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On some intuitionistic fuzzy hyponormal operators

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Abstract

Using the definition of Intuitionistic Fuzzy Hyponormal (IFHN) operator, i.e. $\mathbb{S} \in IFB(\mathbb{H})$ is an IFHN-operator if $\mathcal{P}_{\mu,\nu}(\mathbb{S}^*x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}x,u), \forall x \in \mathbb{H}$ or equivalently $\mathbb{S}^*\mathbb{S} - \mathbb{S}\mathbb{S}^* \geq 0$, we investigate certain properties of IFHN-operators on an IFH-space. The definition of intuitionistic fuzzy class (N) of operators and some spectral properties are introduced. Also, a few theorems are discussed in detail.

Keywords

Intuitionistic fuzzy Banach space, Intuitionistic Fuzzy Hilbert (IFH) space, Intuitionistic Fuzzy Normal Operator (IFN-operator), Intuitionistic Fuzzy Hyponormal Operator (IFHN-operator), Intuitionistic Fuzzy Class (N).

AMS Subject Classification

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1. Introduction

Atanossov[11] introduced the Intuitionistic Fuzzy Set in 1986. The notion of Intuitionistic Fuzzy Metric Space $(\mathcal{S}, M, N, *, \diamond)$ has been introduced by Park [7] in 2004. Saadati and Park [10] introduced modulation of the intuitionistic fuzzy metric space in IFIP-space using continuous t-representable in 2005. In 2009, Goudarzi et al.[6] introduced the new definition of Intuitionistic Fuzzy Normed Spaces and also given the modified definition of Intuitionistic Fuzzy Inner Product Space (IFIP-space). Goudarzi et al.[6] introduced a triplet $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$, where $\mathcal{H}$ is a Real vector space, $\mathcal{T}$ is a continuous t-representable and $\mathcal{F}_{\mu,\nu}$ is an intuitionistic fuzzy set on $\mathcal{H}^2 \times \mathbb{R}$, as an Intuitionistic Fuzzy Hilbert Space in 2009. Majumdar and Samanta [13] gave various definitions of Intuitionistic Fuzzy Inner Product space and some of their properties using $(\mathcal{H}, \mu, \mu^*)$.

In 2018, Radharamani et al. [1], [2] first given intuitionistic fuzzy Hilbert space (IFH-space) definition and introduced Intuitionistic Fuzzy Adjoint & Intuitionistic Fuzzy Self-Adjoint Operators and gave some properties using IFH - space. After that Intuitionistic Fuzzy Normal Operator and their properties were introduced by Radharamani et al. [3] in 2020. An operator $\mathbb{S} \in IFB(\mathcal{H})$ is an IFN-operator if $\mathbb{S}\mathbb{S}^* = \mathbb{S}^*\mathbb{S}$, where $\mathbb{S}^*$ is an IFA-operator. Then, Radharamani et al. [4], [5] given the definition of Intuitionistic Fuzzy Unitary (IFU) operator and Intuitionistic Fuzzy Partial Isometry (IFPI) operator on IFH - space $\mathbb{H}$ and gave some theorems based on these definitions. Also the relation of IFU and IFPI operators with Isometric Isomorphism of $\mathcal{H}$ on to itself was discussed.

The definition of Intuitionistic Fuzzy Hyponormal (IFHN) Operator, $\mathcal{S} \in$ IFB $\mathbb{H})$ is IFHN-operator if $\mathcal{P}_{\mu,\nu}(\mathcal{S}^*x,u) \leq \mathcal{P}_{\mu,\nu}(\mathcal{S}x,u), \forall x \in \mathbb{H}$ or equivalently $\mathcal{S}^*\mathcal{S} - \mathcal{S}\mathcal{S}^* \geq 0$, has been introduced by Radharamani et al. [5] in 2020. A few properties of IFHN - Operator on IFH – Space have been given. Also, some definitions and theorems of intuitionistic fuzzy invariant, eigenvalue, eigenvectors and eigenspaces related to IFHN-operator in IFH-space were discussed.

In this paper, we discussed some properties of Intuitionistic Fuzzy Hyponormal (IFHN) Operator in IFH - Space and also introduced intuitionistic Fuzzy Class (N) of operators and established a few theorems from IFHN – operator on IFH - Space. All are discussed in detail.

2. Preliminaries

Definition 2.1. ***[IFIP-Space]*** *[6] Let* $\mu : \mathcal{V}^2 \times (0,+\infty) \to [0,1]$ *and* $\nu : \mathcal{V}^2 \times (0,+\infty) \to [0,1]$ *be fuzzy sets,* $\ni \mu(x,y,t) +$

$\nu(x,y,t) \leq 1, \forall x,y \in \mathcal{V} \,\&\, t > 0$. *An Intuitionistic Fuzzy Inner Product Space (IFIP-Space) is a triplet* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$, *where* $\mathcal{V}$ *is a Real vector space,* $\mathcal{T}$ *is a continuous t - representable and* $\mathcal{F}_{\mu,\nu}$ *is an intuitionistic fuzzy set on* $\mathcal{V}^2 \times \mathbb{R}$ *satisfying the following conditions for all* $x,y,z \in \mathcal{V}$ *and* $s,r,t \in \mathbb{R}$:

IFI-1: $\mathcal{F}_{\mu,\nu}(x,y,0) = 0 \,\&\, \mathcal{F}_{\mu,\nu}(x,x,t) > 0$, *for every* $t > 0$.

