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On almost contra δgp-continuous functions in topological spaces

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Abstract

The aim of this paper is to introduce a new class of almost contra continuity. The notion of almost contra δgp-continuous functions is introduced and studied.

Keywords

δgp-open set,δgp-closed set,almost contra pre-continuous function,almost contra δgp-continuous function.

AMS Subject Classification

54C08,54C10.

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1. Introduction

Recently, Baker(resp,Ekici,Balasubramanian and Laxmi) introduced and investigated the notions of almost contra continuity [3] (resp, almost contra pre-continuity[10] and almost contra gpr-continuity [4] as a continuation of research done by Dontchev(resp,S.Jafari and T.Noiri and P.Jeyalakshmi) on the notion of contra continuity [9] (resp,contra pre-continuity [16] and contra gpr-continuity [18].

In this paper, we offer a stronger form of almost contra gpr-continuity called almost contra δgp-continuity. Also,some properties and characterizations of the said type of functions are investigated.

Throughout this paper, (U,τ),(V,σ) and (W,η)(or simply U,V and W) represent topological spaces on which no separation axioms are assumed unless explicitly stated and f:(U,τ)$\rightarrow$(V,σ) or simply f:U $\rightarrow$ V denotes a function f of a topological space U into a topological space V. Let M $\subseteq$ U, then cl(M) = $\cap\{$F: M $\subseteq$ F and $F^c \in \tau$ $\}$ is the closure of M. Also,int(M) = $\cup\{$O: O $\subseteq$ M and O $\in \tau$ $\}$ is the interior of M. The class of δgp-open (resp, δgp-closed, open, closed, regular open, regular closed, δ-preopen, δ-semiopen, e^*-open, preopen, semiopen, β-open and clopen) sets of (U,τ) is denoted by δGPO(U) (resp,δGPC(U), O(U), C(U), RO(U), RC(U), δPO(U), δSO(U), e^*O(U), PO(U), SO(U), βO(U) and CO(U)).

2. Preliminaries

Definition 2.1. *A set M $\subseteq$ U is called δ-closed [36] if M = $cl_\delta(M)$ where $cl_\delta(M)$ = $\{$ p $\in$ U :int(cl(G)) $\cap$ M $\neq \phi$, G $\in$ τ and p $\in$ G$\}$. The complement of a δ-closed set is called δ-open*

Definition 2.2. *A set M $\subseteq$ U is called pre-closed [21] (resp, b-closed [1], regular-closed [33], semi-closed [19] and α-closed [22] if cl(int(M)) $\subseteq$ M (resp, cl(int(M)) $\cap$ int(cl(M)) $\subseteq$ M, M = cl(int(M)), int(cl(M)) $\subseteq$ M and cl(int(cl(M))) $\subseteq$ M).*

Definition 2.3. *A set M $\subseteq$ U is called δ-preclosed [27] (resp, e^*-closed [13], δ-semiclosed [26] and a-closed [14]) if cl(int_δ(M)) $\subseteq$ M (resp,int(cl(int_δ(M)) $\subseteq$ M, int(cl_δ(M)) $\subseteq$ M and cl(int(cl_δ(M))) $\subseteq$ M).*

Definition 2.4. *A set M $\subseteq$ U is called:*
(i) δgp-closed [7] (resp, gpr-closed [15] and gp-closed [20]) if pcl(M) $\subseteq$ G whenever M $\subseteq$ G and G is δ-open (resp, regular open and open) in U.
(ii) gδs-closed [5] if scl(M) $\subseteq$ G whenever M $\subseteq$ G and G is δ-open in U

Definition 2.5. *A function f:(U,τ)$\rightarrow$(V,σ) is said to be:*
(i) almost contra continuous [3] (resp,contra R-map [11], δ-continuous [23], almost contra super-continuous [12], almost

contra pre-continuous [10], almost contra gp-continuous, almost contra gpr-continuous [4] and almost contra $g\delta s$-continuous [6]) if $g^{-1}(N)$ is closed (resp, regular closed, δ-open, δ-closed, pre-closed,gp-closed, gpr-closed and $g\delta s$-closed) in (X,τ) for every $N \in RO(Y)$.
(ii) contra continuous [9] (resp,contra pre-continuous [16], contra δgp-continuous [35] and contra gpr-continuous [18]) if $g^{-1}(N)$ is closed (resp,pre-closed,δgp-closed and gpr-closed) in $\cup$ for every $N \in \sigma$.
(iii) perfectly-continuous [24] (resp, almost perfectly-continuous [29]) if $g^{-1}(N) \in CO(\cup)$ for every $N \in \sigma$(resp,$RO(\vee)$.
(iv) R-map [8] if $g^{-1}(N) \in RO(\cup)$ for every $N \in RO(\vee)$

Definition 2.6. *[34] A space $\cup$ is called:*

(i) $T_{\delta gp}$-space[7] if $\delta GPC(\cup)=C(\cup)$.

