Malaya Journal of Matematik, Vol. 8, No. 3, 1206-1212, 2020
https://doi.org/10.26637/MJM0803/0083

On some topological indices of thorn graphs

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Abstract
In this paper, the relation between the reciprocal Randic index, Reduced reciprocal Randic index and Atom-bond connectivity index of a simple connected graph and its thorn graph is stablished and the atom-bond connectivity (ABC) index of a graph G is defined as $ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}$, where $E(G)$ is the edge set and d_u is the degree of vertex u of G [13]. Reciprocal Randic (RR) index of a graph G is defined as $RR(G) = \sum_{uv \in E(G)} \sqrt{d_u d_v}$, where $E(G)$ is the edge set and d_u is the degree of vertex u of G. Reduced Reciprocal Randic (RRR) index of a graph G is defined as $RRR(G) = \sum_{uv \in E(G)} \sqrt{(d_u-1)(d_v-1)}$, where $E(G)$ is the edge set and d_u is the degree of vertex u of G. Results are applied to compute the reciprocal Randic index, Reduced reciprocal Randic index and Atom-bond connectivity index of thorn rings, thorn paths, thorn rods, thorn star, thorn star $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$.

Keywords
Reciprocal Randic Index, Reduced Reciprocal Randic Index and Atom-Bond Connectivity, Degree Distance, Thorn Graph.

AMS Subject Classification
05C10, 05C07, 05C12, 05C76.

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Article History: **19** April **2020**; Accepted **22** July **2020**

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1. Introduction

Let G be an n-vertex simple connected graph with vertex set $V(G) = \{v_1, v_2, \cdots v_n\}$ and let $P = \{p_1, p_2, \cdots p_n\}$ be an n-tuple of nonnegative integers. The thorn graph G_p is the graph obtained by attaching p_i pendent vertices (terminal vertices or vertices of degree one) to the vertex v_i of G, for $i = 1, 2, \ldots, n$. The p_i pendent vertices attached to the vertex v_i are called thorns of v_i. We denote the set of p_i thorns of v_i by V_i, $i = 1, 2, \ldots, n$. Clearly, $V(G_p) = V(G) \bigcup V_1(G) \bigcup V_2(G) \bigcup \cdots \bigcup V_n$. The concept of thorn graphs was introduced by Gutman et al.,[6] and eventually found a variety of chemical applications, see [1, 2, 17–19].

In (Manso et al., [9]), a new topological index (namely Fi index) was proposed to predict the normal boiling point temperatures of hydrocarbons. In the mathematical definition of Fi index two terms are present. Gutman, Furtula and Elphick et al., [7], recently considered one of these terms which is given below:

$$RRR(G) = \sum_{uv \in E(G)} \sqrt{(d_u-1)(d_v-1)},$$

and they named it as reduced reciprocal Randic (RRR) index. In the current study, we are concerned with this recently introduced modified version of the Randic index. The reciprocal Randic index is defined as $RR = \sum_{uv \in E(G)} \sqrt{(d_u)(d_v)}$.

A topological index is a numeric quantity that is mathematically derived in a direct and unambiguous manner from the structural graph of a molecule. It is used in theoretical chemistry for the design of chemical compounds with given physicochemical properties or given pharmacologic and biological activities. Estrada et al. [4] proposed a topological index named atom-bond connectivity (ABC) index using a modification of the Randic connectivity index, found in [12, 16]. Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. For $u \in V(G)$, the degree of u, denoted by d_u, is the number of neighbors of u in G. The ABC index of G is defined as [3, 4]

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}}.$$

2. Main Results

In this section, we establish relation between the reciprocal Randic index, Reduced reciprocal Randic index and Atom-bond connectivity index of a simple connected graph G and its thorn graph G_p, and examine several special cases of the result.

Thorn ring

The m-thorn ring $C_{n,m}$, has a cycle C_n as the parent, and $m-2$ thorns at each cycle vertex, where $m>2$. The 3-thorn ring $C_{6,3}$ is depicted in Fig.1. The m-thorn ring $C_{n,m}$, can be considered as the thorn graph $(C_n)p$, where P is the n-tuple $P=(m-2,m-2,...,m-2)$ [6].

