Malaya Journal of Matematik, Vol. 8, No. 3, 1186-1188, 2020

https://doi.org/10.26637/MJM0803/0078

Eccentric domination decomposition of graphs

K.S. Jinisha Kalaiarasan[1]* and K. Lal Gipson[2]

Abstract

A decomposition $(G_1, G_2.....G_n)$ of G is said to be an eccentric domination decomposition (EDD) if i)$E(G) = E(G_1) \cup E(G_2) \cup .... \cup E(G_n)$ ii) Each $G_i$is connected iii)$\gamma_{ed}(G_i) = i$, $i = 1,2....n$. If a graph G has EDD, we say that G admits eccentric domination decomposition.

Keywords

Decomposition, Domination, Eccentric domination decomposition.

AMS Subject Classification

97K30.

[1] *Research Scholar, Reg No:18213112092026, Department of Mathematics, Nesamony Memorial Christian College, Marthandam.*

[2] *Department of Mathematics, Nesamony Memorial Christian College, Marthandam- 629165, Tamil Nadu, India.*

Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012,Tamil Nadu, India.

***Corresponding author**: [1] jinishakalaiarasan@gmail.com; [2]lalgison@yahoo.com

Article History: Received **24** March **2020**; Accepted **09** July **2020**

Contents

1. Introduction

In this article, all the terminologies from the graph theory are used in the case of Frank Harary [3]. A simple undirected graph without loops or multiple edges are considered here.

The theory of domination is the one of the fastest growing fields of graph theory, Which has been investigated by S.T.Hedetniemi [4]. A set $D \subseteq V(G)$ of vertices in a graph G is a dominating set if every vertex v in $V - D$ is adjacent to a vertex in D. The Minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$.

A set $D \subseteq V$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric point of v in D.

If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set.

An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set. The minimum cardinality of an eccentric dominating is called the $\gamma_{ed}(G)$ is known as minimum eccentric dominating set. This concept was introduced by T.N. Janakiraman, M. Bhanumathi and S. Muthammai [5].

The decomposition of graphs is another important field of graph theory.

Several authors studied various types of decompositions by imposing conditions on G_i in the decomposition.

Let $G = (V, E)$ be a simple connected graph with p vertices and q edges.If $G_1, G_2.....G_n$ are connected edge disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup ..... \cup E(G_n)$ then $(G_1, G_2.....G_n)$ is said to be a Decomposition of G.

Motivated by the concepts of Ascending Domination Decomposition [7] and Continuous Monotonic Decomposition [2] we introduce a new concept Eccentric Domination Decomposition of a graphs.

2. Eccentric Domination Decomposition {EDD}

Definition 2.1. A decomposition $(G_1, G_2.....G_n)$ of G is said to be an Eccentric Domination Decomposition if

i)$E(G) = E(G_1) \cup E(G_2) \cup .... \cup E(G_n)$

ii) Each G_i is connected

iii)$\gamma_{ed}(G_i) = i$, $i = 1,2....n$.

If a graph G has EDD, we say that G admits Eccentric Domination Decomposition.

Theorem 2.2. *$K_{1,n}$ admits Eccentric domination decomposition.*

Proof. Let $G = K_{1,n}$. Let G_1 be a subgraph obtained from $K_{1,n}$ by taking the edge uu_1. Then $\gamma_{ed}(G_1) = 1$. We also see that $G_2 = K_{1,n} - G_1$ and $\gamma_{ed}(G_2) = 2$. Hence $\psi = \{G_1, G_2\}$ is an EDD for $K_{1,n}$. □

Theorem 2.3. *Complete bipartite graph $k_{m,n}$ admits Eccentric Domination Decomposition.*

Proof. Let $V = X \cup Y$ be a bipartition of $k_{m,n}$ with $|X| = m$ and $|Y| = n$. Let $X = \{x_1, x_2, ..., x_m\}$ and $Y = \{y_1, y_2, ..., y_n\}$.Let G_1 be a subgraph obtained from $K_{m,n}$ by taking the edge x_1y_1. Then $\gamma_{ed}(G_1) = 1$. We also see that $G_2 = K_{m,n} - G_1$ and $\gamma_{ed}(G_2) = 2$. Hence $\psi = \{G_1, G_2\}$ is an EDD for $K_{m,n}$. □