(ii) $\delta gpT_{\frac{1}{2}}$-space[7] if $\delta GPC(\cup))=PC(\cup)$.

(iii) preregular$T_{\frac{1}{2}}$-space[15] if $GPRC(\cup)=PC(\cup)$

(iv) hyper connected [32] if every open set is dense.

3. Almost contra δgp-continuous functions

Definition 3.1. *A function f:$\cup \to \vee$ is called almost contra δgp-continuous if the inverse image of every regular open set of $\vee$ is δgp-closed in $\cup$.*

Theorem 3.2. *The following are equivalent for f:$\cup \to \vee$*
(i) f is almost contra δgp-continuous
(ii) For every $M \in RC(\vee)$, $f^{-1}(M) \in \delta GPO(\cup)$.

Proof. Clear □

Remark 3.3. *From Definitions 2.5 and 3.1,we have the following diagram for a function f: $\cup \to \vee$:*

$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 5 \longrightarrow 6 \longrightarrow 7$$
$$\uparrow$$
$$8$$

Notation:1-contra R-map. 2-almost contra-super-continuous. 3-almost contra continuity. 4-almost contra pre continuity. 5- almost contra gp-continuity. 6-almost contra δgp-continuity. 7-almost contra gpr-continuity. 8- contra δgp-continuous.
None of these implications is reversible.

Example 3.4. *Consider $(\cup,\tau)$ and $(\vee,\eta)$ where $\cup = \{p,q,r,s\} = \vee$, $\tau = \{\cup, \phi, \{p\}, \{q\}, \{p,q\}, \{p,q,r\}\}$ and $\eta = \{\vee, \phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}, \{p,q,r\}\}$.*
Define f: $(\cup,\tau) \to (\vee,\eta)$ by f(p) = f(r) = q , f(q) = p and f(s) = r. Clearly f is almost contra δgp-continuous but for $\{q\} \in RO(\vee)$, $f^{-1}(\{q\}) = \{p,r\} \notin GPC(\cup)$. Therefore f is not almost contra gp-continuous. Define g: $(\cup,\tau) \to (\vee,\eta)$ by g(p) = p, g(q) = s, g(r) = r and g(s) = q.Then g is almost contra δgp-continuous but for $\{u\} \in O(Y)$, $g^{-1}(\{p\}) = \{p\} \notin \delta GPC(\cup)$.Therefore g is not contra δgp-continuous. Define h: $\cup \to \vee$ by h(p) = h(q) = q, h(r) = p and h(s) = r.Then h is almost contra gpr-continuous but for $\{q\} \in RO(\vee)$, $h^{-1}(\{q\}) = \{p,q\} \notin \delta GC(\cup)$. Therefore h is not almost contra δgp-continuous

Definition 3.5. *[17] A space $\cup$ is called locally indiscrete if $O(\cup)=RO(\cup)$.*

Theorem 3.6. *Let $\vee$ be a locally indiscrete space.Then every almost contra δgp-continuous function f: $\cup \to \vee$ is contra δgp-continuous.*

Proof. Let $\vee$ be a locally indiscrete space.Let $B \in O(\vee)$,then $B \in RO(\vee)$.As f is almost contra δgp-continuous, $f^{-1}(B) \in \delta GPC(\cup)$. Hence f is contra δgp-continuous □

Theorem 3.7. *Let $\cup$ be a locally indiscrete space,then the following statements are equivalent for any $M \subseteq \cup$:*

(i) M is gpr-closed.

(ii) M is δgp-closed.

(iii) M is gp-closed.

Proof. Follows from the Definition 3.5
As a consequence of Theorem 3.7,we have the following result □

Theorem 3.8. *Let $\cup$ be a locally indiscrete space,then the following properties are equivalent:*

(i) f:$\cup \to \vee$ is almost contra gpr-continuous.

