Malaya Journal of Matematik, Vol. 8, No. 3, 1331-1336, 2020

https://doi.org/10.26637/MJM0803/0105

μ-Pre^*- closed sets in generalized topological spaces

P. Sivagami[1], M. Padmavathi[2*] and G. Hari Siva Annam [3]

Abstract

In this paper, we extend the notion of closed sets in ordinary topological spaces to μ- pre^*- closed sets in generalized topological spaces. Relationship between this closed set and other closed sets are established. Also we have investigated μ- pre^*-open sets, μ- pre^*-closure, μ- pre^*- interior and some of their basic properties.

Keywords

μ- pre^*-closed sets, μ- pre^*- open sets, μ- pre^*- closure, μ- pre^*- interior.

AMS Subject Classification

54A05.

[1,3] *Department of Mathematics, Kamaraj college, Thoothukudi- 628003, Tamil Nadu, India.*

[2] *Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, Tamil Nadu, India.*

[*]**Corresponding author**: [1] sivagamimuthu75@gmail.com; [2] padmavathimariappan@gmail.com; [3]hsannam@yahoo.com

Article History: Received **08** April **2020**; Accepted **09** August **2020**

©2020 MJM.

Contents

1. Introduction

In topological spaces, generalized topology is an important generalization. According to the definition of generalized topological spaces, ϕ may not be come in closed set. In the same way, X may not be come in the open set of generalized topological spaces. In general topology, frequented applications of interior and closure operators give rise to several different new classes of sets.

In 2002, A. Csaszar[3], [4] was initiated the concept and invented many μ - closed sets like μ - pre closed set, μ - closed sets, μ -α - closed sets, μ-β closed sets etc., in generalized topological spaces. He also described some basic operators on a generalized topology. Thereafter in 2007, P. Sivagami[10] developed generalized topological spaces and revealed many more results of it. Then a huge number of papers are devoted to the study of generalized topological spaces like topological spaces. G. Hari siva Annam[6] progressed kuratowski operator in general topology which is motivated to us analysis on μ- pre^*- closed sets in generalized topological spaces. In this paper, we announce the notion of $\mu - pre^*$-closed set in generalized topological spaces, and also come out with some of their properties.

2. Preliminaries:

Definition 2.1. *[3] Let X be a non- empty set and μ be a collection of subsets of X. Then the pair (X, μ) is called as a generalized topological space (short on GTS) on X if μ has the following properties*

(i) $\phi \in \mu$

(ii) Any union of elements of μ belongs to μ.

The elements of μ are called μ- open sets and the complement of μ-open sets are called μ- closed sets. The family of all μ - closed sets in X is denoted by $\mu(X)$.
In GTS (X,μ), M_μ is defined as $M_\mu = \underset{i\in I}{U} U_i$.

If $M_\mu = X$ then (X,μ) is called as a strongly generlized topological space.

Definition 2.2. *[3] For $A \subseteq X$, the generalized closure $c_\mu(A)$ is the intersection of all μ-closed sets containing A, which is the smallest μ-closed set containing A $(A \subseteq c_\mu(A))$ and the generalized interior $i_\mu(A)$, the union of all μ-open sets contained in A, that is the largest μ-open set contained in A $(A \supseteq i_\mu(A))$.*
Note that $c_\mu(A) = X - i_\mu(X–A)$ and $i_\mu(A) = X - c_\mu(X–A)$.

Definition 2.3 (9). *Let $(X,\ \mu)$ be a GTS. Then the subset A of X is called a μ- generalized closed set (in short, $\mu-g$-closed set) iff $c_\mu(A)\ \subseteq\ U$ whenever $A\ \subseteq\ U$ where U is μ-open set in X. The complement of a $\mu-g$- closed set is called a $\mu-g$-open set.*

Result 2.4. *[4] Every μ- closed set is $\mu-g$- closed.*

Definition 2.5. *Let A be a subset of X, then the μ-g closure of A is denoted by $c\mu^*(A)$ and it is defined by the intersection of all μ-g closed set containing A.*

