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k-Lehmer three mean labeling of some graphs

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Abstract

A function h is called k- Lehmer-3 mean graph G with r vertices and s edges, if it is possible to label the vertices $v \in V$ with distinct labels $h(x)$ from $k, k+1, k+2, \ldots, k+s$ in such a way that each edge $e = xy$ is labeled with $h(e) = \left\lceil \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rceil$ (or) $\left\lfloor \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rfloor$ then the edge labels are distinct.In this paper we proved k-Lehmer-three mean labeling of some standard graphs.

Keywords

Lehmer three mean labeling, k- Lehmer three mean labeling, path, comb, caterpillar, kite.

AMS Subject Classification

05C78.

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Contents

1. Introduction

Graphs described here is simple,undirected and connected graphs.Let V(G) and E(G) be stated as the vertex and edge set of graph G.We refer Gallian for more comprehensive survey [1].We follow Harrary for some standard words,expressions and symbols[2],The concept and notation of mean labeling was first introduced by S somasundaram and R Ponraj[3].S Somasundaram,S S Sandya and T Pavithra introduced the concept of Lehmer three mean graph [4]. Here we investigating some more standard graphs in K-Lehmer three mean graphs.

2. Preliminaries

Definition 2.1. *Let G be a (r,s) graph.A function h is called Lehmer three mean labeling of graph G,if it is possible to label the vertices $v \in V$ with distinct labels $h(x)$ from $1, 2, 3, \ldots, s+1$ in such a way that each edge $e = xy$ is labeled with $h(e) = \left\lceil \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rceil$ (or) $\left\lfloor \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rfloor$ then the edge labels are distinct.A graph which admits a Lehmer three mean labeling is called Lehmer three mean graph.*

Definition 2.2. *Let G be a (r,s) graph.A function h is called k-Lehmer three mean labeling of graph G,if it is possible to label the vertices $v \in V$ with distinct labels $h(x)$ from $k, k+1, k+2, \ldots, k+s$ in such a way that each edge $e = xy$ is labeled with $h(e) = \left\lceil \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rceil$ (or) $\left\lfloor \frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2} \right\rfloor$ then the edge labels are distinct.A graph which admits a k-Lehmer three mean labeling is called k-Lehmer three mean graph.*

3. Main Theorem

Theorem 3.1. *The path P_n is k-Lehmer three mean graph for all k and $n \geq 2$.*

Proof. Let $V(P_n) = \{v_j, 1 \leq j \leq n\}$ and
$E(P_n) = \{e_j = (v_j, v_{j+1}); 1 \leq j \leq n-1\}$
We define $h : V(P_n) \to \{k, k+1, k+2, \ldots, k+s\}$ by

$$h(v_j) = k+j-1 \qquad 1 \leq j \leq n$$

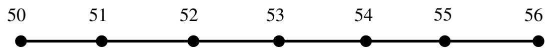
Then edge labels are

$$h^*(e_j) = k+j-1 \qquad 1 \leq j \leq n-1$$

.

Hence path is k-Lehmer three mean graph. □

Example 3.2. 50-Lehmer three mean labeling of P_7.



Theorem 3.3. *A comb $P_n \odot K_1$ is a k-Lehmer three mean graph.*

Proof. Let $V(P_n \odot K_1) = \{u_j v_j, 1 \leq j \leq n\}$ and
$E(P_n \odot K_1) = \{u_j, u_{j+1}; 1 \leq j \leq n-1\} \cup \{u_j v_j, 1 \leq j \leq n\}$
We define $h : V(P_n) \rightarrow \{k, k+1, k+2, \ldots, k+s\}$ by

$$h(u_j) = k + 2j - 2 \qquad 1 \leq j \leq n$$

$$h(v_j) = k + 2j - 1 \qquad 1 \leq j \leq n$$

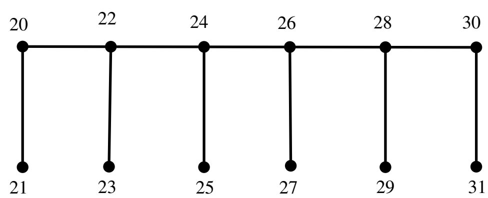
Then edge labels are

$$h^*(u_j, u_{j+1}) = k + 2j - 1 \qquad 1 \leq j \leq n-1$$

$$h^*(u_j, v_j) = k + 2j - 2 \qquad 1 \leq j \leq n-1$$

Hence $P_n \odot K_1$ is k-Lehmer three mean graph. □

Example 3.4. 20-Lehmer three mean labeling of $P_6 \odot K_1$.