IFI-2: $\mathcal{F}_{\mu,\nu}(x,y,t) = \mathcal{F}_{\mu,\nu}(y,x,t)$.

IFI-3: $\mathcal{F}_{\mu,\nu}(x,x,t) \neq H(t)$ *for some* $t \in \mathbb{R}$ *iff* $x \neq 0$,
where $H(t) = \begin{cases} 1, & if \quad t > 0 \\ 0, & if \quad t \leq 0 \end{cases}$

IFI-4: *For any* $\alpha \in \mathbb{R}$,

$$\mathcal{F}_{\mu,\nu}(\alpha x,y,t) = \begin{cases} \mathcal{F}_{\mu,\nu}(x,y,\frac{t}{\alpha}), & \alpha > 0 \\ H(t), & \alpha = 0 \\ \mathcal{N}_s(\mathcal{F}_{\mu,\nu}(x,y,\frac{t}{\alpha})), & \alpha < 0 \end{cases}$$

IFI-5: $\sup\{\mathcal{T}(\mathcal{F}_{\mu,\nu}(x,z,s), \mathcal{F}_{\mu,\nu}(y,z,r))\} = \mathcal{F}_{\mu,\nu}(x+y,y,t)$.

IFI-6: $\mathcal{F}_{\mu,\nu}(x,y,.) : \mathbb{R} \longrightarrow [0,1]$ *is continuous on* $\mathbb{R}\backslash\{0\}$.

IFI-7: $\lim_{t\to 0} \mathcal{F}_{\mu,\nu}(x,y,t) = 1$.

Definition 2.2. ***[Intuitionistic Fuzzy Hilbert Space][1, 6]*** *Let* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFIP-Space with IP:*$\langle x,y\rangle = \sup\{t \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(x,y,t) < 1\}, \forall x,y \in \mathcal{V}$. *If* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *is complete in the norm* $\mathcal{P}_{\mu,\nu}$, *then* $\mathcal{V}$ *is an Intuitionistic fuzzy Hilbert space (IFH-Space).*

Definition 2.3. ***[IFA-operator][2]*** *Let* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-Space and let* $\mathcal{S} \in IFB(\mathcal{V})$. *Then there exists unique* $\mathcal{S}^* \in IFB(\mathcal{V}) \ni \langle \mathcal{S}x,y\rangle = \langle x,\mathcal{S}^*y\rangle, \forall x,y \in \mathcal{V}$.

Definition 2.4. ***[IFSA-operator][2]*** *Let* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-Space with IP:*$\langle x,y\rangle = \sup\{t \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(x,y,t) < 1\}, \forall x,y \in \mathcal{V}$ *and let* $\mathcal{S} \in IFB(\mathcal{V})$. *Then* $\mathcal{S}$ *is intuitionistic fuzzy self-adjoint operator, if* $\mathcal{S} = \mathcal{S}^*$, *where* $\mathcal{S}^*$ *is intuitionistic fuzzy self-adjoint of* $\mathcal{S}$.

Theorem 2.5. *[2] Let* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-Space with IP:*$\langle x,y\rangle = \sup\{t \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(x,y,t) < 1\}, \forall x,y \in \mathcal{V}$ *and let* $\mathcal{S} \in IFB(\mathcal{V})$. *Then* $\mathcal{S}$ *is intuitionistic fuzzy self-adjoint operator.*

Definition 2.6. ***[IFN-operator][3]*** *Let* $(\mathcal{V}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-Space with IP:*$\langle u,v\rangle = \sup\{t \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(u,v,t) < 1\}, \forall u,v \in \mathcal{V}$ *and let* $\mathcal{S} \in IFB(\mathcal{V})$. *Then* $\mathcal{S}$ *is an intuitionistic fuzzy normal operator if it commutes with its intuitionistic fuzzy-adjoint.*

Definition 2.7. ***[IFU-operator][4]*** *Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space with IP:*$\langle u,v\rangle = \sup\{t \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(u,v,t) < 1\}, \forall u,v \in \mathbb{H}$ *and let* $\mathbb{P} \in IFB(\mathbb{H})$. *Then* $\mathbb{P}$ *is intuitionistic fuzzy unitary operator if it satisfies* $\mathbb{P}\mathbb{P}^* = I = \mathbb{P}^*\mathbb{P}$.

Definition 2.8. *[5] Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space and let* $\mathbb{T} \in IFB(\mathcal{H})$. *Then*

(a). A scalar $\lambda, 0 < \lambda < 1$, *is called an eigenvalue of* $\mathbb{T}$ *if* $\exists$ *non-zero* $a \in \mathcal{H}, \ni \mathbb{T}a = \lambda a$.