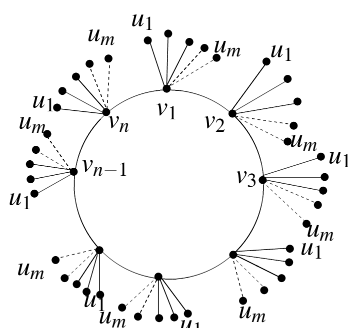

Figure 1: Thorn ring

Theorem 2.1. *The reciprocal Randic index of a thorn ring having n ring vertices is.*

$$RR(C_{n,m}) = (n_i)(m_j)\left(\sqrt{(m_j+2)}\right) + (n_i)(m_j+2).$$

Proof. We separately consider between pair of vertices of C_n between pair of pendent vertices of C_n^* and between pair of vertices of which one belongs to C_n and other wise pendent. Let $v_1, v_2, \cdots, v_n$, be a vertices of cycle, where $i=3,4,\cdots,n$ and let $u_1, u_2, \cdots, u_m$, be a pendent vertices of cycle where $j=1,2,\cdots,m$, (as show in Fig.1). Let $d(v_i)=m_j+2$ and $d(u_j)=1$. Then we have,

$$RR(C_{n,m}) = \sum_{i,j=3,1}^{n_i m_j} \sqrt{(m_j+2)(1)} + \sum_{i=3}^{n_i} \sqrt{(m_j+2)(m_j+2)}$$

$$= (n_i)(m_j)\left(\sqrt{(m_j+2)}\right) + (n_i)\left(\sqrt{(m_j+2)^2}\right)$$

$$= (n_i)(m_j)\left(\sqrt{(m_j+2)}\right) + (n_i)(m_j+2).$$

□

Theorem 2.2. *The reduced reciprocal Randic index of a thorn ring having n ring vertices is.*

$$RRR(C_{n,m}) = (n_i)(m_j+1).$$

Proof. We separately consider between pair of vertices of C_n between pair of pendent vertices of C_n^* and between pair of vertices of which one belongs to C_n and other wise pendent. Let $v_1, v_2, \cdots, v_n$, be a vertices of cycle, where $i=3,4,\cdots,n$ and let $u_1, u_2, \cdots, u_m$, be a pendent vertices of cycle where $j=1,2,\cdots,m$,(as show in Fig.1). Let $d(v_i)=m_j+2$ and $d(u_j)=1$. Then we have,

$$RRR(C_{n,m}) = \sum_{i,j=3,1}^{n_i m_j} \sqrt{(m_j+2-1)(1-1)}$$

$$+ \sum_{i=3}^{n_i} \sqrt{(m_j+2-1)(m_j+2-1)}$$

$$= 0 + \sum_{i=3}^{n_i} \sqrt{(m_j+1)(m_j+1)}$$

$$= (n_i)(m_j+1).$$

□

Theorem 2.3. *The atom-bond connectivity index of a thorn ring having n ring vertices is.*

$$ABC(C_{n,m}) = (n_i)(m_j)\left(\sqrt{\frac{m_j+1}{m_j+2}}\right) + (n_i)\left(\sqrt{\frac{2m_j+2}{(m_j+2)^2}}\right).$$

Proof. We separately consider between pair of vertices of C_n between pair of pendent vertices of C_n^* and between pair of vertices of which one belongs to C_n and other wise pendent. Let $v_1, v_2, \cdots, v_n$, be a vertices of cycle, where $i=3,4,\cdots,n$ and let $u_1, u_2, \cdots, u_m$, be a pendent vertices of cycle where $j=1,2,\cdots,m$,(as show in Fig.1). Let $d(v_i)=m_j+2$ and $d(u_j)=1$. Then we have,

$$ABC(C_{n,m}) = \sum_{i,j=3,1}^{n_i m_j} \sqrt{\frac{(1)+(m_j+2)-2}{(1)(m_j+2)}}$$

