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Fuzzy α-ψ^*-operator in fuzzy automata orbit structure spaces

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Abstract
In this paper, the notion of fuzzy automata orbit structure spaces is introduced and the concepts of fuzzy automata orbit-α-ψ^*-open subsystems and fuzzy automata orbit-α-ψ^*-closed subsystems are studied with suitable examples. Also, the notions of fuzzy automata orbit-α-ψ^*-co-kernel subsystems, fuzzy automata orbit-α-ψ^*-kernel subsystems, fuzzy automata orbit-α-ψ^*-meager* subsystems and fuzzy automata orbit -α-ψ^*-comeager* subsystems are introduced and some of their properties are studied.

Keywords
Fuzzy automata orbit structure spaces, $FA\mathscr{O}_\phi$-α-ψ^*-open subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-closed subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-co-kernel subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-kernel subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-meager* subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-comeager* subsystems and fuzzy automata orbit-α-ψ^*-co-kernel spaces.

AMS Subject Classification
54A40, 03E72.

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Contents

1. Introduction

Zadeh [8] initiated fuzzy set in 1965. In 1968, Chang [1] characterized fuzzy topological space. The notion of an automaton was first fuzzified by Wee [7]. Later, the concepts of fuzzy subsystems and strong fuzzy subsystems of a fuzzy finite state machine (briefly, ffsm) were introduced and studied by Malik and Mordeson [4]. The concept of orbit function in general metric space was introduced by Devaney[3]. In this paper, the notion of fuzzy automata orbit structure spaces is introduced. Also, the notions of fuzzy automata orbit-α-ψ^*-co-kernel subsystems, fuzzy automata orbit-α-ψ^*-kernel subsystems, fuzzy automata orbit-α-ψ^*-meager* subsystems and fuzzy automata orbit -α-ψ^*-comeager* subsystems are introduced and some of their properties are studied.

2. Preliminaries

In this section, some basic concepts of fuzzy automaton, fuzzy orbit under the function, etc. have been recalled. Also, related results and propositions are collected from various research articles

Definition 2.1. [8] Let X be a space of points (objects). A fuzzy set A in X is characterized by a membership function $f_A : X \to [0,1]$.

Definition 2.2. [5] A fuzzy automaton is a triple $M = (Q,X,\delta)$, where Q is a set(of states of M), X is a monoid (the input monoid of M), whose identity shall be denoted as e, and δ is a fuzzy subset of $Q\times X\times Q$, *i.e.*, a map $\delta : Q\times X\times Q \to [0,1]$, such that $\forall q,p \in Q$, $\forall x,y \in X$.
(i) $\delta(q,e,p) = 1$ or 0, according as $q = p$ or $q \neq p$,
(ii) $\delta(q,xy,p) = \vee\{\delta(q,x,r)\wedge\delta(r,y,p) : r \in Q\}$.

Notation 2.1. For any non-empty set of states Q, I^Q denotes the collection of all functions from Q into I, where I is the unit interval $[0,1]$.

Definition 2.3. [6] $\lambda \in I^Q$ is called a fuzzy subsystem of (Q,X,δ) if
$\lambda(q) \geq \lambda(p)\wedge\delta(p,x,q), \forall p,q \in Q, x \in X.$

Proposition 2.1. [6] The function $c : I^Q \to I^Q$ defined as $c(\lambda)(q) = \vee \{ \vee \{ \lambda(p) \wedge \delta(p,x,q) : x \in X \} : p \in Q \}$, $\forall \lambda \in I^Q, \forall q \in Q$. is a kuratowski saturated fuzzy closure operator on Q.

Proposition 2.2. [6] $\lambda \in I^Q$ is a fuzzy subsystem of (Q,X,δ) iff $c(\lambda) = \lambda$. (*i.e.*, iff λ is closed with respect to the fuzzy topology induced by c on Q)

Definition 2.4. [2] A fuzzy subset λ of Q is said to be a generating fuzzy set of M if $c(\lambda) = 1$.

Definition 2.5. [3] Orbit of a point x in X under the mapping f is of

$$O_f(x) = \{x, f(x), f^2(x), \dots\}.$$

3. Fuzzy automata orbit structure spaces

In this section, fuzzy automata orbit structure spaces, $FA\mathscr{O}_\phi$-α-ψ^*-co-kernel subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-kernel subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-meager* subsystems, $FA\mathscr{O}_\phi$-α-ψ^*-comeager* subsystems and fuzzy automata orbit α-ψ^*-co-kernel spaces are introduced and some of their properties are discussed. Further, some equivalent statements are established.

Definition 3.1. Let $M = (Q,X,\delta)$ be a fuzzy automaton, where Q is a set (of states of M), X is a monoid (the input monoid of M), whose identity shall be denoted as e, and δ is a fuzzy subset of $Q \times X \times Q$, *i.e.*, a map $\delta : Q \times X \times Q \to [0,1]$, such that $\forall q, p \in Q, \forall x, y \in X$.

(i) $\delta(q,e,p) = 1$ or 0, according as $q = p$ or $q \neq p$ respectively.