(b). A non-zero vectora $\in \mathcal{H}$ *is called eigenvector of* $\mathbb{T}$, *if there exists* $\lambda, 0 < \lambda < 1$, *such that* $\mathbb{T}a = \lambda a$

Theorem 2.9. *[5] Let* $\mathbb{T}$ *be an IFN - operator on a finite dimensional IFH - space* $\mathcal{H}$ *over* $\mathbb{R}$ *then,*

(i). $\mathbb{T} - \lambda I$ *is Intuitionistic Fuzzy Normal*

(ii). Every eigenvector of $\mathbb{T}$ *is also an eigenvector of* $\mathbb{T}^*$

Definition 2.10. ***[IF-Invariant][5]*** *Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space and let* $\mathbb{T} \in IFB(\mathcal{H})$. *A subspace* $\mathcal{M}$ *of an IFNL-space* $\mathcal{H}$ *is said to be IF-Invariant under* $\mathbb{T}$, *if* $\mathbb{T}\mathcal{M} \subset \mathcal{M}$.

Theorem 2.11. *[5] Let* $\mathcal{M}$ *be a closed subspace of an IFH - space and let* $\mathbb{T} \in IFB(\mathcal{H})$. *Then* $\mathcal{M}$ *is IF-Invariant under* $\mathbb{T}$ *if and only if* $\mathcal{M}^\perp$ *is IF - Invariant under* $\mathbb{T}^*$.

Definition 2.12. *[5] Let* $\mathcal{M}$ *be a closed subspace of an IFH-space and let* $\mathbb{T} \in IFB(\mathcal{H})$. *If both* $\mathcal{M}$ *and* $\mathcal{M}^\perp$ *are IF-Invariant under* $\mathbb{T}$, *then we say that* $\mathcal{M}$ *reduces* $\mathbb{T}$ *(or* $\mathbb{T}$ *is reduced by*$\mathcal{M}$*).*

Theorem 2.13. *[5] A closed subspace* $\mathcal{M}$ *of an IFH - space* $\mathcal{H}$ *reduces an operator* $\mathbb{T}$ *if and only if* $\mathcal{M}$ *is IF-Invariant under* $\mathbb{T}$ *and* $\mathbb{T}^*$.

Definition 2.14. *[5] Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space , let* $\mathbb{T} \in IFB(\mathcal{H})$ *and let* λ *be an eigenvalue of* $\mathbb{T}$. *Then the set of all eigenvectors corresponding to* λ *together with* 0 *vector is called an eigenspace of* $\mathbb{T}$ *corresponding to the eigenvalue* λ *and is denoted by* $\mathcal{M}_\lambda$.

Definition 2.15. ***[Intuitionistic Fuzzy Hyponormal Operator (IFHN-operator)][5]*** *Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space with IP:*$\langle a,b\rangle = \sup\left\{s \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(a,b,s) < 1\right\}, \forall a,b \in \mathcal{H}$ *and let* $\mathbb{T} \in IFB(\mathcal{H})$. *Then* $\mathbb{T}$ *is an Intuitionistic Fuzzy Hyponormal (IFHN) Operator on* $\mathcal{H}$ *if* $\mathcal{P}_{\mu,\nu}(\mathbb{T}^*a,s) \leq \mathcal{P}_{\mu,\nu}(\mathbb{T}a,s), \forall a \in \mathcal{H}$ *or equivalently* $\mathbb{T}^*\mathbb{T} - \mathbb{T}\mathbb{T}^* \geq 0$.

Theorem 2.16. *[5] Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space with IP:*$\langle a,b\rangle = \sup\left\{s \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(a,b,s) < 1\right\}, \forall a,b \in \mathcal{H}$ *and let* $\mathbb{T} \in IFB(\mathcal{H})$ *be an intuitionistic fuzzy hyponormal (IFHN) operator on* $\mathcal{H}$. *Then* $(\mathcal{P}_{\mu,\nu}((\mathbb{T} - zI)a,s) \geq \mathcal{P}_{\mu,\nu}((\mathbb{T}^* - \bar{z}I)a,s)$, $a \in \mathcal{H}$, *i.e.* $\mathbb{T} - zI$ *is an IFHN-operator.*

Theorem 2.17. *[5] Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space and let*$\mathbb{T} \in FB(\mathcal{H})$*.be an IFHN-operator on* $\mathcal{H}$. *Then* $\mathbb{T}a = \lambda a \Rightarrow \mathbb{T}^*a = \bar{\lambda}a$.

Theorem 2.18. *[5] Let* $\mathbb{T} \in IFB(\mathcal{H})$ *be an IFHN-operator iff* $\mathcal{P}_{\mu,\nu}(\mathbb{T}^*a,s) \leq \mathcal{P}_{\mu,\nu}(\mathbb{T}a,s), \forall a \in \mathcal{H}$

Theorem 2.19. *[5] Let* $(\mathcal{H}, \mathcal{F}_{\mu,\nu}, \mathcal{T})$ *be an IFH-space with IP:*$\langle a,b\rangle = \sup\left\{s \in \mathbb{R} : \mathcal{F}_{\mu,\nu}(a,b,s) < 1\right\}, \forall a,b \in \mathcal{H}$ *and let* $\mathbb{T} \in IFB(\mathcal{H})$. *Then* $\mathbb{T}$ *is an Intuitionistic Fuzzy Hyponormal (IFHN) Operator on* $\mathcal{H}$ *with* $\mathcal{M} \subset \mathcal{H}$ *IF-invariant under* $\mathbb{T}$ *also let* $\mathbb{T}_\mathcal{M}$ *be intuitionistic fuzzy hyponormal. Then* $\mathcal{M}$ *reduces* $\mathbb{T}$.