$$+ \sum_{i=3}^{n_i} \sqrt{\frac{(m_j+2)+(m_j+2)-2}{(m_j+2)(m_j+2)}}$$

$$= (n_i)(m_j)\left(\sqrt{\frac{m_j+1}{m_j+2}}\right)$$

$$+ (n_i)\left(\sqrt{\frac{2m_j+2}{(m_j+2)^2}}\right).$$

□

Thorn Path

The thorn path $P_{n,p,u}$ is obtained from the path P_n by adding p neighbors to each of its nonterminal vertices and u neighbors to each of its terminal vertices (see Fig. 2). Consider the path P_n and choose a labeling for its vertices such that its two terminal vertices have numbers 1 and n and its nonterminal vertices have numbers $2,3,...,n-1$ (as shown in Fig. 2). Then, $P_{n,p,u}$ can be considered as the thorn graph $(P_n)p$, where P is the $n-tuple$ $P=(u,p,...,p,u)$ [6].

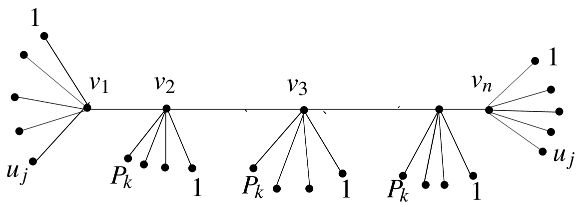

Figure 2: Thorn Path $P_{n,p,u}$

Theorem 2.4. *The reciprocal Randic index of a thorn path, $P_{n,p,u}$, if $n \geq 3$ vertices is.*

$$RR(P_{n,p,u}) = (2u_j)\left(\sqrt{(u_j+1)}\right) + (2)\left(\sqrt{(P_k+2)(u_j+1)}\right) + (n-2)\left(\sqrt{(P_k+2)}\right) + (n-2)(P_k+2).$$

Proof. Let $P_{n,p,u}$, $n \geq 3$. be the thorn path is obtained from the path P_n, by adding P_k, neighbors to each of its nonterminal vertices and u_j, neighbors to each of its terminal vertices (see in Fig. 2). Let $v_1, v_2, \cdots, v_i$, be a vertices of path, where $i = 3,4,\cdots,n$, let $u_1, u_2, \cdots, u_j$, be a pendent vertices, where $j = 1,2,\cdots,n$ and let $p_1, p_2, \cdots, p_k$, be a pendent vertices, where $k = 1,2,\cdots,n$. Let $d(v_1) = d(v_n) = u_j+1$, $d(v_2) = d(v_{n-1}) = P_k+2$, and $d(u_j) = d(p_k) = 1$. Then we have

$$RR(P_{n,p,u}) = \sum_{j=1}^{u_j} \sqrt{(d(u_j))(d(v_1))} + \sum \sqrt{(d(v_1))(d(v_2))}$$
$$+ \sum_{i=2}^{n-2} \sqrt{(d(p_k))(d(v_{n-1}))} + \sum_{i=2}^{n-2} \sqrt{(d(v_2))(d(v_{n-1})}$$
$$+ \sum \sqrt{(d(v_{n-1}))(d(v_n))} + \sum_{j=1}^{u_j} \sqrt{(d(v_n))(d(u_j))}$$
$$= \sum_{j=1}^{u_j} \sqrt{(1)(u_j+1)} + \sum \sqrt{(u_j+1)(P_k+2)}$$
$$+ \sum_{i=2}^{n-2} \sqrt{(1)(P_k+2)} + \sum_{i=2}^{n-2} \sqrt{(P_k+2)(P_k+2)}$$
$$+ \sum \sqrt{(P_k+2)(u_j+1)} + \sum_{j=1}^{u_j} \sqrt{(u_j+1)(1)}$$
$$= (2u_j)\left(\sqrt{(u_j+1)}\right) + (2)\left(\sqrt{(P_k+2)(u_j+1)}\right)$$
$$+ (n-2)\left(\sqrt{(P_k+2)}\right) + (n-2)(P_k+2).$$