Theorem 3.5. *A graph G obtained with pendant edges attached to both sides of each vertex of P_n. Then G is k-Lehmer-3 mean graph.*

Proof. A graph G obtained with pendent edges to both sides of each vertex of P_n.
We define $h : V(G) \rightarrow \{k, k+1, k+2, \ldots, k+s\}$ by

$$h(u_j) = k + 3j - 3 \qquad 1 \leq j \leq n$$

$$h(v_j) = k + 3j - 2 \qquad 1 \leq j \leq n$$

$$h(w_j) = k + 3j - 1 \qquad 1 \leq j \leq n$$

Then edge labels are

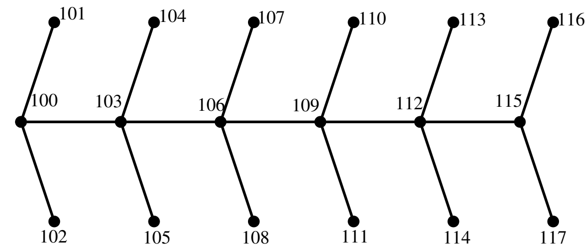
$$h^*(u_j, u_{j+1}) = k + 3j - 1 \qquad 1 \leq j \leq n-1$$

$$h^*(u_j, v_j) = k + 3j - 3 \qquad 1 \leq j \leq n$$

$$h^*(u_j, w_j) = k + 3j - 2 \qquad 1 \leq j \leq n$$

Hence caterpillar is k-Lehmer three mean graph. □

Example 3.6. 100-Lehmer three mean labeling of caterpillar.



Theorem 3.7. *A graph G attaching $K_{1,2}$ to each pendant vertex of a comb forms a k- Lehmer-3 mean graph.*

Proof. Let $V((P_n \odot K_1) \odot K_{1,2}) = \{u_j v_j w_j x_j; 1 \leq j \leq n\}$ and
$E((P_n \odot K_1) \odot K_{1,2}) = \{u_j, u_{j+1}; 1 \leq j \leq n-1\} \cup \{u_j v_j, v_j w_j, v_j x_j; 1 \leq j \leq n\}$
We define $h : V((P_n \odot k_1) \odot k_{1,2}) \rightarrow \{k, k+1, k+2, \ldots, k+s\}$ by

$$h(u_j) = k + 4j - 4 \qquad 1 \leq j \leq n$$

$$h(v_j) = k + 4j - 3 \qquad 1 \leq j \leq n$$

$$h(w_j) = k + 4j - 2 \qquad 1 \leq j \leq n$$

$$h(x_j) = k + 4j - 1 \qquad 1 \leq j \leq n$$

Then edge labels are

$$h^*(u_j, u_{j+1}) = k + 4j - 1 \qquad 1 \leq j \leq n-1$$

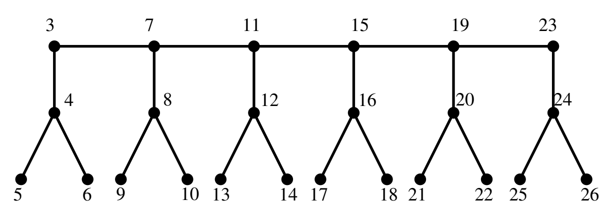
$$h^*(u_j, v_j) = k + 4j - 4 \qquad 1 \leq j \leq n$$

$$h^*(v_j, w_j) = k + 4j - 2 \qquad 1 \leq j \leq n$$

$$h^*(v_j, x_j) = k + 4j - 3 \qquad 1 \leq j \leq n$$

Hence $V((P_n \odot K_1) \odot K_{1,2})$ is k-Lehmer three mean graph. □

Example 3.8. 3- Lehmer three mean labeling of $V((P_6 \odot K_1) \odot K_{1,2})$.



