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Note on fractional integral inequalities using generalized k-fractional integral operator

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Abstract

The aim of this paper is to obtain several fractional integral inequalities involving convex functions by using generalized k-fractional integral operator.

Keywords

Generalized k-fractional integral, convex functions and inequalities.

AMS Subject Classification

26A99, 26D10.

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Contents

1. Introduction

Fractional inequalities play major role in the development of fractional differential, integral equations and other fields of sciences and technology. Recently, a number of mathematician have studied different results about fractional integrals such as Riemann-Liouville, Hadamard, Saigo, Erdeyi-Kober, q-fractional integral and some other operators, see [1, 2, 5, 6, 8–12, 15–18, 20–22]. In [7], authors have studied inequalities using Saigo fractional integral.

Theorem 1.1. *Let f, h be two positive continuous functions on $[0,\infty)$ and $f \leq h$ on $[0,\infty)$. If $\frac{f}{h}$ is decreasing, f is increasing on $[0,\infty)$ and for any convex function ϕ, $\phi(0)=0$, then for $t>0$, $\alpha > max\{0,-\beta\}$, $\beta<1$, $\beta-1<\eta<0$, we have*

$$\frac{I_{0,t}^{\alpha,\beta,\eta}[f(t)]}{I_{0,t}^{\alpha,\beta,\eta}[h(t)]} \geq \frac{I_{0,t}^{\alpha,\beta,\eta}[\phi(f(t))]}{I_{0,t}^{\alpha,\beta,\eta}[\phi(h(t))]}. \tag{1.1}$$

and

Theorem 1.2. *Let f, h be two positive continuous functions on $[0,\infty)$ and $f \leq h$ on $[0,\infty)$. If $\frac{f}{h}$ is decreasing, f is increasing on $[0,\infty)$ and for any convex function ϕ, $\phi(0)=0$, then we have inequality*

$$\frac{I_{0,t}^{\alpha,\beta,\eta}[f(t)]\,I_{0,t}^{\psi,\delta,\zeta}[\phi(h(t))]+I_{0,t}^{\psi,\delta,\zeta}[f(t)]\,I_{0,t}^{\alpha,\beta,\eta}[\phi(h(t))]}{I_{0,t}^{\alpha,\beta,\eta}[h(t)]\,I_{0,t}^{\psi,\delta,\zeta}[\phi(f(t))]+I_{0,t}^{\psi,\delta,\zeta}[h(t)]\,I_{0,t}^{\alpha,\beta,\eta}[\phi(f(t))]} \geq 1, \tag{1.2}$$

where for all $t>0$, $\alpha > max\{0,-\beta\}$, $\psi > max\{0,-\delta\}$, $\beta<1$, $\beta-1<\eta<0$, $\delta<1$, $\delta-1<\zeta<0$.

In the literature, some fractional inequalities are obtain by using Generalized k-fractional integral operator, see [3, 4, 13, 14, 17, 19, 21]. Motivated by above work in this paper we have obtain some new inequalities using generalized k-fractional integral (in terms of Gauss hypergeometric function) operator for convex functions.

2. Preliminaries

Here, we devoted to basic concepts of Generalized k-fractional integral.

Definition 2.1. *Two function x and y are said to synchronous (asynchronous) on $[a,b]$, if*

$$((x(s)-x(t))(y(s)-y(t))) \geq (\leq) 0, \tag{2.1}$$

for all $s,t \in [0,\infty)$.

Definition 2.2. *[14, 23] The function $x(s)$, for all $s > 0$ is said to be in the $L_{p,k}[0,\infty)$, if*

$$L_{p,k}[0,\infty) = \{x : \|x\|_{L_{p,k}[0,\infty)} = \left(\int_0^\infty |x(s)|^p s^k ds\right)^{\frac{1}{p}} < \infty \; 1 \leq p < \infty, k \geq 0\}, \tag{2.2}$$

Definition 2.3. *[14, 23, 24] Let $f \in L_{1,k}[0,\infty)$,. The generalized Riemann-Liouville fractional integral $\mathbb{I}^{\alpha,k} f(x)$ of order $\alpha, k \geq 0$ is defined by*

$$\mathbb{I}^{\alpha,k} f(x) = \frac{(k+1)^{1-\alpha}}{\Gamma(\alpha)} \int_0^x (x^{k+1} - t^{k+1})^{\alpha-1} t^k f(t) dt. \tag{2.3}$$