Theorem 2.20. *[5] Let $\mathbb{T}$ be an IFHN-operator on $\mathcal{H}$ and $\mathcal{M} = \{a \in \mathcal{H} : \mathbb{T}a = \lambda a\}$then $\mathcal{M}$ reduces $\mathbb{T}$ and $\mathbb{T}|_{\mathcal{M}}$is intuitionistic fuzzy hyponormal*

Theorem 2.21. *[5] Let $\mathbb{T}$ be an IFHN-operator on $\mathcal{H}$ and let $\mathcal{M} \subset \mathcal{H}$, IF-invariant under$\mathbb{T}$. Then $\mathbb{T}|_{\mathcal{M}}$ is intuitionistic fuzzy hyponormal.*

3. Main Results

In this section, we introduced a few properties of IFHN-operator on IFH-Space and defined intuitionistic fuzzy class (N) operators in IFH-space. In order to that we first given some basic definitions and theorems.

Definition 3.1. *Let $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ be an IFNL-Space over the field $\mathbb{C}$ where $X = \{0\}$ and $\mathcal{P}_{\mu,\nu} : X \times (0,\infty) \to [0,1]$ and $\mathbb{S} : (X, \mathcal{P}_{\mu,\nu}, \mathcal{T}) \to (X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ be an intuitionistic fuzzy linear operator. A regular value λ of $\mathbb{S}$ is a complex number such that*

(a) $R_\lambda(\mathbb{S})$ exists.

(b) $R_\lambda(\mathbb{S})$ is intuitionistic fuzzy bounded linear operator on Range of $\mathbb{S}_\lambda = \mathbb{S} - \lambda I$.

(c) $R_\lambda(\mathbb{S})$ is defined on a set which is dense in X. where $R_\lambda(\mathbb{S}) = (\mathbb{S}_\lambda)^{-1} = (\mathbb{S} - \lambda I)^{-1}$, called resolvent operator of $\mathbb{S}$.

The set of all regular values λ of $\mathbb{S}$ is called the resolvent set of $\mathbb{S}$. It is denoted by $\rho(\mathbb{S})$.
The complement $\sigma(\mathbb{S})$ of resolvent set $\rho(\mathbb{S})$ in complex plane $\mathbb{C}$ is called the spectrum of $\mathbb{S}$. i.e., $\sigma(\mathbb{S}) = \mathbb{C} - \rho(\mathbb{S})$. A regular value $\lambda \in \sigma(\mathbb{S})$ is called a spectral value of $\mathbb{S}$.

Theorem 3.2. *Let $\mathbb{S}$ be an IFHN-operator on IFH-space $\mathbb{H}$. Then $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = R_{sp}(\mathbb{S})$ (the spectral radius of $\mathbb{S}$).*

Proof. Take $\mathcal{P}_{\mu,\nu}(x,u) = 1$, for $x \in \mathbb{H}$.

$$\begin{aligned}
\mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) &= \langle \mathbb{S}x, \mathbb{S}x\rangle \\
&= \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(\mathbb{S}x, \mathbb{S}x, u) < 1\} \\
&= \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x, x, u) < 1\} \\
&= \langle \mathbb{S}^*\mathbb{S}x, x\rangle \\
&\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x, u)
\end{aligned}$$

$\therefore \mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2x,u) \ldots$ (3.1)
But $\mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2x,u) \leq \mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u)$ ...(3.2)
which implies that $\mathcal{P}^2_{\mu,\nu}(\mathbb{S},u) = \mathcal{P}_{\mu,\nu}(\mathbb{S}^2,u)$. Now,

$$\begin{aligned}
\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^n x,u) &= \langle \mathbb{S}^n x, \mathbb{S}^n x\rangle \\
&= \langle \mathbb{S}^n x, \mathbb{S}\mathbb{S}^{n-1}x\rangle \\
&= \langle \mathbb{S}^*\mathbb{S}^n x, \mathbb{S}^{n-1}x\rangle \\
&\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}^n x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u) \\
&\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}\mathbb{S}^{n-1}x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u) \\
&\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)
\end{aligned}$$

Then $\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^n x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)$.
The above result combined with the equality and induction process yields, $\mathcal{P}_{\mu,\nu}(\mathbb{S}^n,u) = \mathcal{P}^n_{\mu,\nu}(\mathbb{S},u)$, for $n = 1,2,3,\ldots$.