□

Theorem 2.5. *The reduced reciprocal Randic index of a thorn path, $P_{n,p,u}$, if $n \geq 3$ vertices is.*

$$RRR(P_{n,p,u}) = (2)\left(\sqrt{(u_j)(P_k+1)}\right) + (n-2)(P_k+1).$$

Proof. Let $P_{n,p,u}$, $n \geq 3$. be the thorn path is obtained from the path P_n, by adding P_k, neighbors to each of its nonterminal vertices and u_j, neighbors to each of its terminal vertices (see in Fig. 2). Let $v_1, v_2, \cdots, v_i$, be a vertices of path, where $i = 3,4,\cdots,n$, let $u_1, u_2, \cdots, u_j$, be a pendent vertices, where $j = 1,2,\cdots,n$ and let $p_1, p_2, \cdots, p_k$, be a pendent vertices, where $k = 1,2,\cdots,n$. Let $d(v_1) = d(v_n) = u_j+1$, $d(v_2) = d(v_{n-1}) = P_k+2$, and $d(u_j) = d(p_k) = 1$. Then we have

$$RRR(P_{n,p,u}) = \sum_{j=1}^{u_j} \sqrt{(d(u_j)-1)(d(v_1)-1)}$$
$$+ \sum \sqrt{(d(v_1)-1)(d(v_2)-1)}$$
$$+ \sum_{i=2}^{n-2} \sqrt{(d(p_k)-1)(d(v_{n-1})-1)}$$
$$+ \sum_{i=2}^{n-2} \sqrt{(d(v_2)-1)(d(v_{n-1})-1)}$$
$$+ \sum \sqrt{(d(v_{n-1})-1)(d(v_n)-1)}$$
$$+ \sum_{j=1}^{u_j} \sqrt{(d(v_{n-1})-1)(d(u_j)-1)}$$
$$= \sum_{j=1}^{u_j} \sqrt{(1-1)(u_j+1)-1)}$$
$$+ \sum \sqrt{(u_j+1)-1)(p_k+2)-1)}$$
$$+ \sum_{i=2}^{n-2} \sqrt{(1-1)(P_k+2)-1)}$$
$$+ \sum_{i=2}^{n-2} \sqrt{((p_k+2)-1)((p_k+2)-1)}$$
$$+ \sum \sqrt{(P_k+2)-1)(u_j+1)-1)}$$
$$+ \sum_{j=1}^{u_j} \sqrt{(u_j+1)-1)(1-1)}$$
$$= 0 + \sum \sqrt{(u_j)(p_k+1)} + 0$$
$$+ \sum_{i=2}^{n-2} \sqrt{(p_k+1)(p_k+1)} + \sum \sqrt{(P_k+1)(u_j)} + 0$$
$$= (2)\left(\sqrt{(u_j)(P_k+1)}\right) + (n-2)(P_k+1).$$

□

Theorem 2.6. *The atom-bond connectivity index of a thorn path, $P_{n,p,u}$, if $n \geq 3$ vertices is.*

$$ABC(P_{n,p,u}) = (2u_j)\left(\sqrt{\frac{u_j}{u_j+1}}\right) +(2)\left(\sqrt{\frac{u_j+p_k+1}{(u_j+1)(p_k+2)}}\right) +(n-2)\left(\sqrt{\frac{P_k+1}{P_k+2}}\right)+(n-2)\left(\sqrt{\frac{2P_k+2}{(P_k+2)^2}}\right).$$

Proof. Let $P_{n,p,u}$, $n \geq 3$. be the thorn path is obtained from the path P_n, by adding P_k, neighbors to each of its nonterminal vertices and u_j, neighbors to each of its terminal vertices (see in Fig. 2). Let $v_1, v_2, \cdots, v_i$, be a vertices of path, where $i = 3,4,\cdots,n$, let $u_1, u_2, \cdots, u_j$, be a pendent vertices, where $j = 1,2,\cdots,n$ and let $p_1, p_2, \cdots, p_k$, be a pendent vertices, where $k = 1,2,\cdots,n$. Let $d(v_1) = d(v_n) = u_j+1$, $d(v_2) = d(v_{n-1}) = P_k+2$, and $d(u_j) = d(p_k) = 1$. Then we have