Theorem 2.4. *SL_m has an EDD $\psi = \{G_1, G_2 ..... G_n\}$ if and only if SL_m has $\frac{n^2-n+2}{2}$ vertices.*

Proof. Slanting ladder SL_m obtained from two path $u_1, u_2 ..... u_m$ and $v_1, v_2 ..... v_m$ by joining each u_i with v_{i+1}, $1 \le i \le m-1$.
To prove SL_m has an EDD.
Suppose SL_m has $\frac{n^2-n+2}{2}$ vertices.
Let
$G_1 = \{u_1, u_2\}$
$G_2 = \{u_1, v_1, v_2\}$
$G_3 = \{u_2, u_3, u_4, v_2, v_3, v_4\}$
.
.
.
$G_n = \{u_k, u_{k+1} .... u_m, v_k, v_{k+1} .... v_m\}$
clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. We observe that the minimum eccentric dominating set of G_n has n vertices SL_m has$\frac{n^2-n+2}{2}$ vertices. clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. and hence $\psi = \{G_1, G_2 ..... G_n\}$ is an SL_m.
Conversely suppose SL_m has an EDD.
To prove that SL_m has $\frac{n^2-n+2}{2}$ vertices.
Suppose SL_m has no $\frac{n^2-n+2}{2}$ vertices.
The following are the two possilibities.
Case i) In the above construction of $G_1, G_2 ..... G_n$ if we add the vertices $1, 2...n$ in SL_m then there will be remaining 1 to n vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.
Case ii) In the above construction of $G_1, G_2 ..... G_n$ if we remove the vertices $1, 2...n$ in SL_m then there will be remaining 1 to $n-1$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption. □

Theorem 2.5. *TL_m has an EDD $\psi = \{G_1, G_2 ..... G_n\}$ if and only if SL_m has either $\frac{2n^2-6n+8}{2}$ or $\frac{2n^2-6n+10}{2}$ vertices.*

Proof. Triangular Ladder TL_m is a graph obtained from L_m by adding the edges u_iv_{i+1}, $1 \le i \le n-1$. The vertices of L_n are u_i and v_i. u_i and v_i are two path in the graph L_m where $i = \{1, 2...n\}$ To prove TL_m has an EDD.
Suppose TL_m has $\frac{2n^2-6n+8}{2}$ vertices.
Let
$G_1 = \{u_1, v_1\}$
$G_2 = \{u_1, u_2, v_1, v_2\}$
$G_3 = \{u_2, u_3, u_4, v_2, v_3, v_4\}$
.
.
.
$G_n = \{u_k, u_{k+1} .... u_m, v_k, v_{k+1} .... v_m\}$
Clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. We observe that the minimum eccentric dominating set of G_n has n vertices TL_m has $\frac{2n^2-6n+8}{2}$ vertices. Clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. and hence $\psi = \{G_1, G_2 ..... G_n\}$ is an TL_m.
Conversely suppose TL_m has an EDD.
To prove that TL_m has $\frac{2n^2-6n+8}{2}$ vertices.
The following are the two possilibities.
Suppose TL_m has no $\frac{2n^2-6n+8}{2}$ vertices.
Case i) In the above construction of $G_1, G_2 ..... G_n$ if we add the vertices $1, 2...n$ in TL_m then there will be remaining 1 to n vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$.We get contradiction for our assumption.
Case ii) In the above construction of $G_1, G_2 ..... G_n$ if we remove the vertices $1, 2...n$ in TL_m then there will be remaining 1 to n-1 vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.
case iii)
To prove TL_m has an EDD.
Suppose TL_m has $\frac{2n^2-6n+10}{2}$ vertices.
Let
$G_1 = \{u_1, v_1\}$
$G_2 = \{u_1, u_2, v_1, v_2\}$
$G_3 = \{u_2, u_3, u_4, u_5, v_2, v_3, v_4, v_5\}$
.
.
.
$G_n = \{u_k, u_{k+1} .... u_m, v_k, v_{k+1} .... v_m\}$
Clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. We observe that the minimum eccentric dominating set of G_n has n vertices TL_m has $\frac{2n^2-6n+10}{2}$ vertices. Clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2....n$. and hence $\psi = \{G_1, G_2 ..... G_n\}$ is an TL_m.
Conversely suppose TL_m has an EDD.
To prove that TL_m has $\frac{2n^2-6n+10}{2}$ vertices.
Suppose TL_m has no $\frac{2n^2-6n+10}{2}$ vertices.
The following are the two possilibities.
Case iv) In the above construction of $G_1, G_2 ..... G_n$ if we add the vertices $1, 2...n$ in TL_m then there will be remaining 1 to n vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resuting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.
Case v) In the above construction of $G_1, G_2 ..... G_n$ if we re-

