Malaya Journal of Matematik, Vol. 8, No. 3, 1200-1202, 2020

https://doi.org/10.26637/MJM0803/0081

A note on pre generalized b-closed set

J.B. Toranagatti*

Abstract

In this paper, we show that some results obtained in [S. Sekar and R. Brindha, On Pre Generalized B-Closed Set In Topological Spaces, Int. J. Pure Appl. Math., 111(4)(2016),577-586] are incorrect in general,by giving counter examples. Also, we illustrate that Example 3.5,Example 3.13, Example 3.15 and Example 3.17 are incorrect. Moreover, the correction form of the incorrect results of [13] is presented. Finally, we established that the concepts of b-closed sets and pgb-closed sets are same.

Keywords

pg-closed sets, gαb-closed sets, pgb-closed sets, rgb-closed sets, b-closed sets.

AMS Subject Classification

54A05.

*Department of Mathematics,Karnatak University's Karnatak College, Dharwad-580001, India.

***Corresponding author**:jagadeeshbt2000@gmail.com

Article History: Received **24** March **2020**; Accepted **12** July **2020**

Contents

1. Introduction

The concept of b-open set in topological spaces was introduced in 1996 by Andrijevic [2]. Many results had been obtained by using the concept of b-open sets. Also, S. Sekar and R.Brindha [13] discussed and established the concept of pre generalized b-closed sets as a generalization of b-closed sets.

But we observe that some results made in [13] are incorrect.So the aims of this work are, first, to point out that Example 3.5, Example 3.13, Example 3.15 and Example 3.17 of [13] are incorrect, by giving illustrative examples. Second, to show some errors in[Theorem 3.9,Theorem 3.14, Theorem 3.16, Theorem 4.1, Theorem 4.3 and Theorem 4.6] in [13], by presenting examples and then we give corrected form of these results. Finally, to conclude that the notions of b-closed set, gαb-closed set and pgb-closed set are same. This implies that all results in [13–15] are considered as the same well-known results.

2. Preliminaries

Definition 2.1. *A subset M of a space (X,τ) is said to be:*
(1) pre-closed [11] if $cl(int(M)) \subseteq M$
(2) β-closed [3](=semi-preclosed[1]) if $int(cl(int(M))) \subseteq M$.
(3) α-closed [12] if $cl(int(cl(M))) \subseteq M$
(4) g-closed[8] if $cl(M) \subseteq O$ whenever $M \subseteq O$ and O is open in (X,τ).
(5) rgb-closed[10] if $bcl(M) \subseteq O$ whenever $M \subseteq O$ and O is regular open in (X,τ).
(6) pg-closed[9] if $pcl(M) \subseteq O$ whenever $M \subseteq O$ and O is pre-open in (X,τ).
(7) gαb-closed[15] if $bcl(M) \subseteq O$ whenever $M \subseteq O$ and O is α-open in (X,τ)
(8) pgb-closed[13] if $bcl(M) \subseteq O$ whenever $M \subseteq O$ and O is preopen in (X,τ).

Lemma 2.2. *In (X,τ),let $p\in X$ and $M\subseteq X$.Then*
(a)[6] each $\{p\}$ is pre-open or nowhere dense.
(b)[5] If $\{p\}$ is nowhere dense,then $\{p\}$ is α-closed and thus semi-closed, preclosed and β-closed.
(c)[4] $p \in bcl(M)$ if and only if $M \cap U \neq \phi$ for every b-open set U containing p.

Example 2.3. *(Example 3.5[13]:The converse of theorem (3.4) is not true). Let $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$ be a topological space on $X=\{a,b,c\}$.*
The set $\{a,c\}$ is pgb-closed set but it is not a b-closed set.

Example 2.4. *(Example 3.13[13]:The converse of theorem (3.12) is not true). Let* τ=$\{X,\phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$ *be a space on X* =$\{a,b,c\}$.
The set $\{a\}$ *is pgb-closed set but it is not a* gαb*-closed set.*

Example 2.5. *(Example 3.15[13]:The converse of theorem (3.14) is not true) Let* τ=$\{X,\phi,\{a\},\{b\},\{a,b\}\}$ *be a space on X* =$\{a,b,c\}$. *The set* $\{a\}$ *is pgb-closed set but not a rgb-closed set.*

Example 2.6. *(Example 3.17[13] :The converse of theorem (3.16) is not true). Let* $\tau = \{X, \phi, \{a,b\}\}$ *be a topological space on* $X = \{a,b,c\}$.
The set $\{a,b\}$ *is pg-closed set but it is not a pgb-closed set.*

3. Counter Examples

In this section, we point out where the errors occur in [13] and then give counter examples to confirm our claim.