$$ABC(P_{n,p,u}) = \sum_{j=1}^{u_j}\sqrt{\frac{d(u_j)+d(v_1)-2}{d(u_j)d(v_1)}} +\sum\sqrt{\frac{d(v_1)+d(v_2)-2}{d(v_1)d(v_2)}} +\sum_{i=2}^{n-2}\sqrt{\frac{d(p_k)+d(v_{n-1})-2}{d(p_k)d(v_{n-1})}}+\sum_{i=2}^{n-2}\sqrt{\frac{d(v_2)+d(v_{n-1})-2}{d(v_2)d(v_{n-1})}} +\sum\sqrt{\frac{d(v_{n-1})+d(v_n)-2}{d(v_{n-1})d(v_n)}}+\sum_{j=1}^{u_j}\sqrt{\frac{d(v_n)+d(u_j)-2}{d(v_n)d(u_j)}}$$

$$=\sum_{j=1}^{u_j}\sqrt{\frac{(1)+(u_j+1)-2}{(1)(u_j+1)}}+\sum\sqrt{\frac{(u_j+1)+(P_k+2)-2}{(u_j+1)(P_k+2)}} +\sum_{i=2}^{n-2}\sqrt{\frac{(1)+(P_k+2)-2}{(1)(P_k+2)}}+\sum_{i=2}^{n-2}\sqrt{\frac{(P_k+2)+(P_k+2)-2}{(P_k+2)(P_k+2)}} +\sum\sqrt{\frac{(P_k+2)+(u_j+1)-2}{(P_k+2)(u_j+1)}}+\sum_{j=1}^{u_j}\sqrt{\frac{(u_j+1)+(1)-2}{(u_j+1)(1)}}$$

$$=(u_j)\left(\sqrt{\frac{(u_j)}{(u_j+1)}}\right)+\sqrt{\frac{u_j+P_k+1}{(u_j+1)(P_k+2)}} +(n-2)\left(\sqrt{\frac{P_k+1}{P_k+2}}\right)+(n-2)\left(\sqrt{\frac{2P_k+2}{(P_k+2)^2}}\right) +\sqrt{\frac{P_k+u_j+1}{(P_k+2)(u_j+1)}}+(u_j)\left(\sqrt{\frac{u_j}{u_j+1}}\right)$$

$$=(2u_j)\left(\sqrt{\frac{u_j}{u_j+1}}\right)+(2)\left(\sqrt{\frac{u_j+p_k+1}{(u_j+1)(p_k+2)}}\right) (n-2)\left(\sqrt{\frac{P_k+1}{P_k+2}}\right)+(n-2)\left(\sqrt{\frac{2P_k+2}{(P_k+2)^2}}\right).$$

□

Thorn rod

The thorn rod $P_{n,m}$, is a graph which includes a linear chain (termed rod) of n vertices and degree-m terminal vertices at each of the two rod ends, where $m \geq 2$ (see Fig. 3). It is easy to see that, $P_{n,m} \cong P_{n,0,m-1}$ [6].

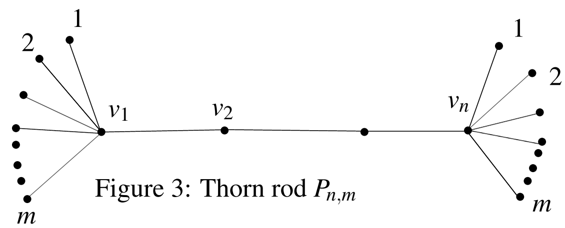

Figure 3: Thorn rod $P_{n,m}$

Corollary 2.7. *The reduced reciprocal Randic index of a thorn rod $P_{n,m}$, if $n \geq 3$. Then*

$$RRR(P_{n,0,m-1}) = (2)(\sqrt{m})(n-2).$$

Corollary 2.8. *The reciprocal Randic index of a thorn rod $P_{n,m}$, if $n \geq 3$. Then*

$$RR(P_{n,0,m-1}) = (2m)(\sqrt{m+1})+(2)\left(\sqrt{2(m+1)}\right)(n-2)(2).$$

Corollary 2.9. *The atom-bond connectivity index of a thorn rod $P_{n,m}$, if $n \geq 3$. Then*

$$ABC(P_{n,0,m-1}) = (2m)\left(\sqrt{\frac{m}{m+1}}\right)+(2)\left(\sqrt{\frac{m+1}{(m+1)(2)}}\right) +(n-2)\left(\sqrt{\frac{2}{4}}\right).$$