$$\because R_{sp}(\mathbb{S}) = \lim_{n\to\infty} {\mathcal{P}_{\mu,\nu}}^{1/n}(\mathbb{S}^n,u)$$

$$R_{sp}(\mathbb{S}) = \lim_{n\to\infty} \mathcal{P}_{\mu,\nu}(\mathbb{S},u)$$

i.e. $R_{sp}(\mathbb{S}) = \mathcal{P}_{\mu,\nu}(\mathbb{S},u)$, where $R_{sp}(\mathbb{S})$ means the spectral radius of $\mathbb{S}$. □

Theorem 3.3. *Let $\mathbb{S}$ be an IFHN-operator on $\mathbb{H}$ and λ_0 be an isolated point in the spectrum of $\mathbb{S}$ then $\lambda_0 \in \sigma_p(\mathbb{S})$, the point spectrum of $\mathbb{S}$.*

Proof. By theorem 2.16, we may assume that $\lambda_0 = 0$, choose $r > 0$ sufficiently small that 0 is the only point of $\sigma(\mathbb{S})$, contained in or on the circle $|\lambda| = r$.
Define $\mathbb{E} = \int_{|\lambda|=r}(\mathbb{S} - \lambda I)^{-1}\,d\lambda$. Then $\mathbb{E}$ is a non-zero projection which commutes with $\mathbb{S}$.
Thus, $\mathbb{E}\mathbb{H}$ is invariant under $\mathbb{S}$ and $\mathbb{S}|_{\mathbb{E}\mathbb{H}}$ is intuitionistic fuzzy hyponormal, also $\sigma(\mathbb{S}|_{\mathbb{E}\mathbb{H}}) = \sigma(\mathbb{S}) \cap \{|z| = r\}$
Also, $\sigma(\mathbb{S}|_{\mathbb{E}\mathbb{H}}) = \{0\}$.
Since, $\mathbb{S}|_{\mathbb{E}\mathbb{H}}$ is an IFHN-operator, we may conclude that $\mathbb{S}|_{\mathbb{E}\mathbb{H}}$ is the zero transformation.
It is clear that $\mathbb{E}\mathbb{H} = \{x \in \mathbb{H} : \mathbb{S}x = 0\} \Rightarrow \mathbb{E}\mathbb{H}$ reduces $\mathbb{S}$. □

Theorem 3.4. *Let $(\mathbb{H}, \mathbb{F}_{\mu,\nu}, \mathcal{T})$ be an IFH - Space with IP : $\langle x,y\rangle = \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(x,y,u) < 1\}, \forall x,y \in \mathbb{H}$. If $\mathbb{S} \in$IFB $(\mathbb{H})$ is an Intuitionistic Fuzzy Hyponormal operator on $\mathbb{H}$ with a single limit point in its spectrum then $\mathbb{S}$ is Intuitionistic Fuzzy Normal.*

Proof. We may assume that by theorem 2.16, the limit point is zero.
By theorem 3.2, there exists an isolated point $\lambda_1 \in \sigma(\mathbb{S}) \ni |\lambda_1| = \mathcal{P}_{\mu,\nu}(\mathbb{S},u)$.
Let $\mathbb{M}_1 = \{x \in \mathbb{H} : \mathbb{S}x = \lambda_1 x\}$; $\mathbb{M}_1$ is not empty by theorem 3.3 and since $\mathbb{M}_1$ reduces $\mathbb{S}$, we conclude from theorem 3.3 that $\mathbb{S}|_{\mathbb{M}_1^\perp}$ does not have λ_1 in its spectrum. By corollary 2.20 that $\mathbb{S}|_{\mathbb{M}_1}$ is intuitionistic fuzzy normal.
We continue in this way, selecting points in $\sigma(\mathbb{S})$ ordered by absolute value, setting $\mathbb{M}_i = \{x \in \mathbb{H} : \mathbb{S}x = \lambda_i x\}$. Then $M_1 \oplus M_2 \oplus M_3 \cdots \oplus M_n$ reduces $\mathbb{S}$ and $\mathbb{S}|_{\mathbb{M}_1 \oplus \mathbb{M}_2 \oplus \mathbb{M}_3 \cdots \oplus \mathbb{M}_n}$ is intuitionistic fuzzy normal. We observe that $\mathbb{S}|_{[\mathbb{M}_1 \oplus \mathbb{M}_2 \oplus \mathbb{M}_3 \cdots \oplus \mathbb{M}_n]^\perp}$ is intuitionistic fuzzy hyponormal with its spectral radius equal to its intuitionistic fuzzy norm.
Thus, since 0 is the only point of $\sigma(\mathbb{S})$, the intuitionistic fuzzy normal operators $\mathbb{S}|_{\mathbb{M}_1 \oplus \mathbb{M}_2 \oplus \mathbb{M}_3 \cdots \oplus \mathbb{M}_n}$ must converge to $\mathbb{S}$ in the uniform operator topology. Hence $\mathbb{S}$ is intuitionistic fuzzy normal. □

Corollary 3.5. *If $\mathbb{S} \in IFB(\mathbb{H})$ is an Intuitionistic Fuzzy Hyponormal, Completely Continuous Operator then $\mathbb{S}$ is Intuitionistic Fuzzy Normal.*