(ii) $\delta(q,xy,p) = \vee\{\delta(q,x,r) \wedge \delta(r,y,p) : r \in Q\}$.

Let $\phi : I^Q \to I^Q$ be any mapping. For any $\lambda \in I^Q$, the fuzzy automata orbit subsystem under the mapping ϕ is denoted by $FA\mathscr{O}_\phi(\lambda)$ and defined as $FA\mathscr{O}_\phi(\lambda) = \{\lambda \wedge \phi(\lambda) \wedge \phi^2(\lambda) \wedge \dots\} \in I^Q$. For all $\mu \in I^Q$ and $q \in Q$, $c(\mu)(p) = \vee_{q\in Q} \{ \vee_{x\in X} \{ (\mu)(q) \wedge \delta(q,x,p)\}\}$ is a Kuratowski saturated fuzzy closure operator on Q. Let $\tau = \{FA\mathscr{O}_\phi(\mu) \in I^Q : c(1_Q - FA\mathscr{O}_\phi(\mu)) = (1_Q - FA\mathscr{O}_\phi(\mu))$, where $(1_Q - FA\mathscr{O}_\phi(\mu))$ is the fuzzy complement of $FA\mathscr{O}_\phi(\mu)\}$ be the collection of fuzzy subsystems which satisfies the following axioms :

(i) $0_Q, 1_Q \in \tau$;

(ii) If $\gamma_1, \gamma_2 \in I^Q$ and $FA\mathscr{O}_\phi(\gamma_1), FA\mathscr{O}_\phi(\gamma_2) \in \tau$, then $FA\mathscr{O}_\phi(\gamma_1) \wedge FA\mathscr{O}_\phi(\gamma_2) \in \tau$;

(iii) If $\gamma_i \in I^Q$ and $FA\mathscr{O}_\phi(\gamma_i) \in \tau$ for each $i \in J$,where J is an indexed set, then $\vee_{i\in J} FA\mathscr{O}_\phi(\gamma_i) \in \tau$.

Then, the ordered pair (Q,τ) is said to be a fuzzy automata orbit structure space iff there exists a fuzzy automaton (Q,X,δ) such that τ is a fuzzy topology associated with (Q,X,δ). Moreover, the members of τ are called the fuzzy automata orbit open subsystems and their fuzzy complements are called the fuzzy automata orbit closed subsystems.

Notation 3.1. Throughout this paper, 0_Q takes the membership value $\mathscr{M}_{0_Q}(q) = 0$, for all $q \in Q$ and 1_Q takes the membership value $\mathscr{M}_{1_Q}(q) = 1$, for all $q \in Q$.

Example 3.1. Let $M = (Q,X,\delta)$ be a fuzzy automaton, where $Q = X = \{0,1,2,\dots\dots\}$ and $\delta : Q \times X \times Q \to [0,1]$ be given by

$$\delta(q,0,p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q,x_0,p) = 0.35$, $\delta(q,x_0,q) = 0.75$, $\delta(p,x_0,p) = 0.8$, $\delta(p,x_0,q) = 0.7$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p, q \in Q$. For other $p, q \in Q$ and $x \in X$, $\delta(p,x,q) = 0$. Let $\lambda,\ \mu \in I^Q$ be defined as follows : $\lambda(1) = 0.45$, $\lambda(2) = 0.55$, $\mu(1) = 0.33$, $\mu(2) = 0.5$ and for other $r \in Q$, $r \neq 1,2$, $\lambda(r) = 0$, $\mu(r) = 0$. Let $\phi : I^Q \to I^Q$ be a mapping defined by

$$\eta_\phi(x) = \begin{cases} \eta(2), & \text{if } x = 1, \\ \eta(1), & \text{if } x = 2, \\ 0, & \text{otherwise} \end{cases}$$

for all $\eta \in I^X$. Then $FA\mathscr{O}_\phi(\lambda)(1) = 0.45$, $FA\mathscr{O}_\phi(\lambda)(2) = 0.45$, $FA\mathscr{O}_\phi(\mu)(1) = 0.33$, $FA\mathscr{O}_\phi(\mu)(2) = 0.33$. The Kuratowski saturated fuzzy closure operator $c : I^Q \to I^Q$ on Q is defined as

$$c(\mu)(p) = \vee_{q\in Q} \{ \vee_{x\in X} \{ (\mu)(q) \wedge \delta(q,x,p)\}\}.$$

It is clear that
$c(FA\mathscr{O}_\phi(\lambda)) = FA\mathscr{O}_\phi(\lambda)$,
$c(FA\mathscr{O}_\phi(\mu)) = FA\mathscr{O}_\phi(\mu)$,
$c(FA\mathscr{O}_\phi(0_Q)) = FA\mathscr{O}_\phi(0_Q) = 0_Q$
and $c(FA\mathscr{O}_\phi(1_Q)) = FA\mathscr{O}_\phi(1_Q) = 1_Q$.
Let $\tau = \{ 0_Q, 1_Q, 1_Q - FA\mathscr{O}_\phi(\lambda), 1_Q - FA\mathscr{O}_\phi(\mu) \}$. Then τ is a fuzzy automata orbit structure on Q and hence the ordered pair (Q,τ) is a fuzzy automata orbit structure space.