Thorn star

The thorn star $S_{n,p,k}$ is obtained from the star S_n by adding p neighbors to the center of the star and k neighbors to its terminal vertices (see Fig.4). Consider the star S_n and choose a labeling for its vertices such that its terminal vertices have numbers $1,2,...,n-1$ and its central vertex has number n as shown in Fig.4. Then, $S_{n,p,k}$ can be considered as the thorn graph $(S_n)P$, where P is the n-tuple $P = (k,k,...,k,p)$ [6].

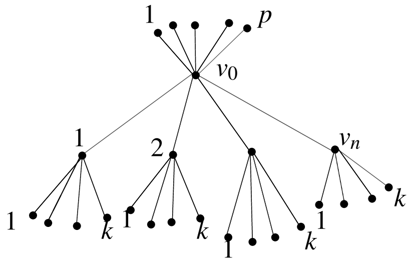

Figure 4: Thorn star $S_{n,p,k}$

Theorem 2.10. *The reciprocal Randic index of a thorn star $S_{n,p,k}$ is.*

$$RR(S_{n,p,k}) = (p)\left(\sqrt{p+v_n}\right) + (n)\left(\sqrt{(p+v_n)(k+1)}\right) + (k)\left(\sqrt{(k+1)}\right).$$

Proof. Let $S_{n,p,k}$ be a thorn star is obtained from the star S_n. Let v_0 be a central vertex of S_n. Let $v_1, v_2, v_3, \cdots, v_n$, be the terminal vertices have number $1, 2, \cdots, n-1, n$. Let $1, 2, \cdots p$ be the pendent vertices , where p is the $n-tuple$, $p = (k, k \cdots, k, p)$. Let $d(v_0) = (p+v_n)$ where $i = 1, 2, \cdots, n$, $d(v_n) = k+1$, $d(p) = d(k) = 1$. Then

$$RR(S_{n,p,k}) = \sum_{i=1}^{p} \sqrt{(d(p))(d(v_0))} + \sum_{i=1}^{n} \sqrt{(d(v_0))(d(v_n))} + \sum_{i=1}^{k} \sqrt{(d(v_n))(d(k))}$$

$$= \sum_{i=1}^{p} \sqrt{(1)(p+v_n)} + \sum_{i=1}^{n} \sqrt{(p+v_n)(k+1)} + \sum_{i=1}^{k} \sqrt{(k+1)(1)}$$

$$= (p)\left(\sqrt{p+v_n}\right) + (n)\left(\sqrt{(p+v_n)(k+1)}\right) + (k)\left(\sqrt{(k+1)}\right).$$

□

Theorem 2.11. *The reduced reciprocal Randic index of a thorn star $S_{n,p,k}$ is.*

$$RRR(S_{n,p,k}) = (n)\left(\sqrt{((p+v_n)-1)(k)}\right).$$

Proof. Let $S_{n,p,k}$ be a thorn star is obtained from the star S_n. Let v_0 be a central vertex of S_n. Let $v_1, v_2, v_3, \cdots, v_n$, be the terminal vertices have number $1, 2, \cdots, n-1, n$. Let $1, 2, \cdots p$ be the pendent vertices, where p is the $n-tuple$, $p = (k, k \cdots, k, p)$. Let $d(v_0) = (p+v_n)$ where $i = 1, 2, \cdots, n$, $d(v_n) = k+1$, $d(p) = d(k) = 1$. Then

$$RRR(S_{n,p,k}) = \sum_{i=1}^{p} \sqrt{(d(p)-1)(d(v_0)-1)} + \sum_{i=1}^{n} \sqrt{(d(v_0)-1)(d(v_n)-1)} + \sum_{i=1}^{k} \sqrt{(d(v_n)-1)(d(k)-1)}$$

$$= \sum_{i=1}^{p} \sqrt{(1-1)((p+v_n)-1)} + \sum_{i=1}^{n} \sqrt{((p+v_n)-1)((k+1)-1)} + \sum_{i=1}^{k} \sqrt{((k+1)-1)(1-1)} = (n)\left(\sqrt{((p+v_n)-1)(k)}\right).$$