Lemma 3.6. *Let $\mathbb{S} \in IFB(\mathbb{H})$ be an IFHN-operator and let $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = 1$. Then there exists a sequence $\{x_n\}$ in $\mathbb{H}$, $\mathcal{P}_{\mu,\nu}(x_n,u) = 1$ such that*

(1) $\mathcal{P}_{\mu,\nu}(\mathbb{S}^* x_n, u) \to 1$

(2) $\mathcal{P}_{\mu,\nu}(\mathbb{S}^m x_n, u) \to 1, m = 1,2,3,...$

(3) $\mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S} x_n - x_n, u) \to 0$

(4) $\mathcal{P}_{\mu,\nu}(\mathbb{S}\mathbb{S}^* x_n - x_n, u) \to 0$

(5) $\mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}^m x_n - \mathbb{S}^{m-1} x_n, u) \to 0, m = 1,2,3,...$

Proof. **Relation (1):**
By definition there exists a sequence $\{x_n\}, \mathcal{P}_{\mu,\nu}(x_n,u) = 1$ such that
$\mathcal{P}_{\mu,\nu}(\mathbb{S}^* x_n, u) \to \mathcal{P}_{\mu,\nu}(\mathbb{S}^*, u) = \mathcal{P}_{\mu,\nu}(\mathbb{S}, u) = 1$.
Relation (2):
By equations (3.1 & 3.2) of theorem (3.2), we have for x, $\mathcal{P}_{\mu,\nu}(x,u) = 1, \mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 x, u)$.
$\because \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^* x_n, u) \leq \mathcal{P}^2_{\mu,\nu}(\mathbb{S} x_n, u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 x_n, u) \leq 1$
We have, $\lim \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 x_n, u) = 1$.
Next, we prove that if $\mathcal{P}_{\mu,\nu}(\mathbb{S}^{k-1} x_n, u) \to 1$ and $\mathcal{P}_{\mu,\nu}(\mathbb{S}^k x_n, u) \to 1$, then $\mathcal{P}_{\mu,\nu}(\mathbb{S}^{k+1} x_n, u) \to 1$, by using induction.
Now, $\mathcal{P}_{\mu,\nu}(\mathbb{S}^2 x_n, u) \leq 1$

$$\begin{aligned}
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 \frac{\mathbb{S}^{k-1} x_n}{\mathbb{S}^{k-1} x_n}, u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 \frac{\mathbb{S}^{k-1} x_n}{\mathbb{S}^{k-1} x_n}, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^{k+1} x_n, u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^{k-1} x_n, u) \leq 1 \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^{k+1} x_n, u) \leq 1
\end{aligned}$$

By induction we have the relation (2).
$\mathcal{P}_{\mu,\nu}(\mathbb{S}^m x_n, u) \longrightarrow 1 \ m = 1,2,3,...$
Relation (3):
From (2), $\mathcal{P}_{\mu,\nu}(\mathbb{S}^m x_n, u) \longrightarrow 1$
Let $m = 1$, then $\mathcal{P}_{\mu,\nu}(\mathbb{S} x_n, u) \longrightarrow 1$

$$\begin{aligned}
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S} x_n, u) \longrightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^*, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S} x_n, u) \longrightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S} x_n, u) - \mathcal{P}_{\mu,\nu}(x_n, u) \longrightarrow 1 - \mathcal{P}_{\mu,\nu}(x_n, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S} x_n - x_n, u) \longrightarrow 0
\end{aligned}$$

Relation (4): From (1),

$$\begin{aligned}
&\mathcal{P}_{\mu,\nu}(\mathbb{S}^* x_n, u) \longrightarrow 1 \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}\mathbb{S}^* x_n, u) \longrightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}\mathbb{S}^* x_n, u) - \mathcal{P}_{\mu,\nu}(x_n, u) \longrightarrow 1 - \mathcal{P}_{\mu,\nu}(x_n, u) \\
&\Rightarrow \mathcal{P}_{\mu,\nu}(\mathbb{S}\mathbb{S}^* x_n - x_n, u) \longrightarrow 0
\end{aligned}$$

Relation (5): Let $y_n(m) = \mathbb{S}^*\mathbb{S}^m x_n - \mathbb{S}^{m-1} x_n$ and $\delta_n(m) = \mathcal{P}^2_{\mu,\nu}(y_n(m), u)$.