(ii) f:$\cup \to \vee$ is almost contra δgp-continuous.

(iii) f:$\cup \to \vee$ is almost contra gp-continuous.

Remark 3.9. *almost contra δgp-continuity and almost contra $g\delta s$-continuity are independent eachother.*

Example 3.10. *In Example 3.4,f is δgp-continuous but it is not a contra δgp-continuous*

Example 3.11. *Consider $(\cup,\tau)$ and $(\vee,\eta)$ as in Example 3.4, Define f: $(\cup,\tau) \to (\vee,\eta)$ by f(p) = q, f(q) = s, f(r) = p and f(s) = r.Then f is almost contra $g\delta s$-continuous but for $\{q\} \in RO(\vee)$, $f^{-1}(\{q\}) = \{p\} \notin \delta GPC(\cup)$. Therefore f is not almost contra δgp-continuous*

Example 3.12. *Consider $(\cup,\tau)$ and $(\vee,\sigma)$ where $\cup = \{p,q,r,s,t\}$, $\vee = \{a,b,c,d\}$, $\tau = \{\cup, \phi, \{p,q\}, \{r,s\}, \{p,q,r,s\}\}$ and $\sigma = \{\vee,\phi,\{a\},\{b\},\{a,b\}, \{a,c\}, \{a,b,c\}\}$.*
Define f: $\cup \to \vee$ by f(p) = a, f(q) = d, f(r) = c and f(s) = b. Clearly f is almost contra δgp-continuous but for $\{a,c\} \in RO(\vee)$, $f^{-1}(\{a,c\}) = \{p,r\} \notin \delta SC(\cup)$.Therefore f is not almost contra $g\delta s$-continuous,

Theorem 3.13. *[34] (1)In extremely disconnected space, every $g\delta s$-closed set is δgp-closed.*
(2)In strongly irresolvable space, every δgp-closed set is $g\delta s$-closed.
As a consequence of Theorem 3.13, we have the following Theorem.

Theorem 3.14. *(1) Let U be extremely disconnected space,then every almost contra gδs-continuous function f:U $\to$ V is almost contra δgp-continuous.*
(2) Let U be strongly irresolvable space.Then every almost contra δgp-continuous function f:U $\to$ V is almost contra gδs-continuous.

Lemma 3.15. *[34] The following are equivalent for any M $\subseteq$ U:*

(1) *M is clopen.*

(2) *M is open and pre-closed.*

(3) *M is open and gp-closed.*

(4) *M is δ-open and δgp-closed.*

(5) *M is regular-open and gpr-closed.*

(6) *M is regular-open and pre-closed.*

(7) *M is δ-open and pre-closed.*

Theorem 3.16. *The following statements are equivalent:*

(1) *f: U $\to$ V is almost perfectly continuous.*

(2) *f: U $\to$ V is almost continuous and almost contra pre-continuous.*

(3) *f: U $\to$ V is almost continuous and almost contra gp-continuous.*

(4) *f: U $\to$ V is δ-continuous and almost contra δgp-continuous.*

(5) *f: U $\to$ V is R-map and almost contra gpr-continuous.*

(6) *f: U $\to$ V is R-map and almost contra pre-continuous.*

(7) *f: U $\to$ V is δ-continuous and almost contra pre-continuous.*

Theorem 3.17. *Let U be a $\delta gpT_{\frac{1}{2}}$-space. Then the following are equivalent:*

(1) *f:U $\to$ V is almost contra pre-continuous.*

(2) *f:U $\to$ V is almost contra gp-continuous.*

(3) *f:U $\to$ V is almost contra δgp-continuous.*

Theorem 3.18. *Let U be a preregular$T_{\frac{1}{2}}$-space. Then the following statements are equivalent:*

(1) *f:U $\to$ V is almost contra pre-continuous.*

(2) *f:U $\to$ V is almost contra gp-continuous.*

(3) *f:U $\to$ V is almost contra δgp-continuous.*

(4) *f:U $\to$ V is almost contra gpr-continuous.*

Theorem 3.19. *Let U be a $T_{\delta gp}$-space. Then the following are equivalent:*

