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The multiplicative reformulated first Zagreb index of some graph operations

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Abstract

In this paper, the multiplicative reformulated first Zagreb index is presented and the sharp upper bound for the multiplicative reformulated first Zagreb index of various graph operations for example, join, composition, cartesian and corona product of graphs are derived.

Keywords

Topological indices and graph operations.

AMS Subject Classification

05C07, 05C76.

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1. Introduction

All graphs observed here are simple, connected and finite. Let $V(G), E(G)$ and $d_G(w)$ indicate the vertex set, the edge set and the degree of a vertex of a graph G respectively. A graph with p vertices and q edges is known as a (p,q) graph.

A topological index of a graph G is a real number which is invariant under automorphism of G and does not depend on the labeling or pictorial representation of a graph.

Gutman et.al.,[1] introduced the first and second Zagreb indices of a graph G as follows:

$$M_1(G) = \sum_{wz\in E(G)} (d_G(w)+d_G(z)) = \sum_{w\in V(G)} d_G^2(w)$$

and

$$M_2(G) = \sum_{wz\in E(G)} d_G(w)d_G(z)$$

Shirdel et.al. in [6] found Hyper-Zagreb index $HM(G)$ which is established as

$$HM(G) = \sum_{wz\in E(G)} [d_G(w)+d_G(z)]^2.$$

Also, they have computed the hyper - Zagreb index of the cartesian product, composition, join and disjunction of graphs.

Milicevic et al [3] reformulated the Zagreb indices in terms of edge degrees instead of vertex degrees, where the degree of an edge $e = wz$ is defined as $d(e) = d(w)+d(z)-2$. Thus, the reformulated first and second Zagreb indices of a graph G are defined as

$$EM_1(G) = \sum_{e\in E(G)} d^2(e) \text{ and } EM_2(G) = \sum_{e\sim f} d(e)d(f)$$

where $e \sim f$ means that the edges e and f share a common vertex in G. That is, they are adjacent.

Nilanjan De. et.al., [4] computed precise formulae for the reformulated first Zagreb index of some graph operations.

Recently, Todeschine et al [7,8] have presented the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$\prod_1 = \prod_1(G) = \prod_{w\in V(G)} d_G(w)^2 = \prod_{wz\in E(G)} \left[d_G(w)+d_G(z)\right]$
and $\prod_2 = \prod_2(G) = \prod_{wz\in E(G)} d_G(w)d_G(z)]$

In this paper, we introduce a new graph invariant namely multiplicative reformulated Zagreb indices, denoted by

$$\prod EM_1(G) = \prod_{wz\in E(G)} (d_G(w)+d_G(z)-2)^2$$

The join $G = G_1 + G_2$ of two graphs G_1 and G_2 is a graph formed from disjoint copies of G_1 and G_2 by connecting each vertex of G_1 to each vertex of G_2.

The composition $G = G_1[G_2]$ of graphs G_1 and G_2 with vertex set $V(G_1)\times V(G_2)$ and $(w_1,z_1)(w_2,z_2) \in G_1[G_2]$ iff $w_1w_2 \in E(G_1)$ or r $w_1 = w_2$ and $z_1z_2 \in E(G_2)$.

The Cartesian product of the graphs G_1 and G_2 is the graph $G_1\Box G_2$ with vertex set $V(G_1)\times V(G_2)$ and for which $(w_1,w_2)(z_1,z_2) \in G_1\Box G_2$ iff $w_1 = z_1$ and $w_2z_2 \in E(G_2)$ or (ii) $w_2 = z_2$ and $w_1z_1 \in E(G_1)$.

The corona product of the graphs G_1 and G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 and $|V(G_1)|$ disjoint copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in i^{th} copy of G_2.

The aim of this paper is to continue this program for computing the sharp upper bound for the multiplicative reformulated first Zagreb index of these operations on graphs and to prove our bound is tight.