Definition 2.4. *[14, 23] Let $k \geq 0, \alpha > 0 \mu > -1$ and $\beta, \eta \in R$. The generalized k-fractional integral $\mathbb{I}^{\alpha,\beta,\eta,\mu}_{t,k}$ (in terms of the Gauss hypergeometric function)of order α for real-valued continuous function $f(t)$ is defined by*

$$\mathbb{I}^{\alpha,\beta,\eta,\mu}_{t,k}[f(t)] = \frac{(k+1)^{\mu+\beta+1} t^{(k+1)(-\alpha-\beta-2\mu)}}{\Gamma(\alpha)} \int_0^t \tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha-1} \times {}_2F_1(\alpha+\beta+\mu, -\eta; \alpha; 1 - (\frac{\tau}{t})^{k+1}) \tau^k f(\tau) d\tau. \tag{2.4}$$

where, the function ${}_2F_1(-)$ in the right-hand side of (2.4) is the Gaussian hypergeometric function defined by

$${}_2F_1(a,b;c;t) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{t^n}{n!}, \tag{2.5}$$

and $(a)_n$ is the Pochhammer symbol

$$(a)_n = a(a+1)...(a+n-1) = \frac{\Gamma(a+n)}{\Gamma(a)}, \; (a)_0 = 1.$$

Consider the function

$$\begin{aligned}
\mathfrak{F}(t,\tau) &= \frac{(k+1)^{\mu+\beta+1} t^{(k+1)(-\alpha-\beta-2\mu)}}{\Gamma(\alpha)} \tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha-1} \times {}_2F_1(\alpha+\beta+\mu, -\eta; \alpha; 1 - (\frac{\tau}{t})^{k+1}) \\
&= \sum_{n=0}^{\infty} \frac{(\alpha+\beta+\mu)_n (-n)_n}{\Gamma(\alpha+n) n!} \times t^{(k+1)(-\alpha-\beta-2\mu-\eta)} \tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha-1+n} (k+1)^{\mu+\beta+1} \\
&= \frac{\tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha-1} (k+1)^{\mu+\beta+1}}{t^{k+1} (\alpha+\beta+2\mu) \Gamma(\alpha)} + \frac{\tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha} (k+1)^{\mu+\beta+1} (\alpha+\beta+\mu)(-n)}{t^{k+1} (\alpha+\beta+2\mu+1) \Gamma(\alpha+1)} + \\
&\quad \frac{\tau^{(k+1)\mu} (t^{k+1} - \tau^{k+1})^{\alpha+1}}{t^{k+1}} \times \frac{(k+1)^{\mu+\beta+1} (\alpha+\beta+\mu)(\alpha+\beta+\mu+1)(-n)(-n+1)}{(\alpha+\beta+2\mu+1) \Gamma(\alpha+2) 2!} + ...
\end{aligned} \tag{2.6}$$

It is clear that $F(t,\tau)$ is positive because for all $\tau \in (0,t)$, $(t > 0)$, since each term of the (2.6) is positive.

3. Fractional integral inequalities involving convex functions

In this section, we prove some fractional integral inequalities involving convex function using generalized k-fractional integral (in terms of Gauss hypergeometric function) operator.

Theorem 3.1. *Let p, r be two positive continuous functions on $[0,\infty)$ and $p \leq r$ on $[0,\infty)$. If $\frac{p}{r}$ is decreasing, p is increasing on $[0,\infty)$ and for any convex function Φ, $\Phi(0) = 0$, then for all $k \geq 0$, $t > 0$, $\pi > max\{0, -\varpi - \nu\}$, $\varpi < 1$, $\nu > -1$, $\varpi - 1 < \theta < 0$, we have,*

$$\frac{\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[p(t)]}{\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[r(t)]} \geq \frac{\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[\Phi(p(t))]}{\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[\Phi(r(t))]}. \tag{3.1}$$