Definition 3.2. Let (Q,τ) be a fuzzy automata orbit structure space. For any $FA\mathscr{O}_\phi(\mu)$, $\mu \in I^Q$, the fuzzy automata orbit interior of $FA\mathscr{O}_\phi(\mu)$ (briefly, $FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi(\mu))$) is defined by

$$FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi(\mu)) = \vee\{\sigma : \sigma \leq FA\mathscr{O}_\phi(\mu) \text{ and each } \sigma \in I^Q \text{ is a fuzzy automata orbit open subsystem in } (Q,\tau) \}.$$

Definition 3.3. Let (Q,τ) be a fuzzy automata orbit structure space. For any $FA\mathscr{O}_\phi(\mu)$, $\mu \in I^Q$, the fuzzy automata orbit closure of $FA\mathscr{O}_\phi(\mu)$ (briefly, $FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi(\mu))$) is defined by

$$FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi(\mu)) = \wedge\{\sigma : \sigma \geq FA\mathscr{O}_\phi(\mu) \text{ and each } \sigma \in I^Q \text{ is a fuzzy automata orbit closed subsystem in } (Q,\tau) \}.$$

Definition 3.4. Let (Q,τ) be a fuzzy automata orbit structure space. For any $\mu \in I^Q$, $FA\mathscr{O}_\phi(\mu)$ is said to be a fuzzy automata orbit-α-open subsystem in (Q,τ) if $FA\mathscr{O}_\phi(\mu) \leq FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi(\mu))))$ and the fuzzy complement of fuzzy automata orbit-α-open subsystem is said to be a fuzzy automata orbit-α-closed subsystem.

Notation 3.2. Let (Q,τ) be a fuzzy automata orbit structure space. Then $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$ will denote the family of all fuzzy automata orbit-α-open subsystems in (Q,τ) and $FA\mathscr{O}_\phi$-$\alpha C(Q,\tau)$ will denote the family of all fuzzy automata orbit-α-closed subsystems in (Q,τ).

Definition 3.5. Let (Q,τ) be a fuzzy automata orbit structure space. A function

$$\psi^* : FA\mathscr{O}_\phi\text{-}\alpha O(Q,\tau) \to I^Q$$

is called a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$, if for each $FA\mathscr{O}_\phi(\mu) \in FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$, $\mu \in I^Q$ with $FA\mathscr{O}_\phi(\mu) \neq 0_Q$, $FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi(\mu)) \leq \psi^*(FA\mathscr{O}_\phi(\mu))$ and $\psi^*(FA\mathscr{O}_\phi(0_Q)) = 0_Q$.

Remark 3.1. It is easy to check that some examples of fuzzy operators on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$ are the well known fuzzy operators viz. $FA\mathscr{O}_\phi int, FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi cl), FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi int)$, $FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi int))$ and $FA\mathscr{O}_\phi cl(FA\mathscr{O}_\phi int(FA\mathscr{O}_\phi cl))$.

Definition 3.6. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Then any fuzzy automata orbit-α-open subsystem $FA\mathscr{O}_\phi(\mu) \in I^X$ is called fuzzy automata orbit-α-ψ^*-open if $FA\mathscr{O}_\phi(\mu) \leq \psi^*(FA\mathscr{O}_\phi(\mu))$. The fuzzy complement of a fuzzy automata orbit-α-ψ^*-open subsystem is said to be a fuzzy automata orbit-α-ψ^*-closed subsystem.

Definition 3.7. Let (Q,τ) be a fuzzy automata orbit structure space and and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Let $FA\mathscr{O}_\phi(\lambda) \in I^Q$ where $\lambda \in I^Q$. Then the fuzzy automata orbit-α-ψ^*-co-kernel of $FA\mathscr{O}_\phi(\lambda)$ is denoted by $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(FA\mathscr{O}_\phi(\lambda))$ and defined as $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(FA\mathscr{O}_\phi(\lambda))$ $= \vee\{FA\mathscr{O}_\phi(\mu) \in I^Q : FA\mathscr{O}_\phi(\mu)$ is fuzzy automata orbit-α-ψ^*-closed and $FA\mathscr{O}_\phi(\mu) \leq FA\mathscr{O}_\phi(\lambda)\}$.

Definition 3.8. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Let $FA\mathscr{O}_\phi(\lambda) \in I^Q$ where $\lambda \in I^Q$. Then the fuzzy automata orbit-α-ψ^*-kernel of $FA\mathscr{O}_\phi(\lambda)$ is denoted by $FA\mathscr{O}_\phi$-α-ψ^*-$ker(FA\mathscr{O}_\phi(\lambda))$ and defined as $FA\mathscr{O}_\phi$-α-ψ^*-$ker(FA\mathscr{O}_\phi(\lambda)) = \wedge\{FA\mathscr{O}_\phi(\mu) \in I^Q : FA\mathscr{O}_\phi(\mu)$ is fuzzy automata orbit -α-ψ^*-open and $FA\mathscr{O}_\phi(\lambda) \leq FA\mathscr{O}_\phi(\mu)\}$.