□

Theorem 2.12. *The atom-bond connectivity index of a thorn star, Sn, p, k is.*

$$ABC(S_{n,p,k}) = (p)\left(\sqrt{\frac{1+(p+v_n)-2)}{p+v_n}}\right) + (n)\left(\sqrt{\frac{(p+v_n)+(k+1)-2}{(p+v_n)(k+1)}}\right) + (k)\left(\sqrt{\frac{k}{k+1}}\right).$$

Proof. Let $S_{n,p,k}$ be a thorn star is obtained from the star S_n. Let v_0 be a central vertex of S_n. Let $v_1, v_2, v_3, \cdots, v_n$, be the terminal vertices have number $1, 2, \cdots, n-1, n$. Let $1, 2, \cdots p$ be the pendent vertices , where p is the $n-tuple$, $P = (k, k \cdots, k, p)$. Let $d(v_0) = (p+v_n)$ where $i = 1, 2, \cdots, n$, $d(v_n) = k+1$, $d(p) = d(k) = 1$. Then we have,

$$ABC(S_{n,p,k}) = \sum_{i=1}^{p} \sqrt{\frac{d(p)+d(v_0)-2}{(d(p))(d(v_0))}} + \sum_{i=1}^{n} \sqrt{\frac{d(v_0)+d(v_n)-2}{(d(v_0))(d(v_n))}} + \sum_{i=1}^{k} \sqrt{\frac{d(v_n)+d(k)-2}{(d(v_n))(d(k))}}$$

$$= \sum_{i=1}^{p} \sqrt{\frac{1+(p+v_n)-2}{(p+v_n)}} + \sum_{i=1}^{n} \sqrt{\frac{(p+v_n)+(k+1)-2}{(p+v_n)(k+1)}} + \sum_{i=1}^{k} \sqrt{\frac{(k+1)+(1)-2}{(k+1)}}$$

$$= (p)\left(\sqrt{\frac{1+(p+v_n)-2)}{p+v_n}}\right) + (n)\left(\sqrt{\frac{(p+v_n)+(k+1)-2}{(p+v_n)(k+1)}}\right) + (k)\left(\sqrt{\frac{k}{k+1}}\right).$$

□

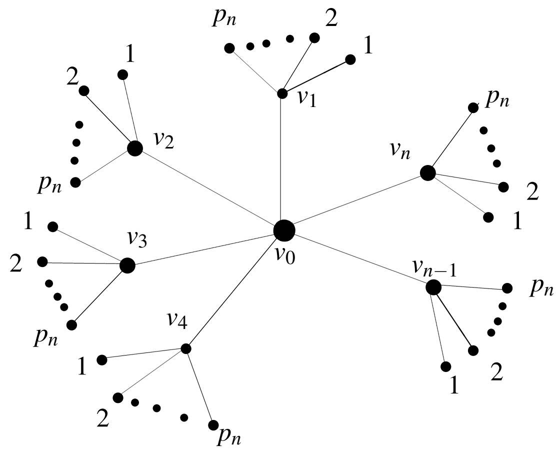

Figure 5: The thorn star $S_n(p_1, p_2, \cdots, p_{n-1})$.

Theorem 2.13. *The reciprocal Randic index of a thorn star $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ is.*

$$RR(S_n(p_1, p_2, \cdots, p_{n-1}, P_n)) = (n)\left(\sqrt{(n)(P_n+1)}\right) + (p)\left(\sqrt{(P_n+1)}\right).$$