2. Main Results

Lemma 2.1. *[2, 5]*

1. $d_{G_1+G_2}(w) = \begin{cases} d_{G_1}(w)+V(G_2), & w\in V(G_2)\\ d_{G_2}(w)+V(G_1), & w\in V(G_2)\end{cases}$

2. $d_{G_1[G_2]}(w,z) = p_2 d_{G_1}(w) + d_{G_2}(z)$

3. $d_{G_1\Box G_2}((w_i,z_j)) = d_{G_1}(w_i)+d_{G_2}(z_j)$, *where* $(w_i,z_j) \in V(G_1\Box G_2)$.

4. $$d_{G_1\odot G_2}(w) = \begin{cases} d_{G_1}(w)+p_2 & if\ w\in V(G_1)\\ d_{G_1}(w)+p_2 & if\ w\in V(G_{2,i})\\ & for\ some\ 0\le i\le p_1-1,\end{cases}$$

   where $w\in V(G_1\odot G_2)$ $G_{2,i}$ *is the ith copy of the graph* G_2 *in* $G_1\odot G_2$.

Lemma 2.2 (Arithmetic geometric Inequality)**.** *Let* $y_1,y_2,\ldots,y_n$ *be non-negative numbers. Then* $\dfrac{y_1+y_2+\cdots+y_n}{n} \ge \sqrt[n]{y_1x_2\cdots y_n}$

3. The multiplicative reformulated first Zagreb index of join of graphs

Theorem 3.1. *Let* $G_i, i = 1,2$ *be a* $(p_i,q_i)-$ *graph. Then*

$$\begin{aligned}&\prod EM_1(G_1+G_2)\\ &\le \left[\frac{EM_1(G_1)+4p_2^2q_1+4p_2(M_1(G_1)-2q_1)}{q_1}\right]^{q_1}\\ &\times\left[\frac{EM_1(G_2)+4p_1^2q_2+4p_1(M_1(G_2)-2q_2)}{q_2}\right]^{q_2}\\ &\times\left[\frac{\begin{array}{c}p_2M_1(G_1)+p_1M_1(G_2)\\ +8q_1q_2+p_1p_2(p_1+p_2-2)^2\\ +4(p_1+p_2-2)(p_1q_2+p_2q_1)\end{array}}{p_1p_2}\right]^{p_1p_2}\end{aligned}$$

Proof. From the definition of the multiplicative first Zagreb index,

$$\begin{aligned}\prod EM_1(G_1+G_2) &= \prod_{wz\in E(G_1+G_2)} [d_{G_1,+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\ &= \prod_{wz\in E(G_1)} [d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\ &\times \prod_{wz\in E(G_2)} [d_{G_1+G_2}(w)+d_{G_1,G_2}(z)-2]^2\\ &\times \prod_{w\in V(G_1)}\prod_{z\in V(G_2)} [d_{G_1+G_2}(w)+d_{G_1,G_2}(z)-2]^2\\ &= A\times B\times C\end{aligned}$$

where A, B and C indicate the products of the above terms in order.
Now we calculate A.

$$\begin{aligned}A &= \prod_{wz\in E(G_1)} [d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\ &= \prod_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)+2p_2-2]^2\\ &\le \left[\frac{\sum_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)+2p_2-2]^2}{q_1}\right]^{q_1}\\ &= \left[\frac{\begin{array}{c}\sum_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)-2]^2+4p_2^2\\ +4p_2[d_{G_1}(w)+d_{G_1}(z)-2]\end{array}}{q_1}\right]^{q_1}\\ &= \left[\frac{EM_1(G_1)+4p_2^2q_1+4p_2(M_1(G_1)-2q_1)}{m_1}\right]^{q_1}\end{aligned}$$

Next we calculate B.

$$\begin{aligned}
B &= \prod_{wz\in E(G_2)} [d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\
&= \prod_{wz\in E(G_2)} [d_{G_2}(w)+d_{G_2}(z)+2p_1-2]^2\\
&\leq \left[\frac{\sum_{wz\in E(G_2)} [d_{G_2}(w)+d_{G_2}(z)+2p_1-2]^2}{q_2}\right]^{q_2}\\
&= \left[\frac{\sum_{wz\in E(G_2)} [d_{G_2}(w)+d_{G_2}(z)-2]^2+4p_1^2 +4p_1[d_{G_2}(w)+d_{G_2}(z)-2]}{q_2}\right]^{q_2}\\
&= \left[\frac{EM_1(G_2)+4p_1^2q_2+4p_1(M_1(G_2)-2q_2)}{q_2}\right]^{q_2}
\end{aligned}$$

