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Half separation axioms in generalized topological spaces

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Abstract
In this paper,we study separations of sets called μ half separated and corresponding to these notions introduced μ half connectedness in generalized topological spaces.

Keywords
μ separated, μ semi-separated, μ pre-separated, μ α-separated, μ β-separated, μ b-separated, μ half separated, μ half α-separated, μ half semi-separated, μ half pre-separated, μ half β-separated sets, μ half b-separated, μ connectedness, μ α-connectedness, μ semi-connectedness, μ pre-connectedness, μ β-connectedness, μ half connectedness, μ half α-connectedness, μ half semi-connectedness, μ half pre-connectedness, μ half β-connectedness.

AMS Subject Classification
54A05, 54D15.

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1. Introduction and Preliminaries

A generalized topology or simply GT μ [2] on a nonempty set X is a collection of subsets of X such that $\phi \in \mu$ and μ is closed under arbitrary union. Elements of μ are called μ-open sets. A subset A of X is said to be μ-closed if $X-A$ is μ-open. The pair (X,μ) is called a generalized topological space (GTS). If A is a subset of a space (X,μ), then $c_\mu(A)$ is the smallest μ closed set containing A and $i_\mu(A)$ is the largest μ open set contained in A. If $\gamma:\ \varphi(X)\rightarrow\varphi(X)$ is a monotonic function defined on a nonempty set X and $\mu=\{A|A\subset\nu(A)\}$,the family of all γ open sets is also a GT [1], $i_\mu=i_\gamma,\ \ c_\mu=c_\gamma$ and $\mu=\{A|A=i_\mu(A)\}$ [3, Corollary 1.3]. The family of all monotonic functions defined on X is denoted by Γ. By a space (X,μ), we will always mean a GTS (X,μ). The concepts of μ semi-connectedness, μ pre-connectedness, μ α-connectedness, μ β-connectedness and μ b-connectedness in general topological spaces are based on the notions of semi-open set, pre-open set, α-open set, β-open set and b-open sets respectively. The classes of μ α-connected, μ semi-connected, μ pre-connected, μ β-connected and μ b-connected topological spaces are subclasses of the class of connected topological spaces. In this article we introduce the notions of μ half connectedness, μ half α-connectedness, μ half semi-connectedness, μ half pre-connectedness, μ half β-connectedness and μ half b-connectedness in general topological spaces. These properties are weaker forms of μ connectedness, μ α-connectedness, μ semi-connectedness, μ pre-connectedness, μ β-connectedness and b-connectedness respectively.

Definition 1.1. *[1] Let (X,μ) be a GTS. The GTS X is called*

(i) μ-connected if there are no nonempty disjoint μ- open subsets P, Q of X such that $P\cup Q=X$.

(ii) μ α- connected if there are no nonempty disjoint μ-α-open subsets P, Q of X such that $P\cup Q=X$.

(iii) μ semi-connected if there are no nonempty disjoint μ-semi-open subsets P, Q of X such that $P\cup Q=X$.

(iv) μ pre-connected if there are no nonempty disjoint μ–pre-open subsets P, Q of X such that $P\cup Q=X$.

(v) μ β*-connected if there are no nonempty disjoint* μ*-*β*-open subsets P, Q of X such that* $P \cup Q = X$.

2. μ -Half separated sets

In this section μ -half separated sets are defined and some of their properties are discussed.

Definition 2.1. *Two non-empty subsets A and B in a GTS X are said to be* μ*-separated (resp.,*μ *semi-separated,* μ *pre-separated,* μ α *–separated,* μ β*-separated,* μ *b-separated) if* $A \cap C_\mu(B) = \phi$ *or* $\phi = C_\mu(A) \cap B$ *(*$A \cap SC_\mu(B) = \phi$ *or* $\phi = SC_\mu(A) \cap B$*,* $A \cap PC_\mu(B) = \phi$ *or* $\phi = PC_\mu(A) \cap B$*,* $A \cap \alpha C_\mu(B) = \phi$ *or* $\phi = \alpha C_\mu(A) \cap B$*,* $A \cap \beta C_\mu(B) = \phi$ *or* $\phi = \beta C_\mu(A) \cap B$*,* $A \cap bC_\mu(B) = \phi$ *or* $\phi = bC_\mu(A) \cap (B)$*).*