Remark 3.1. *(1)Consider* (X,τ)*as in Example 3.5 of [13] .*
Let M=$\{a,c\}$.*Then int(cl(M))*$\cap$*cl(int(M))*= $\{a\} \cap \{a\} = \{a\} \subseteq \{a,c\}$. *Therefore M is b-closed.*
(2)Consider (X,τ)*as in Example 3.13 of [13].*
Let $N = \{a\}$, *bcl(N)* $= \{a\} \subseteq \{a\}, \{a,b\}$ *which are* α*-open sets containing N. Therefore N is* gαb*-closed.*
(3)Consider (X,τ)*as in Example 3.15 of [13].*
Let P=$\{a\}$, *bcl(P)* $= \{a\} \subseteq \{a\}$ *which is regular-open set containing P.Therefore P is rgb-closed.*
(4) Consider (X,τ)*as in Example 3.17 of [13].*
Set $Q = \{a,b\}$, *pcl(Q) = X which is not a subset of preopen set* $\{a,b\} \supseteq Q$. *Therefore Q is not pg-closed.*

Theorem 3.2. *[13]*
(1)[Theorem 3.9]:Every g-closed set is pgb-closed.
(2)[Theorem 3.14]:Every rgb-closed set is pgb-closed.
(3)[Theorem 3.16]:Every pgb-closed set is pg-closed.
(4)[Theorem 4.1]:If A and B are pgb-closed sets in X then A$\cup$*B is pgb-closed set in X.*
(5)[Theorem 4.3]:The intersection of any two subsets of pgb-closed sets in X is pgb-closed set in X.
(6)[Theorem 4.6]:If A is both pre-open and pgb-closed,then A is pre-closed.

Example 3.3. *Consider* (X,τ) *where* $X = \{u,v,w,x\}$ *and*
$\tau = \{X, \phi, \{u\}, \{v\}, \{u,v\}, \{u,w\}, \{u,v,w\}\}$.*then we have*
$PO(X) = \{X, \phi, \{u\}, \{v\}, \{u,v\}, \{u,w\}, \{u,v,w\}, \{u,v,x\}\}$
$BC(X) = \{X, \phi, \{v\}, \{w\}, \{x\}, \{u,w\}, \{v,w\}, \{w,x\}, \{v,x\}, \{u,w,x\}, \{v,w,x\}\}$
$GC(X) = \{X, \phi, \{x\}, \{u,x\}, \{v,x\}, \{w,x\}, \{u,v,x\}, \{u,w,x\}, \{v,w,x\}\}$
$PGC(X) = \{X, \phi, \{w\}, \{x\}, \{v,x\}, \{w,x\}, \{u,w,x\}, \{v,w,x\}\}$
$RGBC(X) = \{X, \phi, \{u\}, \{v\}, \{w\}, \{x\}, \{u,v\}, \{u,w\}, \{u,x\}, \{v,w\}, \{w,x\}, \{v,x\}, \{u,v,w\}, \{u,v,x\},\{u,w,x\}, \{v,w,x\}\}$
$PGBC(X) = \{X, \phi, \{v\}, \{w\}, \{x\}, \{u,w\}, \{v,w\}, \{w,x\}, \{v,x\}, \{u,w,x\}, \{v,w,x\}\}$
(1)Let $M = \{u,v,x\}$, *then M is g-closed but it is not a pgb-closed set in* (X,τ).
(2)Let $N = \{u\}$,*then N is rgb-closed but not a pgb-closed set in* (X,τ).
(3)Let $L = \{v\}$,*then L is pgb-closed but not a pg-closed set in* (X,τ).
(4)The sets $M = \{u,w\}$ *and* $N = \{v,w\}$ *are pgb-closed sets but* $M \cup N = \{u,v,w\}$ *is not a pgb-closed set in* (X,τ).
(5)The sets $L = \{v,x\}$ *and* $M = \{u,x\}$*are pgb-open sets but L* $\cap M = \{x\}$ *is not a pgb-open set in* (X,τ).
(6)The set $\{v\}$ *is both pgb-closed and b-closed but it is not pre-closed in* (X,τ).

4. Corrected Results

The following Theorem is the correction form of [Theorem 3.9, p. 580] in [13].

Theorem 4.1. *The concept of g-closed set is independent of a pgb-closed set.*

Example 4.2. *Consider* (X,τ)*as in Example 3.3*
(1)Let $M = \{v\}$.*Then M is pgb-closed but not a g-closed set.*
(2)Let $N = \{u,v,x\}$.*Then N is g-closed but not a pgb-closed set.*

The following Theorem is the correction form of [Theorem 3.14, p. 581] in [13].

Theorem 4.3. *Every pgb-closed set is rgb-closed but not conversely.*

Proof. Let N$\subseteq$X be a pgb-closed set such that N $\subseteq$ G where G is a regular open set in (X,τ).Since every regular open set is preopen.Then bcl(N)$\subseteq$ G.This means that N is rgb-closed. □

Example 4.4. *Consider* (X,τ)*as in Example 3.3, let* $M = \{u\}$,*then M is rgb-closed but not a pgb-closed set in* (X,τ).

The following Theorem is the correction form of [Theorem 3.16, p.581] in [13].

Theorem 4.5. *Every pg-closed set is pgb-closed but not conversely.*

Proof. Let N $\subseteq$ X be a pg-closed set such that N $\subseteq$ G where G is a preopen set in (X,τ).Since every preclosed set is b-closed it follows that bcl(N) $\subseteq$ pcl(N).Then bcl(N)$\subseteq$pcl(N)$\subseteq$ G,in consequence, N is pgb-closed. □

Example 4.6. *Consider* (X,τ)*as in Example 3.3, let* $N = \{v\}$. *Then N is pgb-closed but not a pg-closed set in* (X,τ).