(1) *f:U $\to$ V is almost contra continuous.*

(2) *f:U $\to$ V is almost contra pre-continuous.*

(3) *f:U $\to$ V is almost contra gp-continuous.*

(4) *f:U $\to$ V is almost contra δgp-continuous.*

(5) *f:U $\to$ V is almost contra gpr-continuous.*

Theorem 3.20. *The following properties are equivalent:.*

(1) *f: U $\to$ V is almost contra δgp-continuous.*

(2) *for every M $\in \beta$O(V), f^{-1}(cl(M)) $\in \delta$GPO(U) .*

(3) *for every M $\in$ SO(V), f^{-1}(cl(M)) $\in \delta$GPO(U).*

(4) *for every M $\in$ PO(V), f^{-1}(int(cl(M)))$\in \delta$GPC(U) .*

(5) *for every M $\in$ O(V), f^{-1}(int(cl(M))) $\in \delta$GPC(U) .*

(6) *for every F $\in$ C(V), f^{-1}(int(cl(F)))$\in \delta$GPO(U)*

Proof. (1)$\to$(2) Let M $\in \beta$O(V),then cl(M)$\in$ RC(V).Then by (1),f^{-1}(cl(M)) is δgp-open in U.
(2)$\to$(3) Obvious.
(3)$\to$(4) Let M $\in$ PO(V).Then V$\setminus$int(cl(M)) is regular closed and hence it is semi-open. By (3), f^{-1}(cl(V$\setminus$int(cl(M))) = f^{-1}(V$\setminus$int(cl(M)) = U$\setminus f^{-1}$(int(cl(M))) $\in \delta$GPO(U). Hence f^{-1}(int(cl(M)))$\in \delta$GPC(U).
(4)$\to$(1)Let H $\in$ RO(V).Then H $\in$ PO(V). By (4), f^{-1}(H) = f^{-1}(int(cl(H))) is δgp-closed in U.
(1)$\to$(5) Let H $\in$ O(V). Since int(cl(H)) is regular open,by (1),f^{-1}(int(cl(G))) is δgp-closed in U.
(5)$\to$(1) Similar to (1)$\to$(5)
(1)$\leftrightarrow$(6) Similar to (1)$\leftrightarrow$(5) ☐

Lemma 3.21. *[25] the following properties hold for any M $\subseteq$ U:*

(1) *αcl(M) = cl(M) for every M $\in \beta$O(U).*

(2) *pcl(M) = cl(M) for every M $\in$ SO(U).*

(3) *scl(M) = int(cl(M)) for every M $\in$ PO(U).*

Theorem 3.22. *The following statements are equivalent:.*

(1) *f: U $\to$ V is almost contra δgp-continuous.*

(2) *for every A $\in \beta$O(V), $f^{-1}(\alpha$cl(A)) $\in \delta$GPO(U).*

(3) *for every A $\in$ SO(V), f^{-1}(pcl(A)) $\in \delta$GPO(U).*

(4) *for every A $\in$ PO(V), f^{-1}(scl(A))) $\in \delta$GPC(U).*

Theorem 3.23. *[7] Let M $\subseteq$ U.Then p $\in \delta$gpcl(M) if and only if H $\cap$ M $\neq \Phi$ for every H $\in \delta$GPO(U,p).*

Recall that for a set M $\subseteq$ U, rker(M)=$\cap${G $\in$RO(U):M $\subseteq$ G } where rker(M) is called the kernel of M [11].

Lemma 3.24. *[11] For any sets M,N$\subseteq$ U, the following hold:*

(1) *p ∈ rker(M) if and only if M ∩ F = ϕ for every F ∈ RC(U,p)*

(2) *M ⊆ rker(M) and M = rker(M) if M ∈ RO(U)*

(3) *If M ⊆ N, then rker(M) ⊆ rker(N).*

Definition 3.25. *[34] A space U is called δgp-additive if δGPC(U) is closed under arbitrary intersections.*

Theorem 3.26. *The following properties are equivalent:*

(1) *f:U → V is almost contra δgp-continuous and U is δgp-additive.*

(2) *For each p ∈ U and each N ∈ RC(V,f(p)),there exists an M ∈ δGPO(U,p) such that f(M) ⊆ N.*

(3) *For each p ∈ U and each B ∈ SO(V,f(p)),there exists an A ∈ δGPO(U,p) such that f(A)⊆cl(B).*