Theorem 2.2. *Let A and B be non-empty sets in a space X. The following statements hold:*
(i) If A and B are μ *half separated (*μ *half semi-separated,* μ *half pre-separated,* μ *half* α*-separated,* μ *half* β*-separated) and* $A_1 \subseteq A$ *and* $B_1 \subseteq B$*, then* A_1 *and* B_1 *are so, respectively.*
(ii) If $A \cap B = \phi$ *and one of A and B is* μ*-closed (*μ *semi-closed,* μ *pre-closed,* μ α*-closed,* μ β*-closed) or* μ*-open (*μ *semi-open,* μ *pre-open,* μ α*-open,* μ β*-open), then A and B are* μ *half-separated (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α*-separated,* μ *half* β*-separated).*
(iii) If one of A and B is μ*-closed (*μ *semi-closed,* μ *pre-closed,* μ α*-closed,* μ β*-closed) or* μ*- open (*μ *semi-open,* μ *pre-open,* μ α*-open,* μ β*-open) and if* $H = A \cap (X - B)$ *and* $G = B \cap (X - A)$*, then H and G are* μ *half-separated (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α*-separated,* μ *half* β*-separated).*

Proof. (i) Claim: A_1 and B_1 are also μ half separated.
$A \cap C_\mu(B) = \phi$ or $C_\mu(A) \cap B = \phi$.
Since $A_1 \subseteq$ A and $B_1 \subseteq$ B, we have $A_1 \cap C_\mu(B_1) \subset A \cap C_\mu(B) = \phi$.
Which implies $A_1 \cap C_\mu(B_1) = \phi$.
Since $A_1 \subseteq A$ and $B_1 \subseteq B$, we have $C_\mu(A_1) \cap B_1 \subset C_\mu(A) \cap B = \phi$.
Which implies $C_\mu(A_1) \cap B_1 = \phi$.
(ii) In case A is μ-open, by $A \cap B = \phi, A \cap C_\mu(B) = \phi$. In case A is μ- closed, $C_\mu(A) \cap B = A \cap B = \phi$. Therefore, A and B are μ half-separated.
(iii) (1) Let A be μ-closed. Then $C_\mu(H) \cap G \subset C_\mu(A) \cap$ (X - A) $= A \cap$ (X - A) $= \phi$.
(2) Let A be μ-open. Then $H \cap C_\mu(G) \subset A \cap C_\mu(X - A) = A \cap (X - A) = \phi$. □

Theorem 2.3. *Let X be a space. If A is a* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected, half* α*-connected,* μ *half* β*-connected) subset of X and H and G are* μ*-half separated (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α *-separated,* μ *half* β *-separated) subsets of X with* $A \subset H \cup G$*, then either* $A \subset H$ *or* $A \subset G$*.*

Proof. Let A be a μ half-connected set and $A \subset H \cup G$. Since H and G are μ half-separated, $G \cap C_\mu(H) = \phi$ or $C_\mu(G) \cap H = \phi$. Let $G \cap C_\mu(H) = \phi$. Since $A = (A \cap H) \cup (A \cap G)$, then $(A \cap G) \cup C_\mu(A \cap H) \subset G \cap C_\mu(H) = \phi$. If $A \cap H$ and $A \cap G$ are non-empty, then A is not μ half-connected, a contradiction. Thus, either $A \cap H = \phi$ or $A \bigcap G = \phi$. This implies that $A \subset H$ or $A \subset G$. □

The following Figure 1 shows the connection between μ separated and μ half separated sets in GTS.

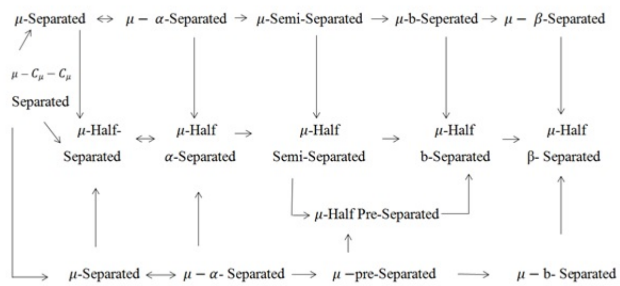

Figure 1

Following examples prove that the reverse implications of the above Figure 1 need not be true.