Proof:- If the function Φ is convex with $\Phi(0) = 0$, then the function $\frac{\Phi(t)}{t}$ is increasing. Since p is increasing, then $\frac{\Phi(p(t))}{r(t)}$ is also increasing. Clearly $\frac{p(t)}{r(t)}$ is decreasing, for all $\tau, \rho \in [0,\infty)$, and

$$\left(\frac{\Phi(p(\tau))}{p(\tau)} - \frac{\Phi(p(\rho))}{p(\rho)}\right)\left(\frac{p(\rho)}{r(\rho)} - \frac{p(\tau)}{r(\tau)}\right) \geq 0, \tag{3.2}$$

which implies that

$$\frac{\Phi(p(\tau))}{p(\tau)} \frac{p(\rho)}{r(\rho)} + \frac{\Phi(p(\rho))}{p(\rho)} \frac{p(\tau)}{r(\tau)} - \frac{\Phi(p(\tau))}{p(\tau)} \frac{p(\tau)}{r(\tau)} - \frac{\Phi(p(\rho))}{p(\rho)} \frac{p(\rho)}{r(\rho)} \geq 0. \tag{3.3}$$

Multiplying equation (3.3) by $r(\tau)r(\rho)$, we have

$$\frac{\Phi(p(\tau))}{p(\tau)} p(\rho) r(\tau) + \frac{\Phi(p(\rho))}{p(\rho)} p(\tau) r(\rho) - \frac{\Phi(p(\tau))}{p(\tau)} p(\tau) r(\rho) - \frac{\Phi(p(\rho))}{p(\rho)} p(\rho) r(\tau) \geq 0. \tag{3.4}$$

Multiplying both sides of (3.4) by $\mathfrak{F}(t,\tau)$ which is positive, and integrating obtained result with respect to τ from 0 to t, we have

$$p(\rho)\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}\left[\frac{\Phi(p(t))}{p(x)} r(t)\right] + \frac{\Phi(p(\rho))}{p(\rho)} r(\rho) \mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[p(t)] - r(\rho)\mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}\left[\frac{\Phi(p(t))}{p(t)} p(t)\right] - \frac{\Phi(p(\rho))}{p(\rho)} p(\rho) \mathbb{I}^{\pi,\varpi,\theta,\nu}_{t,k}[r(t)] \geq 0. \tag{3.5}$$

Multiplying both sides of (3.5) by $\mathfrak{F}(t,\rho)$ which is positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\\&\geq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}p(t)\right]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}p(t)\right]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)].\end{aligned}\tag{3.6}$$

It follows that

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)r(t)}\right]\\&\geq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(f(t))}{p(t)}p(t)\right],\end{aligned}\tag{3.7}$$

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]}\geq\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}p(t)\right]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]}.\tag{3.8}$$

Since $p\leq r$ on $[0,\infty)$ and function $\frac{\Phi(t)}{t}$ is increasing, then for $\tau,\rho\in[0,\infty)$, we have

$$\frac{\Phi(p(\tau))}{p(\tau)}\leq\frac{\Phi(r(\tau))}{r(\tau)}.\tag{3.9}$$

Multiplying (3.9) by $\mathfrak{F}(t,\tau)r(\tau)$ which is positive, we obtain

$$\mathfrak{F}(t,\tau)\frac{\Phi(p(\tau))}{p(\tau)}\leq\mathfrak{F}(t,\tau)\frac{\Phi(r(\tau))}{r(\tau)},\tag{3.10}$$

integrating equation (3.10) on both side with respective τ from 0 to t, we get

$$\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]\leq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(r(t))}{r(t)}r(t)\right].\tag{3.11}$$

Hence, from (3.8) and (3.11) we obtain required inequality (3.1).

Theorem 3.2. *Let p, r be two positive continuous functions on $[0,\infty)$ and $f\leq r$ on $[0,\infty)$. If $\frac{p}{r}$ is decreasing, p is increasing on $[0,\infty)$ and for any convex function Φ, $\Phi(0)=0$, then for all $k\geq 0$, $t>0$, $\pi>max\{0,-\varpi-\nu\},\gamma>max\{0,-\delta-\upsilon\}$ $\varpi,\delta<1$, $\upsilon,\nu>-1$, $\varpi-1<\theta<0$, $\delta-1<\zeta<0$, we have,*

$$\begin{aligned}&\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\phi(r(t))]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\Phi(r(t))]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\Phi(p(t))]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\Phi(p(t))]}\\&\geq 1,\end{aligned}\tag{3.12}$$