(4) *f(δgpcl(C)) ⊆ rker(f(C)) for any M ⊆ U.*

(5) *δgpcl(f^{-1}(D)) ⊆ f^{-1}(rker(D)) for any any D ⊆ V.*

Proof. (1)→(2)Let N ∈ RC(V) such that f(p) ∈ N, then p ∈ f^{-1}(N). By hypothesis, f^{-1}(N) ∈ δGPO(U). Set M =f^{-1}(N), then f(M)=f(f^{-1}(N)) ⊆ N.
(2)→(3) Let B ∈ SO(V) such that f(p)∈ B, then cl(B)∈RC(V). By hypothesis, f^{-1}(cl(B)) ∈ δGPO(U) and p ∈ f^{-1}(cl(B)). Set A = f^{-1}(cl(B)), then f(A)=f(f^{-1}(cl(B))⊆cl(B).
(3)→(4) Let C ⊆ U. Suppose p ∉ f^{-1}[rker(f(C))] which implies f(p) ∉ [rker(f(C))]. Then by Lemma 3.24, there exists a D ∈ RC(V,f(p)) such that f(C) ∩ D = ϕ which implies there exists a D ∈ SO(V,f(p)) such that C∩ f^{-1}(D) = ϕ. Then by (3), there exists a G_p ∈ δGPO(U) such that f(G_p) ⊆ cl(D) = D. Hence f(C∩G_p) ⊆ f(C) ∩ f(G_p) ⊆ f(C) ∩ D = ϕ which implies C ∩ G_p = ϕ. This shows that p ∉ δgpcl(C).
(4)→(5)Let D ⊆ V,then f^{-1}(D) ⊆ U. By (4) and Lemma 3.24, f (δgpcl(f^{-1}(D))) ⊆ rker(f(f^{-1}(D))) ⊆ rker(D). Thus δgpcl(f^{-1}(D)) ⊆ f^{-1}(rker(D)).
(5)→(1): Let H ∈ RO(V). Then by (5) and Lemma 3.24, δgpcl(f^{-1}(H) ⊆ f^{-1}(rker(H)) = f^{-1}(H) and hence δgpcl(f^{-1}(H)) = f^{-1}(H). Since U is δgp-additive, f^{-1}(H) ∈ δGPO(U). □

Theorem 3.27. *The following properties are equivalent:.*

(a) *f: U → V is almost contra δgp-continuous.*

(b) *for every N ∈e^*O(V),f^{-1}(cl_δ(N)) ∈ δGPO(U).*

(c) *for every N ∈ δSO(V,f^{-1}(cl_δ(N)) ∈ δGPO(U).*

(d) *for every N ∈ δPO(V,f^{-1}(int(cl_δ(N))) ∈ δGPC(U).*

(e) *for every N ∈ O(V,f^{-1}(int(cl_δ(N))) ∈ δGPC(U).*

(f) *for every N ∈ C(V,f^{-1}(int(cl_δ(N))) ∈ δGPO(U).*

Proof. Similar to the proof of Theorem 3.20 □

Lemma 3.28. *[2] For a set M ⊆ U,the following properties hold:*

(i) *a-cl(M) = cl_δ(M) for every M ∈ e^*O(U).*

(ii) *δ-pcl(M) = cl_δ(M) for every M ∈ δSO(U).*

(iii) *δ-scl(M) = int(cl_δ(M)) for every M ∈ δPO(U).*

Theorem 3.29. *The following statements are equivalent:*

(i) *f:U → V is almost contra δgp-continuous.*

(ii) *for every H ∈ e^*O(V), f^{-1}(a-cl(H)) ∈ δGPO(U).*

(iii) *for every H ∈ δSO(V), f^{-1}(δ-pcl(H))∈ δGPO(U).*

(iv) *for every H ∈ δPO((V), f^{-1}(δ-scl(H))) ∈δGPC(U).*

Definition 3.30. *A function f:U→V is said to be weakly δgp-continuous if for every open subset H of V, f^{-1}(cl(H)) ∈ δGPO(U).*

Definition 3.31. *[29] A space U is said to be endowed with an almost partition topology if RC(U)=O(U).*

Theorem 3.32. *Every almost contra δgp-continuous function f:(U,τ)→(V,σ) is weakly δgp-continuous.*
If, in addition, σ is almost partition topology, then the converse of the above statement is true.

