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Some properties of P – fuzzy right R–subgroup of R on the algebra A

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Abstract

The aim of this work is to define P – fuzzy right R-subgroup of R(PFRS) using P-fuzzy algebra on the algebra A and investigate main properties. We explore the concept of fuzzy right R-subgroup into P-fuzzy right R-subgroup of R. In general, we study their main properties in detail under $(2,0)$ typed algebra with the help of some interesting examples.

Keywords

P-fuzzy set, P-fuzzy algebra, P – fuzzy subgroup, fuzzy right R-subgroup, P-fuzzy right R – Subgroup of R on the algebra A.

AMS Subject Classification

03E72, 28E10, 08A72.

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Contents

1. Introduction

The concept of fuzzy group theory particularly fuzzy subgroupoid and fuzzy subgroup are introduced by Rosenfeld in 1971. The first stage of the transition from the traditional view to the modern view of uncertainity began in 19th century. The evolution of the modern concept of uncertainity was the publication of a seminal paper by Lotfi A. Zadeh [6], a theory whose objects – fuzzy set – are sets with boundaries that are not precise. The values of membership functions are defined in the unit interval $[0,1]$. For additional details on this theory and its applications, we suggest the reader to refer [1–5].

2. Preliminaries

Definition 2.1. *Let* A *be a nonempty set and* $\mathrm{P}=(\mathrm{P},*,1,\leq)$ *a (2,0) type ordered algebra which satisfies the condition of monoid, poset and isotone.*

Definition 2.2. *A mapping* $\mu: A \to P$ *is a P-fuzzy subset of* $\mathrm{A}\left(\mathrm{P}^{\mathrm{A}}\right)$ *where* $(\mathrm{P},\leq)$ *is a partially ordered set and* X *is a nonvoid set.*

Definition 2.3. *A* $P-$ *fuzzy set* $\mu \in P^A$ *is called a* $\mathrm{P}-$ *fuzzy algebra if it satisfies* $\mathrm{n}-$ *ary and nullary operations.*

Remark 2.1. *If* A *is a group then nullary operation is consequence of* $n-$ *ary operation.*

Definition 2.4. *Let* μ *be a fuzzy subset of* G, *where* G *be a group then* G *is said to be a fuzzy subgroup if it satisfies* $\mu\left(\mathrm{xy}^{-1}\right) \geq \min\{\mu(\mathrm{x}),\mu(\mathrm{y})\}$ *for every* $\mathrm{x}\ \mathrm{y} \in \mathrm{G}$.

Definition 2.5. *let* μ *be a fuzzy set in a near-ring* R, *then* μ *is a fuzzy subnear-ring of* R *if there exists multiplicative inverse and additional inverse with respect to operation min for all* $\mathrm{x},\mathrm{y} \in \mathrm{R}$.

Remark 2.2. *If a fuzzy set* μ *in a near* $-$ *ring* R *satisfies the property* $\mu(x-y) \geq \min\{\mu(x),\ \mu(\mathrm{y})\}$ *then letting* $\mathrm{x}=\mathrm{y}; \mu(0)=\mu(\mathrm{x})$ *for all* $x \in R$.

Definition 2.6. *Consider a near - ring* $(\mathrm{R},+,.)$ *and* $\mu \in \mathrm{R}$ *is called a fuzzy right* R *- Subgroup of* R, *where* μ *is a fuzzy set if for all* $\mathrm{r},\mathrm{x} \in \mathrm{R}$ *under addition* μ *is a fuzzy subgroup and* $\mu(\mathrm{xr}) \geq \mu(\mathrm{x})$.

Example 2.7. *Let* $\mathrm{R}=\{\mathrm{x1},\mathrm{x2},\mathrm{x3},\mathrm{x4}\}$ *be a set with two binary operations as follows.*

+	x1	x2	x3	x4
x1	x1	x2	x3	x4
x2	x2	x1	x4	x3
x3	x3	x4	x2	x1
x4	x4	x3	x1	x2

·	x1	x2	x3	x4
x1	x1	x1	x1	x1
x2	x1	x1	x1	x1
x3	x1	x1	x1	x1
x4	x1	x1	x2	x2

It is clear that the above example is near-ring under $(\mathrm{R},+,\cdot)$, with mapping $\mu:\mathrm{R}\to[0,1]$ defined by $\mu(\mathrm{d})<\mu(\mathrm{b})<\mu(\mathrm{a})$.
$\Rightarrow \mu$ is a fuzzy right R -Subgroup of R.