Proof:- If function Φ is convex with $\Phi(0)=0$, then $\frac{\Phi(t)}{t}$ is increasing. Since p is increasing, then $\frac{\Phi(p(t))}{p(t)}$ is also increasing. Clearly $\frac{p(t)}{r(t)}$ is decreasing, for all $\tau,\rho\in[0,t)$ $t>0$. Multiplying equation (3.5) by $\frac{(k+1)^{\upsilon+\delta+1}t^{(k+1)(-\gamma-\delta-2\upsilon)}}{\Gamma(\gamma)}\rho^{(k+1)\upsilon}\times(t^{k+1}-\rho^{k+1})_2^{\gamma-1}F_1(\gamma+\delta+\upsilon,-\zeta;\gamma;1-(\frac{\rho}{t})^{k+1})$ $(\rho\in(0,t)$, $t>0)$, which remains positive from (2.4). Now integrating obtained result with respect to ρ from 0 to t, we have

$$\begin{aligned}&\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(f(t))}{p(t)}r(t)\right]\\&+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\\&\geq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\Phi(p(t))]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\Phi(p(t))]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[r(t)].\end{aligned}\tag{3.13}$$

Since $p\leq r$ on $[0,\infty)$ and as function $\frac{\Phi(t)}{t}$ is increasing, for $\tau,\rho\in[0,t)$ $t>0$, we have

$$\frac{\Phi(p(\tau))}{p(\tau)}\leq\frac{\Phi(r(\tau))}{r(\tau)}.\tag{3.14}$$

Multiplying both sides of (3.14) by $\mathfrak{F}(t,\tau)r(\tau)$ positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\Phi(p(t))}{p(t)}r(t)\right]\leq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\Phi(p(t))].\tag{3.15}$$

Hence, using (3.13) and (3.15), we obtain required inequality (3.12).

Remark 3.3. *If we put $\pi=\gamma$, $\varpi=\delta$ and $\theta=\zeta$ and $\nu=\upsilon$ in Theorem 3.2 it reduces to the Theorem 3.1.*

Now, we prove our main result.

Theorem 3.4. *Let p, r and q be three positive continuous functions on $[0,\infty)$ and $p\leq r$ on $[0,\infty)$. If $\frac{p}{r}$ is decreasing, p and q are increasing functions on $[0,\infty)$, and for any convex function ϕ such that $\phi(0)=0$, then for all $k\geq 0$, $t>0,\pi>max\{0,-\varpi-\nu\}$, $\varpi<1$, $\nu>-1$, $\varpi-1<\theta<0$, $\varpi-1<\theta<0$, we have,*

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]}\geq\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(p(t))q(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(r(t))q(t)]},\tag{3.16}$$

Proof: Since $p\leq r$ on $[0,\infty)$ and function $\frac{\phi(t)}{t}$ is increasing, then for $\tau,\rho\in[0,t)$, $t>0$, we have

$$\frac{\phi(p(\tau))}{p(\tau)}\leq\frac{\phi(r(\tau))}{r(\tau)}.\tag{3.17}$$

Multiplying both sides of (3.17) by $\mathfrak{F}(t,\tau)r(\tau)$ positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\phi(p(t))}{p(t)}r(t)q(t)\right]\leq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(r(t))q(t)].\tag{3.18}$$

On the other hand, since the fact that the function ϕ is convex with $\phi(0)=0$. Then the function $\frac{\phi(t)}{t}$ is increasing. Since p is increasing, $\frac{\phi(p(t))}{p(t)}$ is also increasing. Clearly we can say that $\frac{p(t)}{r(t)}$ is decreasing, for all $\tau,\rho \in [0,t)$ $t>0$

$$\left(\frac{\phi(p(\tau))}{p(\tau)}q(\tau)-\frac{\phi(p(\rho))}{p(\rho)}q(\rho)\right)(p(\rho)r(\tau)-p(\tau)r(\rho)) \geq 0, \tag{3.19}$$

which implies that

$$\begin{aligned}&\frac{\phi(p(\tau))q(\tau)}{p(\tau)}p(\rho)r(\tau)+\frac{\phi(p(\rho))q(\rho)}{p(\rho)}p(\tau)r(\rho)\\&-\frac{\phi(p(\tau))q(\tau)}{p(\tau)}p(\tau)r(\rho)-\frac{\phi(p(\rho))q(\rho)}{p(\rho)}p(\rho)r(\tau)\geq 0.\end{aligned} \tag{3.20}$$

Hence, we can write

$$\begin{aligned}&p(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\phi(p(t))}{p(t)}r(t)q(t)\right]+\\&\frac{\phi(p(\rho))}{p(\rho)}r(\rho)q(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\\&-r(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(p(t))q(t)]\\&-\frac{\phi(p(\rho))}{p(\rho)}p(\rho)q(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\geq 0,\end{aligned} \tag{3.21}$$

with the same argument as before, we have

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]}\geq\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(p(t))q(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\phi(p(t))}{p(t)}r(t)q(t)\right]}. \tag{3.22}$$

Hence, using equation (3.18) and (3.22), we obtain (3.16).