Example 2.4. *Let* $X = \{a,b,c\}$ *with the general topology* $\mu = \{\phi,\{b\},\{c\},\{b,c\}\}$. $\mu SO = \{\phi,\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\}, X\}$ *and* $\mu\alpha O = \{\phi,\{b\},\{c\},\{b,c\}\}$*. Here* $\{a,b\}$ *and* $\{c\}$ *are* μ *semi-separated sets but not* μ α*-separated sets.*

Example 2.5. *Let* $X = \{a,b,c,d\}$ *with the general topology* $\mu = \{\phi,\{b\},\{d\},\{a,c\},\{b,c\},\{a,b,c\},\{a,b,d\}\}$. μ $bO = \{\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{c,d\},\{a,c\},\{b,d\}, \{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\},X\}$ *and* $\mu SO = \{\phi,\{b\}, \{d\},\{b,c\},\{a,c\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\}, X\}$*. Here* $\{a,b\}$ *and* $\{c,d\}$ *are* μ *b-separated sets but not* μ *semi-separated sets.*

Example 2.6. *Let* $X = \{a,b,c,d\}$ *with the general topology* $\mu = \{\phi,\{a\},\{b,c\},\{a,b,c\}\}$. $\mu\beta O = \{\phi,\{a\},\{b\},\{c\},\{a,b\}, \{b,c\},\{c,d\},\{a,d\},\{a,c\},\{b,d\},\{a,b,d\},\{a,b,c\},\{b,c,d\}, \{a,c,d\},X\}$ *and* $\mu bO = \{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\}, \{a,d\},\{a,c\},\{a,b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},X\}$*. Here* $\{a,b\}$ *and* $\{c,d\}$ *are* μ β*-separated sets but not* μ *b-separated sets.*

Example 2.7. *Let* $X = \{a,b,c,d\}$ *with the general topology* $\mu = \{\phi,\{a\},\{b\},\{a,b\}\}.\mu bO = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\}, \{a,c\},\{a,d\},\{b,d\},\{a,b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},X\}$ *and* $\mu\alpha O = \{\phi,\{a\},\{b\},\{a,b\}\}$*. Here* $\{b\}$ *and* $\{a,c\}$ *are* μ *b-separated sets but not* α*-separated sets.*

Example 2.8. *Let* $X = \{a,b,c\}$ *with the general topology* $\mu = \{\phi,\{b\},\{c\},\{b,c\}\}$. $\mu bO = \{\phi,\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\}, X\}$ *and* $\mu PO = \{\phi,\{b\},\{c\},\{b,c\}\}$*. Here* $\{a,b\}$ *and* $\{c\}$ *are* μ *b-separated sets but not* μ *pre-separated sets.*

Example 2.9. *Let* $X = \{1,2,3,4\}$ *with the general topology* $\mu = \{\phi,\{3\},\{4\},\{1,4\}, \{2,4\},\{1,3,4\},\{1,2,3\}\}.\mu\alpha O =$

$\{\phi,\{3\},\{4\},\{1,4\},\{2,4\},\{1,3,4\},\{1,2,3\}\}$.*Here* $\{1,2,3\}$ *and* $\{4\}$ *are* $\mu\alpha$*-separated sets and* μ *half* α*–separated.*

Example 2.10. *Let* $X=\{a,b,c\}$ *with the general topology* $\mu=\{\phi,\{b\},\{c\},\{b,c\}\}$. $\mu\alpha O=\{\phi,\{b\},\{c\},\{b,c\}\}$*and* $\mu\alpha cl=\{X,\{a,c\},\{a,b\},\{a\}\}$. *Here* $\{a,c\}$ *and* $\{b,d\}$ *are* μ *half* α *separated but not* μ α*-separated.*

Example 2.11. *Let* $X=\{a,b,c,d\}$ *with the general topology* $\mu=\{\phi,\{a\},\{b,c\},\{a,b,c\}\}$.$\mu bO=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},\{a,d\},\{a,b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},X\}$. *Here* $\{a,b,c\}$ *and* $\{d\}$ *are* μ *half b-separated sets but not* $C_\mu-C_\mu$ *separated sets.*

3. μ Half connected sets

In this section we define μ-half connected sets and study some of their properties.