move the vertices $1,2...n$ in TL_m then there will be remaining 1 to n-1 vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resuting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption. □

Theorem 2.6. *$P_p \odot K_1$ has an EDD $\psi = \{G_1, G_2.....G_n\}$ if and only if $P_p \odot K_1$ has $\frac{n^2-n+2}{2}$ vertices.*

Proof. let $P_p = \{u_1, u_2.....u_p\}$ be a path. If we attach the vertices $u'_1, u'_2.....u'_p$ to $u_1, u_2.....u_p$ respectively then we get $P_p \odot K_1$.
To prove $P_p \odot K_1$ has an EDD.
Suppose $P_p \odot K_1$ has $\frac{n^2-n+2}{2}$ vertices.
Let
$G_1 = \{u_1, u'_1\}$
$G_2 = \{u_1, u_2, u'_2\}$
$G_3 = \{u_2, u_3, u_4, u'_3, u'_4\}$
.
.
.
$G_n = \{u_k, u_{k+1}....u_p, u'_{k+1}....u'_p\}$
clearly $\gamma_e d(G_i) = i$, $1 = 1,2....n$. We observe that the minimum eccentric dominating set of G_n has n vertices $P_p \odot K_1$ has $\frac{n^2-n+2}{2}$ vertices.clearly $\gamma_{ed}(G_i) = i$, $1 = 1,2....n$. and hence $\psi = \{G_1, G_2.....G_n\}$ is an $P_p \odot K_1$.
Conversely suppose $P_p \odot K_1$ has an EDD.
To prove that $P_p \odot K_1$ has $\frac{n^2-n+2}{2}$ vertices.
Suppose $P_p \odot K_1$ has no $\frac{n^2-n+2}{2}$ vertices.
The following are the two possibilities.
Case i) In the above construction of $G_1, G_2.....G_n$ if we add the vertices $1,2...n$ in $P_p \odot K_1$ then there will be remaining 1 to n vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resuting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption
Case ii) In the above construction of $G_1, G_2.....G_n$ if we remove the vertices $1,2...n$ in $P_p \odot K_1$ then there will be remaining 1 to n-1 vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i. Therefore the resuting decomposition does not admit EDD. Therefore $\gamma_e d(G_i) \neq i$. We get contradiction for our assumption. □

3. Conclusion

From this paper, we get a knowledge of the eccentric domination decomposition of graphs.

Acknowledgment

The authors wish to thank the anonymous referees for their comments and suggestions.

References

[1] M. Bhanumathi, J. John Flavia, The Minimum Eccentric Dominating Graph, *Procedia Computer Science*, 47(2015), 337–341.
[2] N. Gnanadhas and J. Paul Raj Joseph, Continuous monotonic decomposition of graphs, *International Journal of Management and System*, 16(3)(2000), 333–344.
[3] F. Harary, *Graph Theory*, Narosa Publishing House, New Delhi, 1998.
[4] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekkar, Inc, 1998.
[5] T.N. Janakiraman, M. Bhanumathi and S. Muthammai., Eccentric Domination in Graphs, *International Journal of Engineering Science, Advanced Computing and Bio-Technology*, 2(2000), 55–70.
[6] Juraj Bosak, *Decomposition of Graphs*, Kluwer Academic Publishers, 1990.
[7] K. Lakshmi Prabha, K. Nagarajan, Ascending domination decomposition of graphs, *International Journal of Mathematics and Soft Computing*,4(1)(2014), 119–128.
[8] K. Lakshmi Prabha, K. Nagarajan, Ascending domination Decomposition of Subdivision of graphs, *International Journal of Mathematics and Soft Computing*, 4(1)(2015), 105–114.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