Proof. Consider the star graph S_n and choose a labeling for its vertices such that its terminal vertices have numbers $1, 2, \cdots, v_{n-1}, v_n$ and its central vertex has number v_0. Let $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ denote the thorn star obtained by attaching p_i terminal vertices to the vertex i of S_n for $i = 1, 2, ..., n-1, n$ (see Fig. 5). Let $d(v_0) = n$, $d(v_i) = p_n + 1$ and $d(P_n) = 1$. Then

$$(S_n(p_1, p_2, \cdots, p_{n-1}, p_n)) = \sum_{i=1}^{n} \sqrt{(d(v_0))(d(v_i))} + \sum_{i=1}^{p} \sqrt{(d(v_i))(d(p_n))}$$
$$= \sum_{i=1}^{n} \sqrt{(n)(p_n+1)} + \sum_{i=1}^{p} \sqrt{(p_n+1)(1)}$$
$$= (n)\left(\sqrt{(n)(P_n+1)}\right) + (p)\left(\sqrt{(P_n+1)}\right).$$

□

Theorem 2.14. *The reduced reciprocal Randic index of a thorn star $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ is.*

$$RRR(S_n(p_1, p_2, \cdots, p_{n-1}, p_n)) = (n)\left(\sqrt{(n-1)(P_n)}\right).$$

Proof. Consider the star graph S_n and choose a labeling for its vertices such that its terminal vertices have numbers $1, 2, \cdots, v_{n-1}, v_n$ and its central vertex has number v_0. Let $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ denote the thorn star obtained by attaching p_i terminal vertices to the vertex i of S_n for $i = 1, 2, ..., n-1, n$ (see Fig. 5). Let $d(v_0) = n$, $d(v_i) = p_n + 1$ and $d(p_n) = 1$. Then we have,

$$RRR(S_n(p_1, p_2, \cdots, p_{n-1}, p_n)) = \sum_{i=1}^{n} \sqrt{(d(v_0)-1)(d(v_i)-1)} + \sum_{i=1}^{p} \sqrt{(d(v_i)-1)(d(p_n)-1)}$$
$$= \sum_{i=1}^{n} \sqrt{((n)-1)(p_n+1-1)} + \sum_{i=1}^{p} \sqrt{(p_n+1-1)(1-1)}$$
$$= (n)\left(\sqrt{(n-1)(p_n)}\right).$$

□

Theorem 2.15. *The atom- bond connectivity index of a thorn star $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ is.*

$$ABC(S_n(p_1, p_2, \cdots, p_{n-1}, p_n)) = (n)\left(\sqrt{\frac{n+(p_n+1)-2)}{(n)(p_n+1)}}\right) + (p)\left(\sqrt{\frac{p_n}{(p_n+1)}}\right).$$

Proof. Consider the star graph S_n and choose a labeling for its vertices such that its terminal vertices have numbers $1, 2, \cdots, v_{n-1}, v_n$ and its central vertex has number v_0. Let $S_n(p_1, p_2, \cdots, p_{n-1}, p_n)$ denote the thorn star obtained by attaching p_i terminal vertices to the vertex i of S_n for $i = 1, 2, ..., n-1, n$ (see Fig. 5). Let $d(v_0) = n$, $d(v_i) = p_n + 1$ and $d(p_n) = 1$. Then

$$ABC(S_n(p_1, p_2, \cdots, p_{n-1}, p_n)) = \sum_{i=1}^{n} \left(\sqrt{\frac{d(v_0)+d(v_n)-2}{(d(v_0))(d(v_n))}}\right) + \sum_{i=1}^{p} \left(\sqrt{\frac{d(v_n)+d(p_n)-2}{(d(v_n))(d(p_n))}}\right)$$
$$= \sum_{i=1}^{n} \left(\sqrt{\frac{n+(p_n+1)-2}{(n)(p_n+1)}}\right) + \sum_{i=1}^{p} \left(\sqrt{\frac{(p_n+1)+(1)-2}{(p_n+1)(1)}}\right)$$
$$= (n)\left(\sqrt{\frac{n+(p_n+1)-2)}{(n)(p_n+1)}}\right) + (p)\left(\sqrt{\frac{p_n}{(p_n+1)}}\right).$$

□

3. Acknowledgments

This research is supported by $UGC-SAP-DRS-II, No.F.510/12/DRS-II/2018(SAP-I)$, Dated: April 9^{th}, 2018.

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ISSN(P):2319 – 3786

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