We have,

$$\begin{aligned}
\delta_n(m) =& \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}^m x_n, u) - 2\mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}^m x_n, u) \\
& \mathcal{P}_{\mu,\nu}(\mathbb{S}^{m-1} x_n, u) + \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^{m-1} x_n, u) \\
\leq & \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^m x_n, u) - 2\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^m x_n, u) + \\
& \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^{m-1} x_n, u) \\
=& \mathcal{P}_{\mu,\nu}(\mathbb{S}^{m-1} x_n, u) - \mathcal{P}_{\mu,\nu}(\mathbb{S}^m x_n, u)
\end{aligned}$$

We get from Relation (2) that $\delta_n(m) \longrightarrow 0$ for every m. Hence, proved. □

Theorem 3.7. *Let $(\mathbb{H}, \mathbb{F}_{\mu,\nu}, \mathcal{T})$ be an IFH - Space with IP : $\langle x,y \rangle = \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(x,y,u) < 1\}, \forall x,y \in \mathbb{H}$ and let $\mathbb{S} \in$IFB $(\mathbb{H})$ be an IFHN-operator on H. If $\mathbb{S}^{*p}\mathbb{S}^q$ is Completely Continuous where p and q are positive integers then $\mathbb{S}$ is Intuitionistic Fuzzy Normal.*

Proof. Suppose p and q, the positive integers such that $\mathbb{S}^{*p}\mathbb{S}^q$ is a completely continuous operator. By the above lemma $\mathbb{S}^*\mathbb{S}^q x_n - \mathbb{S}^{q-1} x_n \to 0$, where $\{x_n\}$ is the sequence, same as the one in above lemma.
Clearly, $\{\mathbb{S}^{*p-1}\mathbb{S}^{q-1} x_n\}$ accept a convergent subsequence. From the above lemma and from this statement we find a convergent subsequence of $\{\mathbb{S}^{*p-2}\mathbb{S}^{q-2} x_n\}$. Repeating this process we get a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ which is convergent.
Let $x_0 = \lim x_{n_k}$.
Thus, $\mathbb{S}^*\mathbb{S} x_0 = x_0$ and $\mathbb{S}\mathbb{S}^* x_0 = 0$.
The closed subspace $M_{\mathbb{S}} = \{x, \mathbb{S}\mathbb{S}^* x = x\}$ is a non-zero subspace. By the known result we have that $\mathbb{S}$ has an approximate eigenvalue
$\mathbb{S} y_n - \lambda y_n \to 0$.
Therefore, every sequence of approximate eigen vectors $\{y_n\}$ of $\mathbb{S}$ corresponding to $\overline{\lambda}$ with $\overline{\lambda} = 1$, has a subsequence which is convergent. So that $\overline{\lambda}$ is an eigenvalue of $\mathbb{S}^*$. Thus, λ is an eigenvalue of $\mathbb{S}$.
Let M be the smallest closed linear subspace containing every proper subspace of $\mathbb{S}$ and $N = M^{\perp}$. It is known that N is IF-Invariant for $\mathbb{S}^*$ and thus $\mathbb{S}^{*p}\mathbb{S}^q$ is a Completely Continuous Operator on N. Also, we know that $\mathbb{S}_N$ is fuzzy hyponormal. Hence, from this we get $N = \{0\}$ and $M = \mathbb{H}$. This completes the proof. □

Definition 3.8. *Let $(\mathbb{H}, \mathbb{F}_{\mu,\nu}, \mathcal{T})$ be an IFH - Space with IP : $\langle x,y \rangle = \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(x,y,u) < 1\}, \forall x,y \in \mathbb{H}$. Then $\mathbb{S} \in$IFB $(\mathbb{H})$ is said to be of Intuitionistic Fuzzy Class (N) if $x \in \mathbb{H}, \mathcal{P}_{\mu,\nu}(x,u) = 1$ and $\mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2 x, u)$.*

Lemma 3.9. *Every Intuitionistic Fuzzy Hyponormal Operator is of Intuitionistic Fuzzy Class (N).*

Proof. For $x \in \mathbb{H}, \mathcal{P}_{\mu,\nu}(x,u) = 1$

$$\begin{aligned}\mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) &= \langle \mathbb{S}x, \mathbb{S}x\rangle \\ &= \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(\mathbb{S}x,\mathbb{S}x,u) < 1\}, \forall x \in \mathbb{H} \\ &= \sup\{u \in \mathbb{R} : \mathbb{F}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,x,u) < 1\}, \forall x \in \mathbb{H} \\ &= \langle \mathbb{S}^*\mathbb{S}x, x\rangle \\ &\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u)\end{aligned}$$

$\therefore \mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^2x,u)$ □

Note 3.10. *By the above lemma, it is clear that these operators are extension of intuitionistic fuzzy class of IFHN - Operators.*

Lemma 3.11. *If $\mathbb{S} \in$IFB $(\mathbb{H})$, the IFHN – operator, is of Intuitionistic Fuzzy Class (N) and $(i)\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = 1, (ii)\mathcal{P}^2_{\mu,\nu}(x_n,u) \to 1, (iii)\mathcal{P}_{\mu,\nu}(\mathbb{S}x_n,u) \to 1$, then $\mathcal{P}_{\mu,\nu}(\mathbb{S}^m x_n,u) \to 1 (m = 1,2,3,\dots)$.*

Proof. The Proof of this lemma is given earlier in Relation (2) of lemma 3.6. □

Theorem 3.12. *Let $\mathbb{S} \in$IFB $(\mathbb{H})$ be an IFHN-operator. If $\mathbb{S}$ is of Intuitionistic Fuzzy Class (N) on an IFH - Space, then $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = \lim \mathcal{P}^{1/n}_{\mu,\nu}(\mathbb{S}^n,u) = \delta_r$.*