Remark 3.2. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Let $FA\mathscr{O}_\phi(\lambda) \in I^X$ be a fuzzy automata orbit subsystem in (Q,τ).

(i) $(1_Q - FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-$ker(FA\mathscr{O}_\phi(\lambda))) = FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-$co$-$ker(1_Q - (FA\mathscr{O}_\phi(\lambda)))$.

(ii) $(1_Q - FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-$co$-$ker(FA\mathscr{O}_\phi(\lambda)))$ $= FA\mathscr{O}_\phi$-α-ψ^*-$ker(1_Q - (FA\mathscr{O}_\phi(\lambda)))$.

(iii) $FA\mathscr{O}_\phi$-α-ψ^*-$ker(0_Q) = 0_Q$ and $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(0_Q) = 0_Q$.

(iv) $FA\mathscr{O}_\phi$-α-ψ^*-$ker(1_Q) = 1_Q$ and $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(1_Q) = 1_Q$.

(v) If $FA\mathscr{O}_\phi(\lambda)$ is a fuzzy automata orbit -α-ψ^*-open subsystem then

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi(\lambda)) = FA\mathscr{O}_\phi(\lambda).$$

(vi) If $FA\mathscr{O}_\phi(\lambda)$ is a fuzzy automata orbit -α-ψ^*-closed subsystem then

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda)) = FA\mathscr{O}_\phi(\lambda).$$

(vii) $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(FA\mathscr{O}_\phi(\lambda_1) \vee FA\mathscr{O}_\phi(\lambda_2))$

$$= FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda_1)) \vee FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda_2)).$$

(viii) $FA\mathscr{O}_\phi$-α-ψ^*-$ker(FA\mathscr{O}_\phi(\lambda_1) \wedge FA\mathscr{O}_\phi(\lambda_2))$

$$= FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi(\lambda_1)) \wedge FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi(\lambda_2)).$$

Proof The proof is simple.

Definition 3.9. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Then (Q,τ) is said to be a fuzzy automata orbit -α-ψ^*-co-kernel space if, for any finite collection $\{FA\mathscr{O}_\phi(\lambda_i) : \lambda_i \in I^Q$ and $i = 1,2,...,n$, where each $FA\mathscr{O}_\phi(\lambda_i)$ is a fuzzy automata orbit open subsystem in $(Q,\tau)\}$, $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(FA\mathscr{O}_\phi(\lambda_i)) = 0_Q$, $i = 1,2,...,n$ and $FA\mathscr{O}_\phi$-α-ψ^*-co-$ker(\bigwedge_{i=1}^n FA\mathscr{O}_\phi(\lambda_i)) = 0_Q$.

Example 3.2. Let $M = (Q,X,\delta)$ be a fuzzy automaton, where $Q = X = \{0,1,2,......\}$ and $\delta : Q \times X \times Q \to [0,1]$ be given by

$$\delta(q,0,p) = \begin{cases} 1, & \text{if } q = p \\ 0, & \text{if } q \neq p \end{cases}$$

with $\delta(q,x_0,p) = 0.83$, $\delta(q,x_0,q) = 0.85$, $\delta(p,x_0,p) = 0.93$, $\delta(p,x_0,q) = 0.9$ for fixed $x_0 \in X (x_0 \neq 0)$ and for fixed $p,q \in Q$. For other $p,q \in Q$ and $x \in X$, $\delta(p,x,q) = 0$. Let $\lambda,\mu \in I^Q$ be defined as follows : $\lambda(1) = 0.77$, $\lambda(2) = 0.8$, $\mu(1) = 0.65$, $\mu(2) = 0.75$ and for other $r \in Q$, $r \neq 1,2$, $\lambda(r) = 0$, $\mu(r) = 0$. Let $\phi : I^Q \to I^Q$ be a mapping defined by

$$\eta_\phi(x) = \begin{cases} \eta(2), & \text{if } x = 1, \\ \eta(1), & \text{if } x = 2, \\ 0, & \text{otherwise} \end{cases}$$

for all $\eta \in I^X$. Then $FA\mathscr{O}_\phi(\lambda)(1) = 0.77$, $FA\mathscr{O}_\phi(\lambda)(2) = 0.77$, $FA\mathscr{O}_\phi(\mu)(1) = 0.65$, $FA\mathscr{O}_\phi(\mu)(2) = 0.65$ The Kuratowski saturated fuzzy closure operator

$$c(\mu)(p) = \vee_{q\in Q} \{ \vee_{x\in X} \{ (\mu)(q) \wedge \delta(q,x,p)\}\}.$$