Theorem 3.9. *A graph G attaching each vertex of P_n by the central vertex of $K_{1,2}$. Then G is a k-Lehmer-3 mean labeling.*

Proof. Let $V(P_n \odot K_{1,2}) = \{u_j v_j, w_j; 1 \leq j \leq n\}$ and
$E(P_n \odot K_{1,2}) = \{u_j, u_{j+1}; 1 \leq j \leq n-1\} \cup \{u_j v_j, 1 \leq j \leq n\}$
We define $h : V(P_n \odot k_{1,2}) \rightarrow \{k, k+1, k+2, \ldots, k+s\}$ by

$$h(u_j) = k + 3j - 3 \qquad 1 \leq j \leq n$$

$$h(v_j) = k + 3j - 2 \qquad 1 \leq j \leq n$$

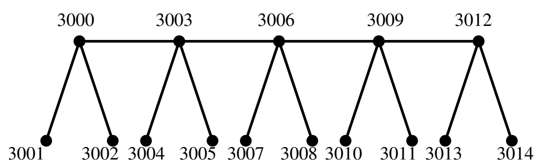
$$h(w_j) = k + 3j - 1 \qquad 1 \leq j \leq n$$

Then edge labels are

$$h^*(u_j,u_{j+1}) = k+3j-1 \qquad 1 \le j \le n-1$$
$$h^*(u_j,v_j) = k+3j-3 \qquad 1 \le j \le n$$
$$h^*(v_j,w_j) = k+3j-2 \qquad 1 \le j \le n$$

Hence $V(P_n \odot K_{1,2})$ is k-Lehmer three mean graph. □

Example 3.10. 3000-Lehmer three mean labeling of $P_5 \odot K_{1,2}$



Theorem 3.11. *A graph G which identifying a pendant vertex P_n and an end vertex C_3.Then G is k-Lehmer three mean labeling.*

Proof. Let P_n be $v_1, v_2, v_3, \ldots, v_n$ and uvx be C_3.
We define $h : V(G) \to \{k, k+1, k+2, \ldots, k+s\}$ by

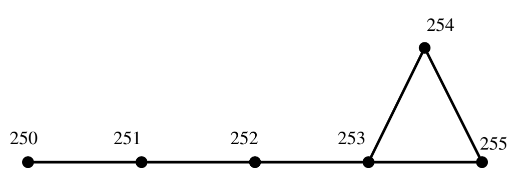
$$h(v_j) = k+j-1 \qquad 1 \le j \le n$$
$$h(u) = k+2n-4$$
$$h(x) = k+2n-3$$

Then edge labels are

$$h^*(v_jv_{j+1}) = k+j-1 \qquad 1 \le j \le n-1$$
$$h^*(v_nu) = k+2n-5$$
$$h^*(v_nx) = k+2n-3$$
$$h^*(ux) = k+2n-4$$

Hence G is a k-Lehmer three mean graph. □

Example 3.12. 250-Lehmer three mean labeling of G.



4. Conclusion

From this paper, we get a knowledge of necessary and sufficient conditions for a graph to be a k-Lehmer three mean labelled and also we have attained some graphs which has k-Lehmer three mean labeling.

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References

[1] J.A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatories*, (2019), DS6.

[2] F. Harary, *Graph theory*, Narosa Publication House Reading, New Delhi, 1988.

[3] S. Somasundaram and R. Ponraj, Mean Labeling of Graphs, *National Academy of Science Letters*, 26(2003), 210–213.

[4] S. Somasundaram, S.S. Sandhya and T.S. Pavithra, Lehmer-3 mean labeling of graphs, *International Journal of Mathematical Forum*, 12(17)(2017), 819–825.

[5] S. Somasundaram and R. Ponraj and S. S. Sandhya, Harmonic mean labeling of graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing*, to appear.

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