Finally, we compute C

$$\begin{aligned}
C &= \prod_{w\in V(G_1)}\prod_{z\in V(G_2)} [d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\
&= \prod_{w\in V(G_1)}\prod_{z\in V(G_2)} [d_{G_1}(w)+p_2+d_{G_2}(z)+p_1-2]^2\\
&\leq \left[\frac{\sum_{w\in V(G_1)}\sum_{z\in V(G_2)} [d_{G_1}(w)+d_{G_2}(z) +p_1+p_2-2]^2}{p_1p_2}\right]^{p_1p_2}\\
&= \left[\frac{\sum_{w\in V(G_1)}\sum_{z\in V(G_2)} [d_{G_1}^2(w)+d_{G_2}^2(z) +2d_{G_1}(w)d_{G_2}(z)+(p_1+p_2-2)^2 +2(d_{G_1}(w)+d_{G_2}(z))(p_1+p_2-2)]}{p_1p_2}\right]^{p_1p_2}\\
&= \left[\frac{p_2M_1(G_1)+p_1M_1(G_2)+8q_1q_2 +p_1p_2(p_1+p_2-2)^2 \; 4(p_1+p_2-2)(p_1q_2+p_2q_1)}{p_1p_2}\right]^{p_1p_2}
\end{aligned}$$

Now using A, B and C we get the desired results. □

Lemma 3.2. *Let $G_i,(i=1,2)$ be two regular graphs of degree r_i.*
Let $G_i,(i=1,2)$ be a $(p_i,q_i)-$ graph. Then
$\prod EM_1(G_1+G_2) = (2r_1+2p_2-2)^{2q_1}$
$\times(2r_2+2p_1-2)^{2q_2}\times(r_1+r_2+p_1+p_2-2)^{2q_1q_2}$.

Proof.

$$\begin{aligned}
&\prod EM_1(G_1+G_2)\\
&= \prod_{wz\in E(G_1)} (d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\
&\times \prod_{wz\in E(G_2)} [d_{G_1+G_2}(w)+d_{G_1+G_2}(z)-2]^2\\
&\times \prod_{w\in V(G_1)}\prod_{z\in V(G_2)} \left[d_{G_1+G_2}(w)+d_{(G_1+G_2)}(z)-2\right]^2\\
&= \prod_{wz\in E(G_1)} (r_1+r_1+2p_2-2)^2\\
&\times \prod_{wz\in E(G_2)} (r_2+r_2+2p_1-2)^2\\
&\times \prod_{wz\in E(G_1)}\prod_{wz\in E(G_2)}(r_1+r_2 +p_1+p_2-2)^2\\
&= (2r_1+2p_2-2)^{2q_1}\\
&\times(2r_2+2p_1-2)^{2q_2}\\
&\times(r_1+r_2+p_1+p_2-2)^{2p_1p_2}
\end{aligned} \tag{3.1}$$

□

Remark 3.3. *We find the upper bound of Lemman 3.2 when G_1 is a regular graph of degree r_1 with p_1 vertices and q_1 edges and G_2 is a regular graph of degree r_2 with p_2 vertices and q_2 edges. Here $q_1=\frac{p_1r_1}{2}, q_2=\frac{p_2r_2}{2}, M_1(G_1)=p_1r_1^2, M_1(G_2)=p_2r_2^2$.*

$$\begin{aligned}
EM_1(G_1) &= q_1(2r_1-2)^2\\
&= 2p_1r_1(r_1-1)\\
EM_1(G_2) &= q_2(2r_2-2)^2\\
&= 2p_2r_2(r_2-1)
\end{aligned}$$

Corollary 3.4. *Using Remark 3.3 in Theorem 3.1, then we get*

$$\begin{aligned}
\prod EM_1(G_1+G_2) &= (2r_1+2p_2-2)^{2q_1}\\
&\times(2r_2+2p_1-2)^{2q_2}\\
&\times(r_1+r_2+p_1+p_2-2)^{2p_1p_2}
\end{aligned} \tag{3.2}$$

From (3.1) and (3.2) the bound is tight.