Result 2.6. *Let A be a subset of X, then $c\mu^*(A) \subseteq c\mu(A)$*

Proof. Suppose $x \notin c\mu(A)$. Then $x \notin F$, for some μ-closed set containing A. By Result 2.4, $x \notin F$, for some μ-g-closed set containing A. Therefore $x \notin c\mu^*(A)$. □

Result 2.7. *If $A \subseteq B$, then $c\mu^*(A) \subseteq c\mu^*(B)$, where A and B are subsets of X.*

Proof. Let $x \notin c\mu^*(B)$. By definition, $x \notin F$, for some μ-g-closed set containing B. Since A ⊆B. Therefore, x ∉ F, for some μ-g-closed set containing A. Hence x ∉ $c\mu^*(A)$. □

Result 2.8. *If A and B are subsets of X, Then $c\mu^*(A\cap B) \subseteq c\mu^*(A)\cap c\mu^*(B)$.*

Proof. We know that, $A\cap B \subseteq A$ and $A\cap B \subseteq B$. By Result 2.7, $c\mu^*(A\cap B) \subseteq c\mu^*(A)$ & $c\mu^*(A\cap B) \subseteq c\mu^*(B)$. Hence $c\mu^*(A\cap B) \subseteq c\mu^*(A)\cap c\mu^*(B)$. □

Result 2.9. *Let A be a subset of X then A is μ- g- closed if and only if $c^*_\mu(A)\ =\ A$.*

Proof. Suppose A is a $\mu-\ g$- closed set of X. By definition of $c^*_\mu(A)$, $c^*_\mu(A) = \cap\{\ F/A \subseteq F$ and F is $\mu-\ g$- closed$\}$. Since A is $\mu-\ g$- closed that implies $c^*_\mu(A)\ =\ A$. Conversely, suppose $c^*_\mu(A)\ =\ A$. By definition of $c^*_\mu(A)$, $c^*_\mu(A)$ *is* $\mu-\ g$-closed. Clearly, A is a $\mu-\ g$- closed set. □

Definition 2.10. *Let $(X,\ \mu)$ be a generalized topological space. Then $A \subseteq\ X$ is said to be*

(i) μ- semi closed [5] if $i_\mu(c_\mu\ (A))\ \subseteq\ A$

(ii) μ- pre closed [3] if $c_\mu\ (i_\mu\ (A))\ \subseteq\ A$

(iii) $\mu-\ \alpha$ closed [4] if $c_\mu\ (i_\mu(c_\mu\ (A)))\ \subseteq\ A$

(iv) $\mu-\ \beta$ closed [4] if $i_\mu\ (c_\mu(i_\mu\ (A)))\ \subseteq\ A$

(v) μ- regular closed [8] if $A = c_\mu\ (i_\mu\ (A))$

(vi) $\mu-\ \pi$ closed if A = union of finitely many μ- regular open sets.

The complement of μ- semi closed (resp., μ- pre closed, μ-α closed, $\mu-\ \beta$ closed, μ- regular closed, $\mu-\ \pi$ closed) is said to be μ- semi open (resp., μ- pre open, $\mu-\ \alpha$ open, $\mu-\beta$ open, μ- regular open, $\mu-\ \pi$ open) in X.

3. $\mu - pre^*$-closed set:

Definition 3.1. *A subset A of a generalized topological space $(X,\ \mu)$ is called a $\mu-\ pre^*$-closed set if $c^*_\mu(i_\mu\ (A)) \subseteq\ A$.*

The collection of all $\mu-pre^*$-closed sets in X is denoted by $P^*c_\mu(X)$ or $pre^*c_\mu(X)$.