It is clear that $c(FA\mathcal{O}_\phi(\lambda))$
$= FA\mathcal{O}_\phi(\lambda)$, $c(FA\mathcal{O}_\phi(\mu))$
$= FA\mathcal{O}_\phi(\mu)$, $c(FA\mathcal{O}_\phi(0_Q)) = FA\mathcal{O}_\phi(0_Q) = 0_Q$ and
$c(FA\mathcal{O}_\phi(1_Q)) = FA\mathcal{O}_\phi(1_Q) = 1_Q$.
Let $\tau = \{\ 0_Q, 1_Q, 1_Q - FA\mathcal{O}_\phi(\lambda), 1_Q - FA\mathcal{O}_\phi(\mu)\ \}$. Then τ is a fuzzy automata orbit structure on Q and hence the ordered pair (Q,τ) is a fuzzy automata orbit structure space. Thus the collection $\{FA\mathcal{O}_\phi(\lambda_i) : \lambda_i \in I^Q$ and $i = 1,2$ is a fuzzy automata orbit -α-open subsystems in $(Q,\tau)\}$. Let $\psi^* = FA\mathcal{O}_\phi int(FA\mathcal{O}_\phi cl)$. Then $FA\mathcal{O}_\phi(\lambda_i)$ is a fuzzy automata orbit -α-ψ^*-open subsystems in $(Q,\tau)\}$. Thus

$$FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$$

and

$$FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(\bigwedge_{i=1,2} FA\mathcal{O}_\phi(\lambda_i)) = 0_Q.$$

Hence (Q,τ) is a fuzzy automata orbit -α-ψ^*-co-kernel space.

Definition 3.10. Let (Q,τ) be any fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathcal{O}_\phi$-$\alpha O(Q,\tau)$. For any $\mu \in I^Q$, $FA\mathcal{O}_\phi(\mu)$ is said to be a fuzzy automata orbit-α-ψ^*-O_δ subsystem (briefly, $FA\mathcal{O}_\phi$-α-ψ^*-$O_\delta S$) if

$$FA\mathcal{O}_\phi(\mu) = \wedge_{i=1}^n \{FA\mathcal{O}_\phi(\mu_i), \mu_i \in I^Q : \text{each } FA\mathcal{O}_\phi(\mu_i) \text{ is a fuzzy automata orbit -}\alpha\text{-}\psi^*\text{-open subsystem}\ \}.$$

The fuzzy complement of a fuzzy automata orbit-α-ψ^*-O_δ subsystem is fuzzy automata orbit -α-ψ^*-C_σ subsystem (briefly, $FA\mathcal{O}_\phi$-α-ψ^*-$C_\sigma S$).

Proposition 3.1. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathcal{O}_\phi$-$\alpha O(Q,\tau)$. Then the following statements are equivalent :

(i) (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space.

(ii) For any fuzzy automata orbit-α-ψ^*-open subsystem $FA\mathcal{O}_\phi(\lambda_i)$ in (Q,τ) where $\lambda_i \in I^Q, i = 1,2,...,n$ and for every fuzzy automata orbit -α-ψ^*-O_δ subsystem $FA\mathcal{O}_\phi(\mu)$ in (Q,τ) where $\mu \in I^Q$, $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$ and
$FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\mu)) = 0_Q$.

(iii) For any fuzzy automata orbit -α-ψ^*-open subsystem $FA\mathcal{O}_\phi(\lambda_i)$ in (Q,τ) where $\lambda_i \in I^Q, i = 1,2,...,n$ and for every fuzzy automata orbit-α-ψ^*-C_σ subsystem
$1_Q - (FA\mathcal{O}_\phi(\mu))$ in (Q,τ)
where $\mu \in I^Q$, $FA\mathcal{O}_\phi$-α-ψ^*-$ker(1_Q - FA\mathcal{O}_\phi(\lambda_i)) = 1_Q$ and $FA\mathcal{O}_\phi$-α-ψ^*-$ker(1_Q - FA\mathcal{O}_\phi(\mu)) = 1_Q$.

Proof. **(i) ⇒ (ii)**
Let (Q,τ) is a fuzzy automata orbit co-kernel space. Then for any finite collection $\{FA\mathcal{O}_\phi(\lambda_i) : \lambda_i \in I^Q$ and $i = 1,2,...,n$, where each $FA\mathcal{O}_\phi(\lambda_i)$ is fuzzy automata orbit-α-ψ^*-open subsystem in $(Q,\tau)\}$, $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$, $i = 1,2,...,n$ and $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(\bigwedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$.

Let $FA\mathcal{O}_\phi(\mu) \in I^Q$ be a $FA\mathcal{O}_\phi$-α-ψ^*-$O_\delta S$. By Definition 3.10,

$$FA\mathcal{O}_\phi(\mu) = \bigwedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i), \lambda_i \in I^Q$$

$$FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathcal{O}_\phi(\mu)) = FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(\wedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i))$$