Now, we give generalization of Theorem 3.3.

Theorem 3.5. *Let p, r and q be three positive continuous functions on $[0,\infty)$ and $p \leq r$ on $[0,\infty)$. If $\frac{p}{r}$ is decreasing, p and q are increasing functions on $[0,\infty)$, and for any convex function ϕ such that $\phi(0)=0$, then we have*

$$\begin{aligned}&\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\phi(r(t))q(t)]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(r(t))q(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\phi(p(t))q(t)]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(p(t))q(t)]}\\&\geq 1,\end{aligned} \tag{3.23}$$

where for all $k\geq 0, t>0$, $\pi>max\{0,-\varpi-\nu\},\gamma>max\{0,-\delta-\upsilon\}$ $\varpi,\delta<1$, $\upsilon,\nu>-1$, $\varpi-1<\theta<0$, $\delta-1<\zeta<0$,

Proof:- Multiplying equation (3.21) by $\frac{(k+1)^{\upsilon+\delta+1}t^{(k+1)(-\gamma-\delta-2\upsilon)}}{\Gamma(\gamma)}\rho^{(k+1)\upsilon}\times$ $(t^{k+1}-\rho^{k+1})_2^{\gamma-1}F_1(\gamma+\delta+\upsilon,-\zeta;\gamma;1-(\frac{\rho}{t})^{k+1})$ $(\rho\in(0,t)$, $t>0)$, which remains positive . Then integrate the resulting identity with respect to ρ from 0 to t, we have

$$\begin{aligned}&\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\phi(p(t))q(t)}{p(t)}r(t)\right]\\&+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}\left[\frac{\phi(p(t))q(t)}{p(t)}r(t)\right]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\\&\geq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\phi(p(t))q(t)]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(p(t))q(t)]\,\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[r(t)],\end{aligned} \tag{3.24}$$

and since $p\leq r$ on $[0,\infty)$ and use the fact that $\frac{\phi(t)q(t)}{t}$ is increasing, we obtain

$$\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}\left[\frac{\phi(p(t))q(t)}{p(t)}r(t)\right]\leq\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[\phi(r(t))q(t)], \tag{3.25}$$

and

$$\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}\left[\frac{\phi(p(t))q(t)}{p(t)}r(t)\right]\leq\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[\phi(r(t))q(t)]. \tag{3.26}$$

Hence, from equation (3.24), (3.25) and (3.26), we obtain (3.23).

Remark 3.6. *If we put $\pi=\gamma$, $\varpi=\delta$ and $\theta=\zeta$ and $\nu=\upsilon$ in Theorem 3.4 it reduces to the Theorem 3.3.*

4. Other fractional integral inequalities

In [10], authors have proved the inequalities using Riemann-Liouville fractional integral. Now, we prove the similar results using generalized k-fractional integral (in terms of Gauss hypergeometric function) operator.

Theorem 4.1. *Let p, q be two positive and continuous functions on $[0,\infty)$ such that p is decreasing and q is increasing on $[0,\infty)$. Then for all $k\geq 0$, $t>0$, $\pi>max\{0,-\varpi-\nu\}$, $\varpi<1$, $\nu>-1$, $\varpi-1<\theta<0$, $l\geq m>0$, and $n>0$ we have*

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^l(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^m(t)]}\geq\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^np^l(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^np^m(t)]}. \tag{4.1}$$

Proof:- Consider $\rho,\tau\in(0,t)$, we have

$$(q^n(\rho)-q^n(\tau))\left(p^l(\tau)p^m(\rho)-p^m(\tau)p^l(\rho)\right)\geq 0,$$

which implies that

$$\begin{aligned}&q^n(\rho)p^l(\tau)p^m(\rho)+q^m(\tau)p^m(\tau)p^l(\rho)\\&\geq q^m(\rho)p^m(\tau)p^l(\rho)+q^m(\tau)p^m(\rho)p^l(\tau),\end{aligned} \tag{4.2}$$

Multiplying both sides of (4.2) by $\mathfrak{F}(t,\tau)$ which is positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\begin{aligned}&q^n(\rho)p^m(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^l(t)]+p^l(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^np^m(t)]\\&\geq g^r(\rho)f^p(\rho)I_{t,k}^{\alpha,\beta,\eta,\mu}[f^q(t)]+p^m(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^np^l(t)].\end{aligned}$$

$$\tag{4.3}$$

Now, multiplying both side of (4.3) by $\mathfrak{F}(t,\rho)$ which is positive from (2.4). Now integrating obtained result with respect to ρ from 0 to t, we have

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^m(t)]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^l(t)]\\&\geq \mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^l(t)]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^m(t)],\end{aligned}\tag{4.4}$$

which gives the inequality 4.1.