Definition 3.1. *A subset A of a space X is said to be* μ*-half connected (resp.,* μ*half semi-connected,* μ *half pre-connected,* μ *half* α *-connected,* μ *half* β *-connected) if A is not the union of two non-empty* μ *half separated (resp.,*μ *half semi-separated,* μ *half pre-separated,* μ *half* α *-separated,* μ *half* β *-separated) sets in X (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α *-separated,* μ *half* β *-separated).*

Example 3.2. *Let* $X=\{1,2,3\}$ *with the topology* $\mu=\{\phi,\{2\},\{3\},\{2,3\}\}$. $S=\{1,2,3\}\subset X$ *and* $\mu SO=\{\phi,\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\},X\}$. *Take* $A=\{1,2\}$ *and* $B=\{3\}$. $S=A\cup B=\{1,2,3\}\subset X$. *S cannot be written as the union of two disjoint non-empty* μ*-semi open sets. Hence S is* μ*-semi connected.*

Example 3.3. *Let* $X=\{a,b,c,d\}$ *with the topology* $\mu=\{\phi,\{a\},\{b\},\{a,b\}$ *and* $A=\{a,b,c\}\subset X$. $\mu bO=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\},\{a,d\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\},X\}$.*Take* $C=\{a,b\}$ *and* $D=\{b,c\}$. *Now* $A=C\cup D$. *A cannot be written as the union of two disjoint non-empty* μ *half open sets. Hence A is* μ *half semi connected.*

Theorem 3.4. *A space X is* μ*-half-connected (resp.,* μ*-half semiconnected,* μ*-half pre connected,* μ*- half* α*-connected,* μ*- half* β*-connected) iff it cannot be expressed as the disjoint union of non-empty* μ*-open (resp.,* μ*- semi-open,* μ*- pre-open,* μ α*-open,* $\mu\beta$*-open) set and a non-empty* μ*-closed (resp.,* μ*-semi-closed,* μ*- pre-closed,* μ α*-closed,* μ β*-closed) set.*

Proof. Take X as a μ- half-connected space. If $X=U\cup F$, where $U\cap F=\phi$, U is a non-empty μ -open set and F is a non-empty μ- closed set in X. Since F is a μ -closed set in X, $U\cap C_\mu(F)=\phi$ and so U and F are μ- half -separated. Hence, X is not μ- half -connected space, a contradiction. Conversely, if X is not a half-connected space, then there exist half-separated sets U and V in X which are nonempty such that $X=U\cup V$. Let $U\cap C_\mu(V)=\phi$. Set $A=X-C_\mu(V)$ and $F=C_\mu(V)$. Then $A\cup F=X$ and $A\cap F=\phi$. Also A is a non-empty μ-open set and F is a non-empty μ- closed set. ☐

Theorem 3.5. *Let X be a space. If A is a* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected) subset of X and H and G are* μ*-half separated (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α*-separated,* μ *half* β*-separated) subsets of X with* $A\subset H\cup G$*, then either* $A\subset H$ *or* $A\subset G$*.*

Proof. Let A be a μ half-connected set and $A\subset H\cup G$. Since H and G are μ half-separated, by Definition 2.1 $G\cap c_\mu(H)=\phi$ or $c_\mu(G)\cap H=\phi$.Let $G\cap c_\mu(H)=\phi$. Since $A=(A\cap H)\cup(A\cap G)$, then $(A\cap G)\cup cl(A\cap H)\subset G\cap c_\mu(H)=\phi$.If $A\cap H$ and $A\cap G$ are non- empty, then A is not μ half-connected, this is a contradiction. Hence, either $A\cap H=\phi$ or $A\cap G=\phi$.Suppose that $A\cap H=\phi\Rightarrow A=A\cap G\Rightarrow A\subset G$ or $A\cap G=\phi\Rightarrow A=A\cap H\Rightarrow A\subset H$.This implies that $A\subset H$ or $A\subset G$. ☐