Since (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space,
$FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(\wedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$.
Then $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\mu)) = 0_Q$.
(ii) ⇒ (iii)
For $\lambda_i \in I^Q, i \in J$, let $FA\mathcal{O}_\phi(\lambda_i)$ be any fuzzy automata orbit-α-ψ^*-open subsystem in (Q,τ) with $FA\mathcal{O}_\phi$-α-ψ^*-$ker(1_Q - FA\mathcal{O}_\phi(\lambda_i)) = 1_Q$. Thus $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$. By (ii), for every fuzzy automata orbit -α-ψ^*-O_δ subsystem $FA\mathcal{O}_\phi(\mu)$ in (Q,τ) where $(\mu \in I^Q)$, $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\mu)) = 0_Q$. Then

$$1_Q - FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathcal{O}_\phi(\mu)) = 1_Q$$
$$FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(1_Q - FA\mathcal{O}_\phi(\mu)) = 1_Q$$

(iii) ⇒ (i)
Let $FA\mathcal{O}_\phi$-α-ψ^*-$ker(1_Q - FA\mathcal{O}_\phi(\lambda_i)) = 1_Q$, $\lambda_i \in I^Q, i = 1,2,...,n$. This implies that,

$$FA\mathcal{O}_\phi co\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q. \tag{3.1}$$

Let $FA\mathcal{O}_\phi(\mu)$ be $FA\mathcal{O}_\phi$-α-ψ^*-$O_\delta S$.
Then $FA\mathcal{O}_\phi(\mu) = \wedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i)$, where each $FA\mathcal{O}_\phi(\lambda_i)$ is a fuzzy automata orbit-α-ψ^*-open subsystem in (Q,τ). Thus $1_Q - FA\mathcal{O}_\phi(\mu))$ is $FA\mathcal{O}_\phi$-α-ψ^*-$C_\sigma S$.
By (iii), $FA\mathcal{O}_\phi$-α-ψ^*-$ker(1_Q - FA\mathcal{O}_\phi(\mu)) = 1_Q$.
Then $1_Q - (FA\mathcal{O}_\phi$-$\alpha$-$\psi^*$-$ker(1_Q - FA\mathcal{O}_\phi(\mu))) = 1_Q - 1_Q$. Thus $FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\mu)) = 0_Q$. Hence

$$FA\mathcal{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(\wedge_{i=1}^n FA\mathcal{O}_\phi(\lambda_i)) = 0_Q. \tag{3.2}$$

From Equations 3.1 and 3.2, we have (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space. □

Proposition 3.2. Let (Q,τ) be a fuzzy automata orbit-α-ψ^*-co-kernel space where ψ^* is a fuzzy operator on $FA\mathcal{O}_\phi$-$\alpha O(Q,\tau)$. For $\mu \in I^Q$, if $FA\mathcal{O}_\phi(\mu) \in I^Q$, is $FA\mathcal{O}_\phi$-α-ψ^*-$C_\sigma S$, then $FA\mathcal{O}_\phi(\mu) \wedge \{1_Q - FA\mathcal{O}_\phi(\gamma)\} \neq 0_Q$, for every fuzzy automata orbit-α-ψ^*-open subsystem $FA\mathcal{O}_\phi(\gamma) \neq 1_Q$, $\gamma \in I^Q$ of (Q,τ).

Proof. Let (Q,τ) be a fuzzy automata orbit-α-ψ^*-co-kernel space. Then for any finite collection $\{FA\mathcal{O}_\phi(\lambda_i) : \lambda_i \in I^Q$ and $i = 1,2,...,n$, where each $FA\mathcal{O}_\phi(\lambda_i)$ is a fuzzy automata orbit-α-ψ^*-open subsystem in $(Q,\tau)\}$,
$FA\mathcal{O}_\phi$-α-ψ^*-co-$ker(FA\mathcal{O}_\phi(\lambda_i)) = 0_Q$ and

$FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(\bigwedge_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i)) = 0_Q$. Let $FA\mathscr{O}_\phi(\mu)$ be $FA\mathscr{O}_\phi$-α-ψ^*-$C_\sigma S$. Then by Definition 3.10,

$$FA\mathscr{O}_\phi(\mu) = \vee_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i), \lambda_i \in I^Q$$

$$\text{then} \quad (1_Q - FA\mathscr{O}_\phi(\mu)) = 1_Q - \vee_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i)$$

$$\text{thus} \quad (1_Q - FA\mathscr{O}_\phi(\mu)) = \wedge_{i=1}^{n}(1_Q - FA\mathscr{O}_\phi(\lambda_i))$$

Since $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(\wedge_{i=1}^{n}\{1_Q - FA\mathscr{O}_\phi(\lambda_i)\}) = 0_Q$,

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(1_Q - (FA\mathscr{O}_\phi(\mu))) = 0_Q \qquad (3.3)$$