4. The multiplicative reformulated first Zagreb index of composition of graphs

Theorem 4.1. *Let $G_i, i = 1,2$ be a $(p_i, q_i)-$ graph. Then*

$$\prod EM_1(G_1[G_2]) \leq \left[\frac{4p_2^2 q_2 M_1(G_1) + p_1 EM_1(G_2) + 8p_2 q_1 M_1(G_2) - 16p_2 q_1 q_2}{p_1 q_2}\right]^{p_1 q_2} \times \left[\frac{\begin{array}{r} p_2^4 EM_1(G_1) + 2p_2(M_1(G_1) - 2q_1) \\ (4p_2 q_2 + 2p_2^2(p_2 - 1)) \\ +2p_2 q_1 M_1(G_2) + 8q_1 q_2^2 + 16p_2 q_1 q_2(p_2 - 1) \\ +4p_2^2 q_1 (p_2 - 1)^2 \end{array}}{q_1 p_2^2}\right]^{q_1 p_2^2}$$

Proof.

$$\begin{aligned} \prod EM_1(G_1[G_2]) &= \prod_{(w,k)(z,l)\in E(G_1[G_2])} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}(z,l) - 2\right]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}(z,l) - 2\right]^2 \\ &\times \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}(z,l) - 2\right]^2 \\ &= A \times B, \end{aligned}$$

where A and B indicate the products of the above terms in order.
Now we compute A.

$$\begin{aligned} A &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}[(w,l) - 2\right]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} \left[2p_2 d_{G_1}(w) + d_{G_2}(k) + d_{G_2}(l) - 2\right]^2 \\ &\leq \left[\frac{\sum_{w\in V(G_1)} \sum_{kl\in E(G_2)} \left[2p_2 d_{G_1}(w) + d_{G_2}(k) + d_{G_2}(l) - 2\right]^2}{p_1 q_2}\right]^{p_1 q_2} \\ &= \left[\frac{4p_2^2 q_2 M_1(G_1) + p_1 EM_1(G_2) + 8p_2 q_1 M_1(G_2) - 16p_2 q_1 q_2}{p_1 q_2}\right]^{p_1 q_2} \end{aligned}$$

$$\begin{aligned} B &= \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}[(z,l) - 2\right]^2 \\ &\leq \left[\frac{\sum_{k\in V(G_2)} \sum_{l\in V(G_2)} \sum_{wz\in E(G_1)} [p_2(d_{G_1}(w) + d_{G_1}(z) - 2) + (d_{G_2}(k) + d_{G_2}(l) - 2) + 2p_2]^2}{p_2^2 q_1}\right]^{p_2^2 q_1} \\ &= \left[\frac{\begin{array}{r} \sum_{k\in V(G_2)} \sum_{l\in V(G_2)} \sum_{wz\in E(G_1)} [p_2^2(d_{G_1}(w) + d_{G_1}(z) - 2)^2 \\ +2p_2(d_{G_1}(w) + d_{G_1}(z) - 2) \\ d_{G_2}(k) + d_{G_2}(l) + 2(p_2 - 1)) + d_{G_2}^2(k) \\ +d_{G_2}^2(l) + 2d_{G_2}(k)d_{G_2}(l) \\ +4(p_2 - 1)(d_{G_2}(k) + d_{G_2}(l)) + 4(p_2 - 1)^2] \end{array}}{q_1 p_2^2}\right]^{q_1 p_2^2} \\ &= \left[\frac{\begin{array}{l} p_2^4 EM_1(G_1) + 2p_2(M_1(G_1) - 2q_1) \\ (4p_2 q_2 + 2p_2^2(p_2 - 1)) + 2p_2 q_1 M_1(G_2) \\ +8q_1 q_2^2 + 16p_2 q_1 q_2(p_2 - 1) + 4p_2^2 q_1(p_2 - 1)^2 \end{array}}{q_1 p_2^2}\right]^{q_1 p_2^2} \end{aligned}$$