Theorem 4.2. *Let p, q be two positive and continuous functions on* $[0,\infty)$ *such that p is decreasing and q is increasing on* $[0,\infty)$. *Then for all* $k\geq 0$, $t>0$, $\pi > max\{0,-\varpi-\nu\}, \gamma > max\{0,-\delta-\upsilon\}$ $\varpi,\delta<1$, $\upsilon,\nu>-1$, $\varpi-1<\theta<0$, $\delta-1<\zeta<0$, $l\geq m>0$, *and* $n>0$ *we have*

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^l(t)]\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[q^n p^m(t)]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p^l(t)]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^m(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^m(t)]\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[q^n p^m(t)]+\mathbb{I}_{t,k}^{\gamma,\delta,\zeta,\upsilon}[p^m(t)]\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^m(t)]}\geq 1.\tag{4.5}$$

Proof:- Multiplying equation (4.3) by $\frac{(k+1)^{\upsilon+\delta+1}t^{(k+1)(-\gamma-\delta-2\upsilon)}}{\Gamma(\gamma)}\rho^{(k+1)\upsilon}\times$ $(t^{k+1}-\rho^{k+1})^{\gamma-1}_2F_1(\gamma+\delta+\upsilon,-\zeta;\gamma;1-(\frac{\rho}{t})^{k+1})$ $(\rho\in(0,t)$, $t>0)$, which remains positive. Then integrate the resulting identity with respect to ρ from 0 to t, we obtain the result 4.5.

Theorem 4.3. *Let p, q be two positive and continuous functions on* $[0,\infty)$ *such that p is decreasing and q is increasing on* $[0,\infty)$, *Such that*

$$(p^n(\tau)q^n(\rho)-p^n(\rho)q^n(\tau))\left(p^{l-n}(\tau)-p^{l-n}(\tau)\right)\geq 0,$$

then we have

$$\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^{n+l}(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p^{n+m}(t)]}\geq\frac{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^l(t)]}{\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q^n p^m(t)]},\tag{4.6}$$

where Then for all $k\geq 0$, $t>0, \pi > max\{0,-\varpi-\nu\}$, $\varpi<1$, $\nu>-1$, $\varpi-1<\theta<0$, $\varpi-1<\theta<0$, $l\geq m>0$, $n>0$.

Proof:- Consider $\tau,\rho\in(0,t)$, we get

$$(p^n(\tau)q^n(\rho)-p^n(\rho)q^n(\tau))\left(p^m(\rho)p^l(\tau)-p^m(\tau)p^l(\rho)\right)\geq 0.$$

and using the same arguments as in Theorem [4.1], we obtain the result.

Theorem 4.4. *Let p, q and r be three function on* $[0,\infty)$ *such that*

$$(p(\tau)-p(\rho))(q(\tau)-q(\rho))(r(\tau)+r(\rho))\tag{4.7}$$

then for all τ,ρ, $k\geq 0$, $t>0$, $\pi>max\{0,-\varpi-\nu\}$, $\varpi<1$, $\nu>-1$, $\varpi-1<\theta<0$, *we have,*

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pqr(t)]\Lambda_{t,k}^{\pi,\varpi,\theta,\nu}\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pq(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\\&\geq \mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pr(t)]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)]\end{aligned}\tag{4.8}$$

where, $\Lambda_{t,k}^{\pi,\varpi,\theta,\nu}=(k+1^{-\nu-\varpi})t^{(k+1)(\nu+\varpi)}\frac{\Gamma(1-\varpi)\Gamma(\pi+\nu+\theta+1)}{\Gamma(1+\nu)\Gamma(1-\varpi+\theta)}$