As a contrary, suppose that

$$FA\mathscr{O}_\phi(\mu) \wedge \{1_Q - FA\mathscr{O}_\phi(\gamma)\} = 0_Q$$

then,

$$1_Q - (FA\mathscr{O}_\phi(\mu) \wedge (1_Q - FA\mathscr{O}_\phi(\gamma))) = (1_Q - 0_Q)$$

and therefore

$$(1_Q - FA\mathscr{O}_\phi(\mu)) \vee FA\mathscr{O}_\phi(\gamma) = 1_Q.$$

Now, $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$\{(1_Q - FA\mathscr{O}_\phi(\mu)) \vee FA\mathscr{O}_\phi(\gamma)\}$
$= FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*(1_Q);
then$FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$\{1_Q - FA\mathscr{O}_\phi(\mu)\}$
$\vee FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$\{FA\mathscr{O}_\phi(\gamma)\} = 1_Q$;
which implies that $0_Q \vee FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-*co-ker*$\{FA\mathscr{O}_\phi(\gamma)\} = 1_Q$ {by 3.3 };
therefore FA$\mathscr{O}_\phi$-α-ψ^*-co-ker$\{$FA$\mathscr{O}_\phi(\gamma)\} = 1_Q$.
which is not possible as $FA\mathscr{O}_\phi(\gamma) \neq 1_Q$. Hence,

$$FA\mathscr{O}_\phi(\mu) \wedge \{1_Q - FA\mathscr{O}_\phi(\gamma)\} \neq 0_Q.$$

□

Definition 3.11. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. A fuzzy automata orbit subsystem $FA\mathscr{O}_\phi(\lambda)$, $\lambda \in I^X$ is said to be a fuzzy automata orbit-α-ψ^*-meager* subsystem if

$$FA\mathscr{O}_\phi(\lambda) = \vee_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i), \lambda_i \in I^Q$$

with $FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(FA\mathscr{O}_\phi(\lambda_i))) = 0_Q$. The fuzzy complement of a fuzzy automata orbit-α-ψ^*-meager* subsystem is a fuzzy automata orbit-α-ψ^*-comeager* subsystem.

Proposition 3.3. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. If (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space, then for every fuzzy automata orbit-α-ψ^*-meager* subsystem $FA\mathscr{O}_\phi(\lambda)$, $\lambda \in I^Q$,

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda))) = 0_Q.$$

Proof. Let $FA\mathscr{O}_\phi(\lambda), \lambda \in I^Q$ be a fuzzy automata orbit-α-ψ^*-meager* subsystem. By Definition 3.11,
$FA\mathscr{O}_\phi(\lambda) = \vee_{i=1}^{n}(FA\mathscr{O}_\phi(\lambda_i)), \lambda_i \in I^Q$ with

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda_i))) = 0_Q.$$

Now, $FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(FA\mathscr{O}_\phi(\lambda)))$
$= FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(\vee_{i=1}^{n}(FA\mathscr{O}_\phi(\lambda_i))))$
$= FA\mathscr{O}_\phi$-α-ψ^*-*ker*(0_Q)
$= 0_Q$.

Hence for every fuzzy automata orbit-α-ψ^*-meager* subsystem $FA\mathscr{O}_\phi(\lambda), \lambda \in I^Q$,

$$FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}ker(FA\mathscr{O}_\phi\text{-}\alpha\text{-}\psi^*\text{-}co\text{-}ker(FA\mathscr{O}_\phi(\lambda))) = 0_Q.$$

□

Proposition 3.4. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. If (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space, then for every fuzzy automata orbit-α-ψ^*-comeager* subsystem $FA\mathscr{O}_\phi(\lambda)$,
$FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi(\lambda))) = 1_Q$.

Proof. Proof is similar to the proof of Proposition 3.3. □

Proposition 3.5. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Then the
following statements are equivalent:

(i) (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space.

(ii) $FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(\vee_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i)) = 1_Q$, for every fuzzy automata orbit-α-ψ^*-closed subsystem $FA\mathscr{O}_\phi(\lambda_i)$, $\lambda_i \in I^Q, i = 1,2,...,n$ with $FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi(\lambda_i)) = 1_Q$.

Proof. **(i) ⇒ (ii)**
Let $\{FA\mathscr{O}_\phi(\lambda_i) \in I^Q, \lambda_i \in I^Q, i = 1,2,...,n\}$ be the collection of fuzzy automata orbit
-α-ψ^*-closed subsystems with $FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(FA\mathscr{O}_\phi(\lambda_i)) = 1_Q$. Then $\{1_Q - FA\mathscr{O}_\phi(\lambda_i), \lambda_i \in I^Q, i = 1,2,...,n\}$ is the collection of fuzzy automata orbit-α-ψ^*-open subsystems with $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(1_Q - FA\mathscr{O}_\phi(\lambda_i)) = 0_Q$. Since (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space,
$FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(\wedge_{i=1}^{n}(1_Q - FA\mathscr{O}_\phi(\lambda_i))) = 0_Q$
$1_Q - \{FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-*co-ker*$(\wedge_{i=1}^{n}(1_Q - FA\mathscr{O}_\phi(\lambda_i)))\}$
$= (1_Q - 0_Q)$
$FA\mathscr{O}_\phi$-α-ψ^*-*ker*$(\vee_{i=1}^{n} FA\mathscr{O}_\phi(\lambda_i)) = 1_Q$. **(ii) ⇒ (i)**

On taking fuzzy complement of (ii), we get (Q,τ) is a fuzzy automata orbit-α-ψ^*-co-kernel space. □

Proposition 3.6. Let (Q,τ) be a fuzzy automata orbit structure space and ψ^* be a fuzzy operator on $FA\mathscr{O}_\phi$-$\alpha O(Q,\tau)$. Let $\lambda \in I^Q$, $FA\mathscr{O}_\phi(\lambda)$ be a fuzzy automata orbit subsystem in (Q,τ). If $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(FA\mathscr{O}_\phi(\lambda)) = 0_Q$ then $(FA\mathscr{O}_\phi(\gamma) \vee FA\mathscr{O}_\phi(\lambda)) \neq 1_Q$, for every fuzzy automata orbit -α-ψ^*-open subsystem $(FA\mathscr{O}_\phi(\gamma), FA\mathscr{O}_\phi(\gamma)) \neq 1_Q, \gamma \in I^Q$.