Theorem 3.2. *Arbitrary intersection of $\mu-\ pre^*$-closed sets is $\mu-pre^*$-closed.*

Proof. Let $\{\ F_\alpha\ \}$ be the collection of $\mu-pre^*$-closed sets. Then $c^*_\mu(i_\mu\ (F_\alpha))\ \subseteq\ F_\alpha$, for each α. Now, $c^*_\mu(i_\mu\ (\cap F_\alpha))\ \subseteq c^*_\mu(\cap\, i\mu(F_\alpha))\ \subseteq \cap\, c^*_\mu(i_\mu(F_\alpha))\ \subseteq \cap F_\alpha$. Therefore $\cap F_\alpha$ *is* $\mu-pre^*$-closed. □

Remark 3.3. *Union of two $\mu-\ pre^*$-closed sets need not be $\mu-pre^*$-closed set as shown in the following example. And hence finite union of $\mu-\ pre^*$-closed sets need not be $\mu-pre^*$-closed set.*

Example 3.4. *Consider the GTS (X, μ), where X= { a, b, c, d} and μ={ ϕ,{ b} ,{ a, b} , { a,d} , { a,b,d} } Then { a,c} and { b,c} are $\mu-pre^*$-closed but { a,c} ∪ { b,c} = { a,b,c} is not $\mu-pre^*$-closed.*

Theorem 3.5. *Every μ-pre – closed set is $\mu-\ pre^*$-closed.*

Proof. Suppose A is a μ-pre- closed set. That implies $c_\mu(i_\mu(A)) \subseteq A$. Now, $c^*_\mu(i_\mu\ (A))\ \subseteq\ c_\mu(i_\mu(A))\ \subseteq\ A$.
It follows that, A is $\mu-\ pre^*$-closed. □

Theorem 3.6. *Every $\mu-g$- closed set is $\mu-\ pre^*$-closed.*

Proof. By Result: 2.9, A is $\mu-g$- closed then $c^*_\mu(A)$= A. Now, $c^*_\mu(i_\mu\ (A))\ \subseteq\ c^*_\mu(A)\ =\ A$. Therefore A is $\mu-\ pre^*$-closed. But the converse is not true. □

Example 3.7. *Let X= { a, b, c, d} and μ = { ϕ , { b} , { a , c} , { a , d} , { a, b, c} , { a, b, d} , { a , c, d} , { b , c, d} , X} Then { a, b} and { c, d} are $\mu-\ pre^*$-closed but not $\mu-g$- closed.*

Theorem 3.8. *Every μ- closed set is $\mu-\ pre^*$-closed.*

Proof. Let A be a μ- closed set. By result 2.4, A is a μ- g-closed set. By theorem 3.6, the result is clearly true.
But the converse is not true. □

Example 3.9. *Consider the GTS $(X,\ \mu)$, where X= { a, b, c, d} and μ={ ϕ , { a, b} , { a , c} , { b, c} , { a, b, c} } . The sets { a, b, d} , { a, c, d} and { b, c, d} are $\mu-\ pre^*$-closed but not μ- closed.*

Theorem 3.10. *Every $\mu-\alpha$- closed set is $\mu-\ pre^*$-closed.*

Proof. Suppose A is a $\mu-\alpha$- closed set which implies $c_\mu\ (i_\mu\ (c_\mu\ (A))) \subseteq A$. Now, $c^*_\mu(i_\mu(A))\ \subseteq\ c_\mu\ (i_\mu(c_\mu\ (A)))\ \subseteq\ A$. Therefore we have A is $\mu-\ pre^*$-closed. But the converse is not true. □

Example 3.11. *Let X= $\{a, b, c, d\}$ and $\mu = \{\phi, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Then $\{a, b, d\}$, $\{a, c, d\}$ are $\mu-$ pre*-closed but not $\mu-\alpha$- closed.*

Theorem 3.12. *Every μ-regular closed set is $\mu-$ pre*-closed.*

Proof. Let A be a μ-regular closed set then $c_\mu(i_\mu(A)) = A$. Since $c^*_\mu(i_\mu(A)) \subseteq c_\mu(i_\mu(A)) = A$. Clearly, the result is true. □

Remark 3.13. *The converse of the above theorem is not true may be seen by the following example.*