Proof. Let H ∈ O(V), then cl(H)∈RC(V). By hypothesis, f^{-1}(cl(H)) is δgp-open in U. Therefore f is weakly δgp-continuous.
Conversely, let σ be almost partition topology and N ∈ RC(V). Then N ∈ O(V).The weakly δgp-continuity of f implies f^{-1}(cl(N)) = f^{-1}(N) ∈ δGPO(U). □

Theorem 3.33. (i) *If f: U → V is almost contra δgp-continuous and g: V → W is contra R-map, then (g∘f):U → W is almost contra δgp-continuous*

(ii) *If f: U → V is contra δgp-continuous and g: V → W is almost continuous, then (g∘f):U → W is almost contra δgp-continuous*

(iii) *If f: U → V is δgp-irresolute and g: V → W is almost contra δgp-continuous,then (g∘f):U → W is almost contra δgp-continuous*

Proof. (i) Let N ∈ RO(W). Then g^{-1}(N) ∈ RO(V) since g is contra R-map. The almost contra δgp-continuity of f implies $f^{-1}[g^{-1}$(N))] = $(g \circ f)^{-1}$((N)) ∈ δGPC(U). Hence g∘f is almost contra δgp-continuous.
The proofs of (ii) and (iii) are similar to (i). □

Definition 3.34. *[35] A function f:U → V is called pre δgp-closed if f(M) ∈ δGPC(V) for every M ∈ δGPC(U) .*

Theorem 3.35. *Let f:U → V be pre δgp-closed surjection and g: V→ W be a function such that g∘f:U→W is almost contra δgp-continuous,then g is almost contra δgp-continuous.*

Proof. Let B ∈ RO(W). Then $(g\circ f)^{-1}$(B) = $f^{-1}(g^{-1}$(B)) is δgp-closed in U. As f is pre δgp-closed surjection, f($f^{-1}(g^{-1}$(B))) = $(g)^{-1}$(B) is δgp-closed in V. Therefore g is almost contra δgp-continuous. □

Theorem 3.36. *If the graph function g:U→U×V of f: U → V, defined by g(p)=(p,f(p)) for each p ∈ U is almost contra δgp-continuous, then f is almost contra δgp-continuous.*

Proof. Let N ∈ RO(V), then U × N ∈ RO(U×V). The almost contra δgp-continuity of g implies f^{-1}(N) = g^{-1}(X×N) ∈ δGPC(U). Therefore f is almost contra δgp-continuous

Recall that for a function f:U → V, the subset G_f = {(x,f(x)):x ∈U} ⊂ U×V is said to be graph of f. □

Definition 3.37. *A graph G_f of a f:U → V is said to be δgp-closed graph if for each (p,q) ∉ G_f, there exist M ∈ δGPO(X,p) and N ∈ O(V,q) such that (U × V) ∩ G_f = ϕ.*

As a consequence of Definition 3.37 and the fact that for any subsets A ⊆ U and B ⊆ V , (A × B) ∩ G_f = ϕ if and only if f(A) ∩ B = ϕ,we have the following result.

Lemma 3.38. *For a graph G_f of a f:U → V, the following statements are equivalent:*
(1) G_f is δgp-closed in U × V
(2)For each (p,q) ∉G_f, there exist M ∈ δGPO(U,p) and N ∈ O(U,q) such that f(M) ∩ N = ϕ.

Definition 3.39. *A space U is called δgp-T_1 space if for any pair of distinct points p and q, there exist G,H ∈ δGPO(U) such that p ∈ G, q ∉ G and q ∈ H, p ∉ H.*

Theorem 3.40. *If f:U → V has a δgp-closed graph G_f. Then U is δgp-T_1 if f is injective.*

Proof. Let f be an injection and x_1,x_2 ∈ U with $x_1 \neq x_2$.Then f(x_1)≠f(x_2) so that (x_1,f(x_2)) ∉ G_f.By theorem , there exist M ∈ δ GPO(U,x_1) and N ∈ O(V,f(x_2)) such that f(M) ∩ N = ϕ.Then f(x_2) ∉ f(M) implies x_2 ∉ M and it follows that U is δgp-T_1. □

Theorem 3.41. *If f:U → V has a δgp-closed graph G_f. Then V is T_1 if f is surjective.*

Proof. Let f be a surjection and y_1,y_2 ∈ V with $y_1 \neq y_2$.Then f(p) = y_2 for some p ∈ U and (p,y_2) ∉ G_f.By Lemma 4.14, there exist M ∈ δGPO(U,p) and N ∈ O(V,y_1) such that f(M) ∩ N = ϕ. It follows that y_2 ∉ N.Hence V is T_1. □