Proof. Let $FA\mathscr{O}$-α-ψ^*-*co-ker*$(FA\mathscr{O}_\phi(\lambda)) = 0_Q$ and
$FA\mathscr{O}_\phi(\gamma) \neq 1_Q$, be a fuzzy automata orbit-α-ψ^*-open subsystem. As a contrary assume that,

$$FA\mathscr{O}_\phi(\gamma) \vee FA\mathscr{O}_\phi(\lambda) = 1_Q,$$

for every fuzzy automata orbit-α-ψ^*-open subsystem $FA\mathscr{O}_\phi(\gamma)$, $FA\mathscr{O}_\phi(\gamma) \neq 1_Q$. Then,

$FA\mathscr{O}_\phi(\gamma) \vee FA\mathscr{O}_\phi(\lambda) = 1_Q$,

implies that $1_Q - \{FA\mathscr{O}_\phi(\gamma) \vee FA\mathscr{O}_\phi(\lambda)\} = 1_Q - 1_Q$,

and so $\{1_Q - FA\mathscr{O}_\phi(\gamma)\} \wedge \{1_Q - FA\mathscr{O}_\phi(\lambda)\} = 0_Q$,

thus $1_Q - FA\mathscr{O}_\phi(\gamma) \leq 1_Q - \{1_Q - FA\mathscr{O}_\phi(\lambda)\}$,

therefore $1_Q - FA\mathscr{O}_\phi(\gamma) \leq FA\mathscr{O}_\phi(\lambda)$,

Hence $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(1_Q - FA\mathscr{O}_\phi(\gamma)) \leq FA\mathscr{O}_\phi$-$\alpha$-$\psi^*$-*co-ker*$(FA\mathscr{O}_\phi(\lambda))$.

By assumption, $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(1_Q - FA\mathscr{O}_\phi(\gamma)) = 0_Q$. Since $1_Q - FA\mathscr{O}_\phi(\gamma)$ is fuzzy automata orbit-α-ψ^*-closed subsystem, $FA\mathscr{O}_\phi$-α-ψ^*-*co-ker*$(1_Q - FA\mathscr{O}_\phi(\gamma)) = 1_Q - FA\mathscr{O}_\phi(\gamma)$. Thus, $1_Q - FA\mathscr{O}_\phi(\gamma) = 0_Q$. This implies that, $FA\mathscr{O}_\phi(\gamma) = 1_Q$, which is a contradiction to our assumption. Hence $FA\mathscr{O}_\phi(\gamma) \vee FA\mathscr{O}_\phi(\lambda) \neq 1_Q$.

□

4. Conclusion

In this paper, the concept of fuzzy automata orbit structure spaces is introduced and some of its properties are studied. Also, the concepts of fuzzy automata orbit-α-ψ^*-co-kernel subsystems, fuzzy automata orbit-α-ψ^*-kernel subsystems, fuzzy automata orbit-α-ψ^*-meager* subsystems and fuzzy automata orbit-α-ψ^*-comeager* subsystems are introduced. The notion of fuzzy automata orbit-α-ψ^*-co-kernel spaces is introduced and some equivalent statements are discussed.

References

[1] C. L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.*, 24(1968), 182–190.

[2] P. Das, A fuzzy topology associated with a fuzzy finite state machine, *Fuzzy Sets and Systems*, 105(1999), 469–479.

[3] R. L. Devaney, *Introduction to Chaotic Dynamical Systems*, Redwood City, Calif, Addison-Wesley, 1986.

[4] D. S. Malik and J. N. Morderson, Algebraic fuzzy automata theory, *Arabian J. Sci. Eng.*, 25(2000), 23–50.

[5] A. K. Srivastava and S. P. Tiwari, A topology for fuzzy automata, Proc. AFSS *International Conference on Fuzzy Systems, Lecture Notes in Artificial Intelligence*, Springer-Verlag, 2275 (2002), 485–491.

[6] A. K. Srivastava and S. P. Tiwari, On relationships among fuzzy approximation operators, fuzzy topology and fuzzy automata, *Fuzzy Sets and Systems*, 138(2003), 197–204.

[7] W. G. Wee, *On Generalizations of Adaptive Algorithm and Application of Fuzzy Sets Concept to Pattern Classification*, Ph. D. Thesis, Purdue University, 1967.

[8] L. A. Zadeh, Fuzzy Sets, *Information and Control*, 8(1965), 338–353.

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