Proof:- From condition (4.7), for any τ,ρ, we have

$$\begin{aligned}&p(\tau)q(\tau)r(\tau)+p(\tau)q(\tau)r(\rho)\\&+p(\rho)q(\rho)r(\tau)+p(\rho)q(\rho)r(\rho)\\&\geq p(\tau)q(\rho)r(\tau)+p(\rho)q(\tau)r(\tau)\\&+p(\rho)q(\tau)r(\rho)+p(\tau)q(\rho)r(\rho).\end{aligned}\tag{4.9}$$

Multiplying both side of equation (4.9) by $\mathfrak{F}(t,\tau)$ which is positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pqr(t)]+r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pq(t)]\\&+p(\rho)q(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]+p(\rho)q(\rho)r(\rho)\Lambda_{t,k}^{\pi,\varpi,\theta,\nu}\\&\geq q(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pr(t)]+f(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)]\\&+p(\rho)r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q(t)]+q(\rho)r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\end{aligned}\tag{4.10}$$

Again, multiplying both side of equation (4.10) $\mathfrak{F}(t,\rho)$ which is positive, and integrating obtained result with respect to ρ from 0 to t, we have

$$\begin{aligned}&\Lambda_{t,k}^{\pi,\varpi,\theta,\nu}\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pqr(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pq(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pq(t)]+\Lambda_{t,k}^{\pi,\varpi,\theta,\nu}\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pqr(t)]\\&\geq \mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\\&+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pr(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)].\end{aligned}\tag{4.11}$$

This compete the proof of inequality (4.8).

Theorem 4.5. *Let p, q and r be three function on* $[0,\infty)$ *such that*

$$(p(\tau)-p(\rho))(q(\tau)+q(\rho))(r(\tau)+r(\rho))\geq 0\tag{4.12}$$

then for all τ,ρ, $k\geq 0$, $t>0$, $\pi>max\{0,-\varpi-\nu\}$, $\varpi<1$, $\nu>-1$, $\varpi-1<\theta<0$, *we have,*

$$\begin{aligned}&\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[q(t)]\\&\geq \mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[qr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[p(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,\nu}[pq(t)]\end{aligned}\tag{4.13}$$

Proof:- From condition (4.12), for any τ, ρ, we have

$$\begin{aligned} & p(\tau)q(\tau)r(\tau)+p(\tau)q(\tau)r(\rho) \\ & +p(\tau)q(\rho)r(\tau)+p(\tau)q(\rho)r(\rho) \\ & \geq p(\rho)q(\tau)r(\tau)+p(\rho)q(\tau)r(\rho) \\ & +p(\rho)q(\rho)r(\tau)+p(\rho)q(\rho)r(\rho). \end{aligned} \tag{4.14}$$

Multiplying both side of equation (4.14)by $\mathfrak{F}(t,\tau)$ which is positive, and integrating obtained result with respect to τ from 0 to t, we have

$$\begin{aligned} & \mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pqr(t)]+r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pq(t)] \\ & +g(\rho)\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pr(t)]+q(\rho)r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[p(t)] \\ & \geq r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[qr(t)]+r(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pq(t)] \\ & +p(\rho)q(\rho)\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[r(\rho)]+p(\rho)q(\rho)r(\rho)\Lambda_{t,k}^{\pi,\varpi,\theta,v}. \end{aligned} \tag{4.15}$$

With the same argument in inequality (4.11), we obtain

$$\begin{aligned} & \Lambda_{t,k}^{\pi,\varpi,\theta,v}\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pqr(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pq(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[r(t)] \\ & +\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[q(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[p(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[qr(t)] \\ & \geq \mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[qr(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[q(t)]+\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pq(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[r(t)] \\ & +\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[r(t)]\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pq(t)]+\Lambda_{t,k}^{\pi,\varpi,\theta,v}\,\mathbb{I}_{t,k}^{\pi,\varpi,\theta,v}[pqr(t)]. \end{aligned} \tag{4.16}$$

where,$\Lambda_{t,k}^{\pi,\varpi,\theta,v}$ is as in theorem 4.4. This compete the proof of inequality (4.13).

5. Concluding Remarks

In this study, we presented generalized k-fractional integral operator operators. We established some fractional integral inequalities involving convex functions by considering generalized k-fractional integral operator. Here, we briefly consider some implication of our main results. The inequalities proposed in this paper give some contribution in the fields of fractional calculus and Generalized k-fractional integral operators. Moreover, they are expected to led to some application for finding uniqueness of solutions in fractional differential equations.

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