Using A and B, we get the required results. □

Lemma 4.2. *Let $G_i, i = 1,2$ be two regular graphs of degree r_i and let $G_i, i = 1,2$ be a $(p_i q_i)$-graph. Then $\prod EM_1(G_1[G_2]) = (2p_2 r_1 + 2r_2 - 2)^{2\left(2p_1 q_2 + p_2^2 q_1\right)}$.*

Proof.

$$\begin{aligned} &\prod EM_1(G_1[G_2]) \\ &= \prod_{u\in V(G_1)} \prod_{kl\in E(G_2)} \left[d_{G_1[G_2]}(w,k) + d_{G_1,[G_2]}(w,l)_{-}2\right]^2 \\ &\times \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} \left[d_{G_1[G_2]}(w,k) + d_{G_1[G_2]}(z,l) - 2\right]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} (2p_2 r_1 + 2r_2 - 2)^2 \\ &\times \prod_{k\in V(G_2)} \prod_{l\in V(G_2)} \prod_{wz\in E(G_1)} (2p_2 r_1 + 2r_2 - 2)^2 \\ &= (2p_2 r_1 + 2r_2 - 2)^{2p_1 q_2} \times (2p_2 r_1 + 2r_2 - 2)^{2p_2^2 q_1} \\ &= [2(p_2 r_1 + r_2 - 1)]^{2(p_1 q_2 + p_2^2 q_1)} \end{aligned} \tag{4.1}$$

□

Corollary 4.3. *Using the Remark 3.3 in Theorem 4.1, we have*

$$\prod EM_1(G_1[G_2]) \leq [2(p_2 r_1 + r_2 - 2)]^{2\left(p_1 q_2 + p_2^2 q_1\right)} \tag{4.2}$$

From (4.1) and (4.2) our bound is tight.

5. The multiplicative reformulated first Zagreb index of cartesian product of graphs

Theorem 5.1. *Let $G_i, i = 1,2$ be a $(p_i, q_i)-$graph. Then*

$$\prod EM_1(G_1 \square G_2) \le \left[\frac{p_1 EM_1(G_2) + 8q_1 M_1(G_2) + 4q_2 M_1(G_1) - 16q_1q_2}{p_1q_2}\right]^{p_1q_2} \times \left[\frac{p_2 EM_1(G_1) + 8q_2 M_1(G_1) + 4q_1 M_1(G_2) - 16q_1q_2}{p_2q_1}\right]^{p_2q_1}$$

Proof.

$$\begin{aligned}\prod EM_1(G_1 \square G_2) &= \prod_{(w,k)(z,l)\in E(G_1\square G_2)} [d_{G_1\square G_2}(w,k) + d_{G_1\square G_2}(z,l) - 2]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_1\square G_2}(w,k) + d_{G_1\square G_2}(w,l) - 2]^2 \\ &\times \prod_{k\in V(G_2)} \prod_{wz\in E(G_1)} [d_{G_1\square G_2}(w,k) + d_{G_1,G_2}(z,k) - 2]^2 \\ &= A \times B\end{aligned}$$

where A and B indicate the products of the above terms in order.
Now we calculate A.

$$\begin{aligned}A &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_1\square G_2}(w,k) + d_{G_1\square G_2}(w,l) - 2]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [2d_{G_1}(w) + d_{G_2}(k) + d_{G_2}(l) - 2]^2 \\ &\le \left[\frac{\sum_{w\in V(G_1)} \sum_{kl\in E(G_2)} [2d_{G_1}(w) + d_{G_2}(k) + d_{G_2}(l) - 2]^2}{p_1q_2}\right]^{p_1q_2} \\ &= \left[\frac{\sum_{w\in V(G_1)} \sum_{kl\in E(G_2)} [4d_{G_1}^2(w) + (d_{G_2}(k) + d_{G_2}(l) - 2)^2 + 4d_{G_1}(w)(d_{G_2}(k) + d_{G_2}(l) - 2)]}{p_1q_2}\right]^{p_1q_2} \\ &= \left[\frac{p_1 EM_1(G_2) + 8q_1 M_1(G_2) + 4q_2 M_1(G_1) - 16q_1q_2}{p_1q_2}\right]^{p_1q_2}\end{aligned}$$