The following Theorem is the correction form of [Theorem 4.6, p. 583] in [13].

Theorem 4.7. *If M is both preopen pgb-closed set, then it is b-closed.*

Proof. Let M $\subseteq$ X be both preopen pgb-closed. Then bcl(M)$\subseteq$ M but M$\subseteq$ bcl(M) is always true it follows that M is b-closed. □

The following Theorem is the correction form of [Theorem 4.1, p. 582] in [13].

Remark 4.8. *The union of any two pgb-closed sets need not be pgb-closed.*

Example 4.9. *Consider* (X,τ)*as in Example 3.3. Then the sets* $M = \{u,w\}$ *and* $N = \{v,w\}$ *are pgb-closed but* $M \cup N = \{u,v,w\}$ *is not a pgb-closed set in* (X,τ)*.*

The following Theorem is the correction form of [Theorem 5.2, p. 583] in [13].

Remark 4.10. *The intersection of any two pgb-open sets need not be pgb-open.*

Example 4.11. *Consider* (X,τ)*as in Example 3.3 Then the sets* $C=\{v,x\}$ *and* $D=\{u,x\}$ *are pgb-open sets but* $C \cap D = \{x\}$ *is not a pgb-open set in* (X,τ)*.*

Theorem 4.12. *Let* (X,τ) *be a topological space and* $N \subseteq X$*.Then the following statements are equivalent:*
(i) N is pgb-closed.
(ii) N is $g\alpha b$*-closed.*
(iii) N is b-closed.

Proof. (i)$\Rightarrow$(ii) Since all α-open set is preopen, then all pgb-closed set is a gαb-closed set.
(ii)$\Rightarrow$(iii):Let N $\subseteq$ X be gαb-closed set and u $\in$ bcl(N).To prove that u $\in$ N.
By Lemma 2.9, $\{u\}$ is either pre-open or nowhere dense.
(a)If $\{u\}$ is pre-open which implies $\{u\}$ is b-open.Then $\{u\}$ $\cap$ N $\neq \phi$ and hence by Theorem 2.11, u $\in$ N.
(b)If $\{u\}$ is nowhere dense. Then by Lemma 2.10,$\{u\}$ is α-closed. Hence X-$\{u\}$ is α-open. Suppose that u $\notin$ N, then N $\subseteq$ (X-$\{u\}$) and since N is gαb-closed,then we have bcl(N)$\subseteq$(X-$\{u\}$). Thus u $\notin$ bcl(N) which is a contradiction and hence u $\in$ N. Therefore bcl(N)$\subseteq$N, in consequence, N is b-closed.
(iii)$\Rightarrow$(i):Obvious □

References

[1] D. Andrijivic, On semi pre open sets, *Mat. Vesnic*, 38(1986), 24–32.

[2] D. Andrijivic, On b-open sets, *Mat. Vesnic*, 48(1996), 59–64.

[3] M.E. Abd. El-Monsef, S.N. El-Deeb and R.A. Mahmoud, β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77–90.

[4] A.A. El-Atik, H.M. Abu Donia, A.S. Salama, On b-connectedness and b-disconnectedness and their applications, *J. Egyptian Math. Soc.*, 21(2013), 63–67.

[5] J. Cao, M. Ganster, I. Reilly and M. Steiner, δ-closure, θ-closure and generalized closed sets, *Applied General Topology, Universidad Politecnica de Valencia*, 6(2005), 79–86.

[6] D. Jankovic and I.L. Reilly, On semi-separation properties, *Indian J. Pure Appl. Math.*, 16(1985), 957–964.

[7] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36–41.

[8] N. Levine, Generalized closed sets in topology, *Rent. Circ. Mat. Palermo*, 19(1970), 89–96.

[9] H. Maki, J. Umehara, and T. Noiri, Every topological space is pre-$T_{\frac{1}{2}}$, *Mem. Fac. Sci. Kochi Univ. Math.*, 17(1996), 33–42.

[10] K. Mariappa and S. Sekar, On regular generalized b-closed sets, *Int. J. Math. Anal.*, 7(2013), 613–624.

[11] A. S. Mashhour, M. E. Abd El-Monsef, and S. N. EL-Deeb,On pre-continuous and weak pre continuous mappings, *Proc. Math and Phys.Soc. Egypt*, (53)(1982), 47–53.

[12] O.Njastad, On some classes of nearly open sets, *Pacific. J. Math.*, (15)(1965), 961–970.

[13] S. Sekar and R. Brindha, On pre generalized b-closed set in topological spaces, *Int. J. Pure Appl. Math.*, 111(2016), 577–586.

[14] S. Sekar and R. Brindha, Almost contra pre generalized b-continuous functions in topological spaces, *Malaya J.Mat.*, 5(2017), 194–201.

[15] L. Vinayagamoorthi and N. Nagaveni, On generalized αb-closed set, *Proceeding ICMD-Allahabad, Pusbha Publication*(1),(2011), 1–10.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