3. P – fuzzy right R– subgroup of R

Definition 3.1. *Let* $(\mathrm{R},*)$ *be a near* $-$ *ring. A* P *- fuzzy set* $\mu\in \mathrm{m\hat{I}P^A}$ *in* R *is called a* P *- fuzzy right* R*– Subgroup(PFRS) of* R *on the algebra A if (i)* μ *is a* P *-fuzzy subgroup of* (R, *) *(ii)* $\mu(\mathrm{xr})\geq\mu(\mathrm{x})$ *for all* $\mathrm{r,x}\in\mathrm{R}$.

Example 3.2. *Let* $*\in\{+,\cdot\}$ *be a binary operation defined on* $\mathrm{R}=\{\mathrm{p,q,r,s}\}$.

+	p	q	r	s
p	p	q	r	s
q	q	q	q	s
r	r	q	r	p
s	s	s	p	s

·	p	q	r	s
p	p	p	p	p
q	p	q	q	q
r	p	q	r	r
s	p	q	r	s

$\Rightarrow(\mathrm{R},*)$ is a near-ring. Define $\mu:\mathrm{R}\to[0,1]$ by $\mu(\mathrm{c})=\mu(\mathrm{a})>\mu(\mathrm{b})>\mu(\mathrm{c})>\mu(\mathrm{d})$. Then μ is a P - fuzzy right R-Subgroup(PFRS) of R on the algebra A.

Theorem 3.3. *If* μ *is a PFRS of a near - ring* R*, then the set* $\mathrm{R}_{\mu\mathrm{P}}=\{\mathrm{x}\in\mathrm{R}/\mu(\mathrm{x})=\mu(0)\}$ *is a right* R*– subgroup of* R *on the algebra* A.

Proof. Let $\mathrm{x,y}\in\mathrm{R}_{\mu\mathrm{P}}$, then $\mu(\mathrm{x})=\mu(\mathrm{y})=\mu(0)$ since μ is a PFRS

$$\Rightarrow\{-,.\}\in\mathrm{f}$$
$$\Rightarrow\mu(\mathrm{f(x,y)})\geq\min\{\mu(\mathrm{x}),\mu(\mathrm{y})\},\ \text{for}$$

operation $\cdot$

$$\mu(\mathrm{xy})\geq\min\{\mu(\mathrm{x}),\mu(\mathrm{y})\}$$
$$\geq\min\{\mu(0),\mu(0)\}$$
$$\Rightarrow\mu(\mathrm{xy})=\mu(0)$$
$$\Rightarrow\mathrm{xy}\in\mathrm{R}_{\mu\mathrm{P}}.$$

Similarly for operation -

$$\Rightarrow\mu(\mathrm{x-y})=\mu(0)$$
$$\Rightarrow\mathrm{x-y}\in\mathrm{R}_{\mu\mathrm{P}}.$$

□

Definition 3.4. *A PFRS* μ *of a near ring* R *on the algebra A is normal if* $\mu(\mathrm{x})=1$.

Theorem 3.5. *Let* μ *be a PFRS of a near - ring* R *and let* μ^+ *be a fuzzy set in* R *defined by* $\mu^+(\mathrm{x})=\mu(\mathrm{x})+1-\mu(0)$ *for all* $\mathrm{x}\in\mathrm{R}$. *Then* μ^+ *is a* P *-normal fuzzy right* R *- subgroup of* R *on the algebra A containing* μ.

Proof. Let $x,y\in\mathrm{R}$ then

$$\mathrm{Min}\left\{\mu^+(\mathrm{x}),\mu^+(\mathrm{y})\right\}=$$

$\mu^+(\mathrm{x-y})$ and for all $\mathrm{x,y}\in\mathrm{R}$

$$\mu^+(\mathrm{xr})=\mu(\mathrm{xr})+1-\mu(0)$$
$$\geq\mu(\mathrm{x})+1-\mu(0)=\mu^+(\mathrm{x})$$
$$\mu^+(\mathrm{rx})=\mu(\mathrm{rx})+1-\mu(0)$$
$$\geq\mu(\mathrm{x})+1-\mu(0)=\mu^+(\mathrm{x})$$

$\Rightarrow\mu^+$ is a fuzzy right R– subgroup of R on the algebra A. Clearly $\mu^+(0)=1$ and $\mu\subset\mu^+$. □

Theorem 3.6. *If* μ *is a PFRS of R on the algebra A satisfying* $\mu^+(\mathrm{x})=0$ *for some* $\mathrm{x}\in\mathrm{R}$ *then* $\mu(\mathrm{x})=0$ *also.*

Proof. If $\mu^+(\mathrm{x})=\mu(\mathrm{x})+1-\mu(0)$ for all $\mathrm{x}\in\mathrm{R}$

$$0=\mu(x)+1-\mu(0)\quad(\text{ since }\mu(0)=0)$$
$$0=\mu(x)+1-1$$
$$\Rightarrow\mu(x)=0.$$

□

Theorem 3.7. *A PFRS* μ *of a near* $-$ *ring* R *on the algebra A is normal if and only if* $\mu^+=\mu$.