Corollary 3.42. *If f:U → V has a δgp-closed graph G_f. If f is bijective, then both U and U are δgp-T_1*

Proof. Follows from Theorems 3.40 and 3.41 □

Definition 3.43. *[30] A space U is said to be weakly Hausdorff if every point of U is expressed by the intersection of regular closed sets of of U*

Theorem 3.44. *If an injective f:U → V is almost contra δgp-continuous and V is weakly Hausdorff, then U is δgp-T_1.*

Proof. Let V be weakly Hausdorff and p,q ∈ V with p ≠ q. Then there exist A and B ∈ RC(V) such that f(p) ∈ A, f(q) ∈ B and A ∩ B = ϕ. The almost contra δgp-continuity of f implies f^{-1}(A) and f^{-1}(B) ∈ δGPO(U) such that p ∈ f^{-1}(A), q ∈ f^{-1}(B) and f^{-1}(A) ∩ f^{-1}(B) = ϕ. This shows that U is δgp-T_1. □

Definition 3.45. *A space U is said to be:*
(i) δgp-connected [35] if U is not the union of two disjoint non empty δgp-open sets.
(ii)δgp-ultra connected if every two non-void δgp-closed subsets of U intersect.

Theorem 3.46. *If a surjective f:U → V is almost contra δgp-continuous.Then*
(1)V is connected if U is δgp-connected.
(2) V is hyper connected if U is δgp-ultra connected.

Proof. (1) On the contrary assume that V is not a connected space,then their exist P(≠ϕ) and Q(≠ϕ) ∈ O(V) such that P ∩ Q =ϕ and V = P ∪ Q. Also, P and Q ∈CO(V). Since f is almost contra δgp-continuous, f^{-1}(P), f^{-1}(Q) ∈ δGPO(U), f^{-1}(P) ∩ f^{-1}(Q) = ϕ and U = f^{-1}(P) ∪ f^{-1}(Q).This shows that U is not δgp-connected. (2) Similar to (1) □

Definition 3.47. *A space U is called:*
(i)[31]ultra Hausdorff if for each p,q∈ U with p ≠ q, there exist disjoint clopen sets A and B ∈ CO(U) such that p ∈ A, q ∈ B and A ∩ B = ϕ.
(ii)[35]δgp-Hausdroff if for each p,q∈ U with p ≠ q, there exist disjoint clopen sets A and B ∈ δGPO(U) such that p ∈ A, q ∈ B and A ∩ B = ϕ

Theorem 3.48. *If an injective f:U → V is almost contra δgp-continuous and V is ultra Hausdorff, then U is δgp-Hausdroff.*

Proof. Let f be injective and and p,q ∈ V with p ≠ q.Then f(p) ≠ f(q). Since V is ultra Hausdorff,there exist M and N ∈ CO(V) such that p ∈ M , q ∈ N and M ∩ N =ϕ.The almost contra δgp-continuity of f implies f^{-1}(M) and f^{-1}(N) ∈ δGPO(U) such that p ∈f^{-1}(M) and q ∈ f^{-1}(N) and f^{-1}(M) ∩ f^{-1}(N) =ϕ.Hence U is δgp-Hausdroff □

Definition 3.49. *A space U is called:*
(i)[31] Ultra normal if every pair of disjoint closed sets can be separated by disjoint clopen sets.
(ii)δgp-normal if every pair of disjoint closed sets can be separated by disjoint δgp-open sets.

Theorem 3.50. *If f:U → V is almost contra δgp-continuous closed injection and V is ultra normal, then U is δgp-normal.*

Proof. Let f be a closed injection and E , F ∈ C(U) with E ∩ F = ϕ. Then f(E),f(F) ∈ C(V) and f(E)∩f(F) = ϕ. Since V is ultra normal, there exists disjoint clopen sets M and N in V such that f(E) ⊂ M and f(F)⊂ N. This implies E ⊂ f^{-1}(M) and F ⊂ f^{-1}(N). Since f is an almost δgp-continuous injection, f^{-1}(M) and f^{-1}(N)∈ δGPO(U) such that f^{-1}(M) ∩ f^{-1}(N) =ϕ. Therefore U is δgb-normal. □

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