Now we compute B.

$$\begin{aligned}B &= \prod_{k\in V(G_2)} \prod_{wz\in E(G_1)} [d_{G_1\square G_2}(w,k) + d_{G_1\square G_2}(z,k) - 2]^2 \\ &= \prod_{k\in V(G_2)} \prod_{wz\in E(G_1)} [2d_{G_2}(k) + d_{G_1}(w) + d_{G_1}(z) - 2]^2 \\ &\le \left[\frac{\sum_{k\in V(G_2)} \sum_{wz\in E(G_1)} [2d_{G_2}(k) + d_{G_1}(w) + d_{G_1}(z) - 2]^2}{p_2q_1}\right]^{p_2q_1} \\ &= \left[\frac{\sum_{k\in V(G_2)} \sum_{wz\in E(G_1)} [4d_{G_2}^2(k) + (d_{G_1}(w) + d_{G_1}(z) - 2)^2 + 4d_{G_2}(k)(d_{G_1}(w) + d_{G_1}(z) - 2)]}{p_2q_1}\right]^{p_2q_1} \\ &= \left[\frac{p_2 EM_1(G_1) + 8q_2 M_1(G_1) + 4q_1 M_1(G_2) - 16q_1q_2}{p_2q_1}\right]^{p_2q_1}\end{aligned}$$

Using A and B we get the required result. □

Lemma 5.2. *Let $G_{i_1} i = 1,2$ be two regular graphs of degree r_i and let $G_i; = 1,2$ be a $(p_i, q_i) -$ graph. Then $\prod EM_1(G_1 \square G_2) = 2(r_1 + r_2 - 2)^{2(p_1q_2 + p_2q_1)}$*

Proof.

$$\begin{aligned}\prod EM_1(G_1 \square G_2) &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_1\square G_2}(w,k) + d_{G_1\square G_2}(w,l) - 2]^2 \\ &\times \prod_{k\in V(G_2)} \prod_{wz\in E(G_1)} [d_{G_1\square G2}(w,k) + d_{G_1\square G_2}(z,k) - 2]^2 \\ &= \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} (r_1 + r_2 + r_1 + r_2 - 2)^2 \\ &\times \prod_{k\in V(G_2)} \prod_{wz\in E(G_2)} (r_1 + r_2 + r_1 + r_2 - 2)^2 \\ &= (2r_1 + 2r_2 - 2)^{2p_1q_2} \times (2r_1 + 2r_2 - 2)^{2p_2q_1} \\ &= [2(r_1 + r_2 - 1)]^{2(p_1q_2 + p_2q_1)} \qquad (5.1)\end{aligned}$$

□

Corollary 5.3. *Using Remark 3.3 in Theorem 5.1, we get*

$$\prod EM_1(G_1 \square G_2) \le [2(r_1 + r_2 - 2)]^{2(p_1q_2 + p_2q_1)} \qquad (5.2)$$

From (5.1) and (5.2) the bound is tight.

6. The multiplicative reformulated first Zagreb index of corona product of graphs

Theorem 6.1. *Let $G_i, i=1,2$ be a (p_i,q_i)-graph. Then*

$$\prod EM_1(G_1 \odot G_2) \leq \left[\frac{EM_1(G_1)+4p_2^2q_1+4p_2M_1(G_1)-8p_2q_1}{q_1}\right]^{q_1} \times \left[\frac{HM(G_2)}{q_2}\right]^{p_1q_2} \times \left[\frac{p_2M_1(G_1)+p_1M_1(G_2)+4p_1p_2(p_2-1)^2 +4p_2q_1(p_2-1)+4p_1q_2(p_2-1)+8q_1q_2}{p_1p_2}\right]^{p_1p_2}$$

Proof.

