cl_j(A)=A\cup m_x$-$D_j(A)$.
vi)It follows from (v) and the fact that if A is an m-j open set if and only if m_x-$b_j(A)=\emptyset$.
vii)m_x-$Fr_j(A)=m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A)=m_x$-$Cl_j(A)\cap m_x$-$Cl_j(X-A)$.
viii)It is similar to (vii)and by the definition of m-j frontier of A.
ix)m_x-$Cl_j(m_x$-$Fr_j(A))=m_x$-$Cl_j(m_x$-$Cl_j(A)\cap m_x$-$Cl_j(X-A))=m_x$-$Cl_j(m_x$-$Cl_j(A))\cap m_x$-$Cl_j(m_x$-$Cl_j(X-A))=m_x$-$Fr_j(A)$.
x)m_x-$Fr_j(m_x$-$Fr_j(A))=m_x$-$Cl_j(m_x$-$Fr_j(A))\cap m_x$-$Cl_j(X\backslash m_x$-$Fr_j(A)\subseteq m_x$-$Cl_j(m_x$-$Fr_j(A))=m_x$-$Fr_j(A)$.
xi)It is similar to (x).
xii)m_x-$Fr_j(m_x$-$Cl_j(A))=m_x$-$Cl_j(m_x$-$Cl_j(A))\backslash m_x$-$Int_j(m_x$-$Cl_j(A))=m_x$-$Cl_j(A)\backslash m_x$-$Int_j(m_x$-$Cl_j(A))=m_x$-$Cl_j(A)\backslash m_x$-$Int_j(A)=m_x$-$Fr_j(A)$.
xiii)$A\backslash m_x$-$Fr_j(A)=A\ \ m_x$-$Cl_j(A)\ \ m_x$-$Int_j(A)=m_x$-$Int_j(A)$. □

The converse part of (i) and (iv) of the above theorem are not true by the following example. Consider $X=\{p,q,r,s\}$ with minimal structure given by $m_x=\{\emptyset,X,\{p\},\{s\},\{q,r\},\{p,q\},\{p,r,s\}\}$. The corresponding m-j open set is given by m-$jO(X)=\{\emptyset,X,\{p\},\{s\},\{p,q\},\{q,r\},\{p,s\},\{p,q,r\},\{p,r,s\},\{p,q,s\},\{q,r,s\}\}$. If $A=\{p,r\}$ then m_x-$Fr(A)=\{b,c\}\not\subset\{c\}=m_x$-$Fr_j(A)$.
Consider $X=\{p,q,r\}$ with minimal structure given by $m_x=\{\emptyset,X,\{p\},\{q\},\{q,r\},\{p,q\}\}$ and the corresponding m-j open set is given by m-$jO(X)=\{\emptyset,X,\{p,q\},\{q,r\},\{p\},\{q\}\}$.If $A=\{a,b\}$, then m_x-$Fr_j(A)=\{c\}$ and m_x-$b_j(A)=\emptyset$. Hence m_x-$Fr_j(A)\not\subset m_x$-$b_j(A)$.

Definition 4.7. *Let A be the subset of minimal structure* (X,m_x)*, then* m_x-$Ext_j(A) = m_x$-$Int_j(X\backslash A)$ *is said to be m-j exterior of A.*

Example 4.8. *Consider* $X = \{p,q,r\}$ *and* $m_x = \{\emptyset, X, \{p\}, \{r\}, \{p,r\}\}$ *and the corresponding m-j open set is given by m-*$jO(X) = \{\emptyset, X, \{p\}, \{q\}, \{r\}, \{p,q\}, \{p,r\}\}$*. Let* $A = \{q\}$*, then* m_x-$Ext_j(A) = \{p,r\}$.