Theorem 3.6. *If A and B are* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected) sets of a space X and A and B are not half* μ*-separated (resp.,* μ *half semi-separated,* μ *half pre-separated,* μ *half* α *-separated,* μ *half* β *-separated) then* $A\cap B$ *is* μ *half connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected).*

Proof. Let A and B be μ half-connected sets in X.Suppose $A\cup B$ is not μ half connected. Then there exist two non-empty μ half-separated sets G and H such that $A\cup B=G\cup H$. Since G and H are μ half-separated, $G\cap c_\mu(H)=\phi$ or $c_\mu(G)\cap H=\phi$. Suppose that $G\cap cl(H)=\phi$, since A and B are μ half-connected by Theorem 3.5, either $A\subset G$ or $A\subset H$. Similarly, $B\subset G$ or $B\subset H$.If $A\subset G$ and $B\subset G$, then H is empty, this is a contradiction. If $B\subset G$ and $A\subset H$, then $c_\mu(A)\cap B\subset c_\mu(H)\cap G=\phi$.Therefore, A and B are μ half-separated. This is a contradiction. Hence $A\cup B$ is μ half -connected. ☐

Theorem 3.7. *If* $\{M_i : i\in I\}$ *is a non-empty family of* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected)sets of a space X and* $\cap_{i\in I}M_i\neq\phi$ *, then* $\cup_{i\in I}M_i\neq\phi$ *is* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected).*

Proof. If $\cup_{i\in I}M_i$ is not μ half-connected, then $\cup_{i\in I}M_i=H\cup G$, where H and G are non-empty μ half-separated sets in X. Since $\cap_{i\in I}M_i\neq\phi$, we have a point $x\in\cap_{i\in I}M_i$. Since $x\in\cup_{i\in I}M_i$, either x ∈ H or x ∈ G. Suppose that x ∈ H. By Theorem 3.5, $M_i\subset H$ for all i. Thus, $\cup_{i\in I}M_i\subset H$. Then G is empty. This contradiction to $\cup_{i\in I}M_i$ is not μ half connected. Hence $\cup_{i\in I}M_i$ is μ half-connected. ☐

Theorem 3.8. *Let X be a space,* $\{A_\alpha:\alpha\in\Delta\}$ *be a family of* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected) sets and A is* μ *half-connected (resp.,* μ *half semi-connected,* μ *half pre connected,* μ *half* α*-connected,* μ *half* β*-connected).If*

$A \cap A_\alpha \neq \phi$, for every $\alpha \in \Delta$, then $A \cup (\cup_{\alpha\in\Delta} A_\alpha)$ is μ half-connected (resp., μ half semi-connected, μ half pre connected, μ half α-connected, μ half β-connected).

Proof. Since $A \cap A_\alpha \neq \phi$ for each, $\alpha \in \Delta$ by Theorem 2.2, $A \cup A_\alpha$ is μ half-connected for each $\alpha \in \Delta$. Now, $A \cup (\cup A_\alpha) = \cup(A \cup A_\alpha)$ and $\phi \neq A \subset \cap_{\alpha\in\Delta}(A \cup A_\alpha)$. Hence by Theorem 3.5, $A \cup (\cup A_\alpha)$ is μ half-connected. □

Theorem 3.9. *Let X be a μ T_0 (resp., μ sT_0, μ pT_0, μ αT_0, μ βT_0) topological space where $|X| \geq 2$. Then X is not μ half-connected (resp., μ half semi-connected, μ half pre connected, μ half α-connected, μ half β-connected).*

Proof. Let x, y be distinct points of X. Then there exists a μ-open set U such that $x \in U$and $y \notin U$ or $y \in U$ and $x \notin U$. Let $x \in U$ and $y \notin U$.Then $y \in X - U$, and $X - U$ is μ-closed.Then, $X = U \cup (X - U)$.By Theorem 3.4, X is not μ half-connected. □

4. Conclusion

In this article μ half separated sets and μ half connected sets were introduced and separation axioms were discussed in generalized topological space.

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