$$\prod EM_1(G_1,\odot G_2) = \prod_{wz\in E(G_1)} [d_{G1}(w)+d_{G_1}(z)+2p_2-2)^2 \times \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_2}(k)+d_{G_2}(l)]^2 \times \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} [d_{G_1}(w)+d_{G_2}(k)+p_2-1]^2 = A\times B\times C$$

where A,B and C are the products of the about terms in order.
Now calculate A,

$$A = \prod_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)+2n_2-2]^2 \leq \left[\frac{\sum_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)+2n_2-2]^2}{q_1}\right]^{q_1} = \left[\frac{\sum_{wz\in E(G_1)} [(d_{G_1}(w)+d_{G_1}(z)-2)^2 +4p_2^2+4p_2(d_{G_1}(w)+d_{G_1}(z)-2)]}{q_1}\right]^{q_1} = \left[\frac{EM_1(G_1)+4p_2^2q_1+4p_2M_1(G_1)-8p_2q_1}{q_1}\right]^{q_1}$$

Next compute B

$$B = \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_2}(k)+d_{G_2}(l)]^2 \leq \left[\frac{\sum_{w\in V(G_1)} \sum_{kl\in E(G_2)} [d_{G_2}(k)+d_{G_2}(l)]^2}{p_1q_2}\right]^{p_1q_2} = \left[\frac{p_1HM(G_2)}{p_1q_2}\right]^{p_1q_2} = \left[\frac{HM(G_2)}{q_2}\right]^{p_1q_2}$$

Finally, compute C

$$C = \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} [d_{G_1}(w)+d_{G_2}(k)+p_2-1]^2 \leq \left[\frac{\sum_{w\in V(G_1)} \sum_{x\in V(G_2)} (d_{G_1}(w)+d_{G_2}(k)+p_2-1)^2}{p_1p_2}\right]^{p_1p_2}$$
$$= \left[\frac{\sum_{w\in V(G_1)} \sum_{k\in V(G_2)} [d_{G_1}^2(w)+d_{G_2}^2(k) +(p_2-1)^2+2(p_2-1)d_{G_1}(w) +2(p_2-1)d_{G_2}(k)+2d_{G_1}(w)d_{G_2}(k)]}{p_1p_2}\right]^{p_1p_2}$$
$$= \left[\frac{p_2M_1(G_1)+p_1M_1G_2) +p_1p_2(p_2-1)^2+4p_2q_1(p_2-1) +4p_1q_2(p_2-1)+8q_1q_2}{p_1p_2}\right]^{p_1p_2}$$

The required result is obtained by multiplying A,B and C. □

Lemma 6.2. *Let $G_i, i=1,2$ be two regular graph of degree r_i, and*
let $G_i, i=1,2$ be a $(p_i,q_i)-$ graph. Then

$$\prod EM_1(G_1\odot G_2) = [2(r_1+p_2-1)]^{2q_1} \times (2r_2)^{2p_1q_2} \times (r_1+r_2+p_2-1)^{2p_1p_2}$$

Proof.

$$\begin{aligned}
&\prod EM_1(G_1 \odot G_2) \\
&= \prod_{wz\in E(G_1)} [d_{G_1}(w)+d_{G_1}(z)+2p_2-2]^2 \\
&\times \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} [d_{G_2}(k)+d_{G_2}(l)]^2 \\
&\times \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} [d_{G_1}(w)+d_{G_2}(k)+p_2-1]^2 \\
&= \prod_{wz\in E(G_1)} (2r_1+2p_2-2)^2 \times \prod_{w\in V(G_1)} \prod_{kl\in E(G_2)} (2r_2)^2 \\
&\times \prod_{w\in V(G_1)} \prod_{k\in V(G_2)} (r_1+r_2+p_2-1)^2 \\
&= [2(r_1+p_2-1)]^{2q_1} \times (2r_2)^{2p_1q_2}(r_1+r_2+p_2-1)^{2p_1p_2}
\end{aligned} \tag{6.1}$$

□

Corollary 6.3. *Using Remark 3.3 in Theorem 6.1, we get*

$$\begin{aligned}
\prod EM_1(G_1 \odot G_2) \le [2(r_1+r_2-1)]^{2q_1} \times (2p_2)^{2p_1q_2} \\
\times (r_1+r_2+p_2-1)^{2p_1p_2}
\end{aligned} \tag{6.2}$$

From (6.1) and (6.2) the bound is tight.

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