Proof. For every n, lemma 3.11 leads the relation $\mathcal{P}_{\mu,\nu}(\mathbb{S}^n,u) = \mathcal{P}^n_{\mu,\nu}(\mathbb{S},u)$. Now,

$$\begin{aligned}\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^n x,u) &= \langle \mathbb{S}^n x, \mathbb{S}^n x\rangle \\ &= \langle \mathbb{S}^n x, \mathbb{S}\mathbb{S}^{n-1}x\rangle \\ &= \langle \mathbb{S}^*\mathbb{S}^n x, \mathbb{S}^{n-1}x\rangle \\ &\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}^n x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u) \\ &\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}\mathbb{S}^{n-1}x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u) \\ &\leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1}x,u)\end{aligned}$$

Then $\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^n,u) \leq \mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1},u)\mathcal{P}_{\mu,\nu}(\mathbb{S}^{n-1},u)$
The above result combined with the equality and induction process yields,$\mathcal{P}_{\mu,\nu}(\mathbb{S}^n,u) = \mathcal{P}^n_{\mu,\nu}(\mathbb{S},u)$ for $n = 1,2,3,\dots$
Therefore, we get $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = \lim \mathcal{P}^{1/n}_{\mu,\nu}(\mathbb{S}^n,u) = \delta_r$. □

Lemma 3.13. *Let $\mathbb{S} \in$IFB $(\mathbb{H})$ be an IFHN-operator on IFH - Space. If $\mathbb{S}$ is of Intuitionistic Fuzzy Class (N) on an IFH - Space and $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = 1$, then $M_{\mathbb{S}^*} = \{x : \mathbb{S}\mathbb{S}^* = x\}$ is IF-invariant under $\mathbb{S}$.*

Proof. Let $x \in M_{\mathbb{S}^*}, \mathcal{P}_{\mu,\nu}(x,u) = 1$. Then

$$\begin{aligned}\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x - x,u) &= \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) \\ &\qquad \mathcal{P}_{\mu,\nu}(x,u) + \mathcal{P}^2_{\mu,\nu}(x,u) \\ &= \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}^2_{\mu,\nu}(\mathbb{S}x,u) + 1 \\ &= \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}^2_{\mu,\nu}(\mathbb{S}\mathbb{S}^*\mathbb{S}x,u) + 1 \\ &\leq \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}^2_{\mu,\nu}(\mathbb{S}\mathbb{S}\mathbb{S}^*x,u) + 1 \\ &= \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^2\frac{\mathbb{S}^*x}{\mathcal{P}_{\mu,\nu}(\mathbb{S}^*x,u)}) \\ &\qquad \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*x,u) + 1 \\ &\leq \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - 2\mathcal{P}^4_{\mu,\nu}(\mathbb{S}\mathbb{S}^*x,u) \\ &\qquad \frac{1}{\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*x,u)} + 1 \\ &\leq \mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x,u) - \frac{2}{\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*x,u)} + 1 \\ &\leq 0\end{aligned}$$

Thus, $\mathcal{P}^2_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x - x,u) \leq 0 \to \mathcal{P}_{\mu,\nu}(\mathbb{S}^*\mathbb{S}x - x,u) = 0$.
It is clear that $\mathbb{S}x = \mathbb{S}\mathbb{S}^*(\mathbb{S}x) = \mathbb{S}(\mathbb{S}^*\mathbb{S}x)$, which shows that $\mathbb{S}x \in M_{S^*}$. □

Theorem 3.14. *If an IFHN-operator $\mathbb{S} \in$IFB $(\mathbb{H})$ is of Intuitionistic Fuzzy Class (N) on an IFH - Space and $\mathbb{S}^*$ is Completely Continuous for some $k \geq 1$, then $\mathbb{S}$ is an IFN-operator.*

Proof. From Completely Continuous property of $\mathbb{S}^k$ (for $\mathcal{P}_{\mu,\nu}(\mathbb{S},u) = 1$), the subspace $M_{\mathbb{S}^*} = \{x : \mathbb{S}\mathbb{S}^* = x\}$ is not $\{0\}$. Also, since $M_{\mathbb{S}^*}$ is IF-Invariant under $\mathbb{S}^k$, it is finite dimensional. $\mathbb{S}^k$ is IF-isometric isomorphism and Completely Continuous. Then $M_{\mathbb{S}^*}$ reduces $\mathbb{S}$. Considering the subspace $M^{\perp}_{\mathbb{S}^*}$ and continuing in this way we get that $\mathbb{S}$ is an IFN-operator. □

4. Conclusion

Applying the definition of Intuitionistic fuzzy Hyponormal operator (IFHN - Operator) in IFH – Space, the new hypotheses were given, which assume the original aspect in our discussion. A few more properties have been explored from IFHN - operator. As the definition of Intuitionistic Fuzzy Class (N) of operators on an Intuitionistic Fuzzy Banach Space is additionally given, the results of this paper will be accessible for further research to develop application side of Functional Analysis.

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