Theorem 4.9. *For the subset A and B of X, the following conditions are true*

i)m_x-$Ext(A) \subseteq m_x$-$Ext_j(A)$ *where* $Ext(A)$ *denotes the exterior of A.*

ii) m_x-$Ext_j(A)$ *is open.*

iii) m_x-$Ext_j(A) = m_x$-$Int_j(X\backslash A) = X\backslash m_x$-$Cl_j(A)$.

iv) m_x-$Ext_j(m_x$-$Ext_j(A)) = m_x$-$Int_j(m_x$-$Cl_j(A))$.

v) If $A \subseteq B$*,then* m_x-$Ext_j(A) \subseteq m_x$-$Ext_j(B)$.

vi) m_x-$Ext_j(A\cup B) \subseteq m_x$-$Ext_j(A) \cap m_x$-$Ext_j(B)$.

vii) m_x-$Ext_j(A) \cup m_x$-$Ext_j(B) \subseteq m_x$-$Ext_j(A\cap B)$.

viii) m_x-$Ext_j(X) = \emptyset$.

ix) m_x-$Ext_j(\emptyset) = X$.

x) m_x-$Ext_j(A) = m_x$-$Ext_j(X\backslash m_x$-$Ext_j(A))$.

xi) m_x-$Int_j(A) = m_x$-$Ext_j(m_x$-$Ext_j(A))$.

xii) $X = m_x$-$Int_j(A) \cup m_x$-$Ext_j(A) \cup m_x$-$Fr_j(A)$.

Proof. i) Since every interior point of A is the j-interior of A in minimal structure, we have m_x-$Int(A) \subseteq m_x$-$Int_j(A)$. Therefore, m_x-$Ext(A) = m_x$-$Int(X\backslash A)$
$\subseteq m_x$-$Int_j(X\backslash A) = m_x$-$Ext_j(A)$.
ii) Since m_x-$Ext_j(A) = m_x$-$Int_j(X\backslash A)$, then m_x-$Ext_j(A)$ is open.
iii) It is obvious that m_x-$Ext_j(A) = m_x$-$Int_j(X\backslash A) = X\backslash m_x$-$Cl_j(A)$.
iv) Since m_x-$Int_j(X\backslash A) = X\backslash m_x$-$Cl_j(A)$, we have m_x-$Ext_j(m_x$-$Ext_j(A)) = m_x$-$Ext_j(m_x$-$Int_j(X\backslash A)) = m_x$-$Int_j(X\backslash m_x$-$Int_j(X\backslash A)) = m_x$-$Int_j(m_x$-$Cl_j(A))$.
v) Since $A \subseteq B$, we have m_x-$Int_j(A) \subseteq m_x$-$Int_j(B)$. Therefore by definition we have m_x-$Ext_j(A) \subseteq m_x$-$Ext_j(B)$.
vi) Using m_x-$Int_j(A) \cup m_x$-$Int_j(B) \subset m_x$-$Int_j(A\cup B)$, we have m_x-$Ext_j(A\cup B) = m_x$-$Int_j(X-(A\cup B)) = m_x$-$Int_j((X-A)\cap(X-B)) \subseteq m_x$-$Int_j(X-A) \cap m_x$-$Int_j(X-B) = m_x$-$Ext_j(A) \cap m_x$-$Ext_j(B)$.
vii) The proof is similar to (vi).
viii) m_x-$Ext_j(X) = m_x$-$Int_j(X^c) = \emptyset$.
ix) m_x-$Ext_j(\emptyset) = m_x$-$Int_j(\emptyset^c) = X$.
x) Since m_x-$Int_j(m_x$-$Int_j(A)) = m_x$-$Int_j(A)$, by the definition of j-exterior of A we have m_x-$Ext_j(A) = m_x$-$Ext_j(X\backslash m_x$-$Ext_j(A))$.
xi) Since m_x-$Ext_j(A) = m_x$-$Int_j(X\backslash A)$, we have m_x-$Int_j(A) = m_x$-$Ext_j(m_x$-$Ext_j(A))$.
xii) It is obvious from the definition that $X = m_x$-$Int_j(A) \cup m_x$-$Ext_j(A) \cup m_x$-$Fr_j(A)$. □

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