Proof. Real Part If $\mu^+=\mu$

$$\mu^+(\mathrm{x})=\mu(\mathrm{x})=\mu(\mathrm{x})+1-\mu(0)$$
$$\mu^+(\mathrm{x})=\mu(\mathrm{x})+1-\mu(0)$$

By Theorem 3.6, A PFRS μ of a near - ring R is normal. Conversely,

$$\mu^+(\mathrm{x})=\mu(\mathrm{x})+1-\mu(0)=\mu(\mathrm{x})\ \text{for all}\ \mathrm{x}\in\mathrm{R}$$
$$\Rightarrow\mu^+=\mu.$$

□

Theorem 3.8. *If μ is a PFRS of a near - ring on the algebra* A *then* $(\mu^{+})^{+} = \mu^{+}$.

Proof. For any $x \in R$

$$\begin{aligned}(\mu^{+})^{+}(x) &= \mu^{+}(x) + 1 - \mu^{+}(0)\\ &= \mu^{+}(x).\end{aligned}$$

□

Theorem 3.9. *If μ is a PFRS of a near - ring* R *on the algebra* A *then* $(\mu^{+})^{+} = \mu$.

Proof. By definition, if $x \in R, \mu(x) = 1$ since μ is a P− normal fuzzy right R subgroup

$$\begin{aligned}(\mu^{+})^{+}(x) &= \mu^{+}(x) + 1 - \mu^{+}(0)\\ &= \mu(x) + 1 - \mu(0) + 1 - \mu^{+}(0).\end{aligned}$$

$$(\mu^{+})^{+}(x) = \mu(x).$$

□

Theorem 3.10. *Let μ be a PFRS of near - ring* R *on the algebra A, if* $v \in R$ *satisfying* $v^{+} \subset \mu$ *then μ is normal.*

Proof. Since $v^{+} \subset \mu$ Then $1 = v^{+}(0) \leq \mu(0)$ where $\mu(0) = 1$. □

Theorem 3.11. *Let μ be a PFRS of a near ring* R *on the algebra* A *and let* $f : [0, \mu(0)] \to [0, 1]$ *be an increasing function. Define a fuzzy set* $\mu_{fp} : R \to [0, 1]$ *by* $\mu_{fp}(x) = f(\mu(x))$ *for all* $x \in R$*. Then* μ_{jp} *is a fuzzy right R− subgroup of r In particular if* $f(\mu(0)) = 1$ *then* μ_{fp} *is normal and if* $f(t) \geq t$ *for all* $t \in [0, \mu(0)]$ *then* $\mu \subseteq \mu_{fp}$.

Theorem 3.12. *Let μ be a non constant normal fuzzy right* R− *subgroup of* R *on the algebra A, which is maximum in PFRS under set inclusion, then μ has the value of 0 and 1.*

Theorem 3.13. *Let μ be a PFRS of a near ring* R *on the algebra* A *and* 1etμ^{0} *be a P-fuzzy set in* R *defined by* $\mu^{0}(x) = \mu(x)/\mu(0)$ *for all* $x \in R$. *Then* μ^{0} *is a normal PFRS of* R *containing* μ.

Proof.

For any $x, y \in R$

$$\begin{aligned}\min\{\mu^{0}(x), \mu^{0}(y)\}\\ &= \min\{\mu(x)/\mu(0), \mu(y)/\mu(0)\}\\ &= \mu^{0}(x - y)\end{aligned}$$

Also

$$\begin{aligned}\Rightarrow \mu^{0}(xr) &= (1/\mu^{0})\, \mu(xr)\\ &\geq \mu(x)/\mu^{0}\\ &= \mu^{0}(x)\end{aligned}$$

$$\begin{aligned}\mu^{0}(rx) &= (1/\mu^{0})\, \mu(rx)\\ &\geq \mu(x)/\mu^{0}\end{aligned}$$

Similarly, $= \mu^{0}(x)$

$$\Rightarrow \mu^{0}(0) = 1 \text{ and } \mu \subseteq \mu^{0}$$

$$\Rightarrow \mu \subset \mu^{0}.$$

$\Rightarrow \mu^{0}$ is a normal PFRS of R on the algebra A containing μ □

4. Conclusion

The research work on fuzzy right *R*- subgroup is extended to partially ordered fuzzy right *R*– subgroup on the algebra *A* using near ring. Also in future the research can be extended to *P*– fuzzy ideal with near ring.

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