Example 3.14. *Let X= $\{a, b, c, d\}$ and $\mu = \{\phi, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$ Here $\{a\}$, $\{b\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, d\}$ and $\{a, c, d\}$ are $\mu-$ pre*-closed but not μ-regular closed.*

Theorem 3.15. *Every $\mu-\pi$- closed set is $\mu-$ pre*-closed.*

Proof. Suppose A is $\mu-\pi$- closed, then A=$\cap F_\alpha$, where F_α is μ-regular closed. By theorem 3.12, F_α is $\mu-$ pre*-closed and theorem 3.2, $\cap F_\alpha$ is $\mu-$ pre*-closed. Hence A is $\mu-$ pre*-closed. But the converse is not true as shown in the ensuing example. □

Example 3.16. *Consider the GTS (X, μ), where X= $\{a, b, c, d\}$ and μ= $\{\phi, \{b\}, \{a, b\}, \{a, d\}$ $\{a, b, d\}\}$. The sets $\{a, c\}$ $\{c, d\}$ and $\{b, c, d\}$ are $\mu-$ pre*-closed but not a $\mu-\pi$- closed set.*

4. $\mu-$ *pre**-closure:

Definition 4.1. *Let (X, μ) be a GTS and $A \subseteq X$. Then the $\mu-$ pre*-closure of A, denoted by $pre^*\ c_\mu(A)$ or $p^*\ c_\mu(A)$, is the intersection of all $\mu-$ pre*-closed sets containing A ($A \subseteq p^*\ c_\mu(A)$).*

ie) $pre^*\ c_\mu(A) = \cap\{F\ /\ A \subseteq F$ and F is $\mu-$ pre*-closed$\}$

$pre^* c\mu(A)$ is the smallest $\mu-pre^*$ closed set containing A.

Theorem 4.2. *Let A be a subset of X then A is $\mu-$ pre*-closed if $pre^*\ c_\mu(A)$ = A.*

Proof. Suppose A is $\mu-$ pre*-closed. By definition, $pre^* c_\mu(A) = \cap\{F\ /\ F \subseteq A$, F is $\mu-$ pre*-closed $\}$ then $pre^*\ c_\mu(A)$= A. Conversely, suppose $pre^*\ c_\mu(A)$= A. By definition $pre^* c_\mu(A) = \cap\{F/F \subseteq A$, F is $\mu-$ pre*-closed $\}$ and By theorem 3.2, $pre^*\ c_\mu(A)$ *is* $\mu-$ pre*-closed. Clearly, A is $\mu-$ pre*-closed. □

Theorem 4.3. *If A and B are subsets of (X, μ) then the following results hold*

i) $pre^*\ c_\mu(X) = X$

ii) $A \subseteq pre^*\ c_\mu(A)$

iii) If $A \subseteq B$ then $pre^*\ c_\mu(A) \subseteq pre^*\ c_\mu(B)$

iv) $A \subseteq pre^*\ c_\mu(A) \subseteq pre\ c_\mu(A) \subseteq c_\mu(A)$

v) $pre^*\ c_\mu(pre^*\ c_\mu(A)) = pre^*\ c_\mu(A)$

Proof. (i) follows from definition. (ii) Suppose $x \notin pre^*\ c_\mu(A)$ then $x \notin \cap\{F : A \subseteq F$ and F is $\mu-$ pre*-closed$\}$ That implies $x \notin F$, for some $\mu-$ pre*-closed set containing A. Hence $x \notin A$. Therefore $A \subseteq pre^*\ c_\mu(A)$. (iii) Let $A \subseteq B$, suppose $x \notin pre^* c_\mu(B)$ then $x \notin \cap F$ such that F is the $\mu-$ pre*-closed set containing B. $x \notin F$, for some $\mu-$ pre*-closed set containing B. Since $A \subseteq B$ Therefore $x \notin F$, for some $\mu-$ pre*-closed set containing A. Hence $x \notin pre^*\ c_\mu(A)$. Thus, the result is true.

(iv) Assume $x \notin c_\mu(A)$, implies $x \notin \cap\{F : A \subseteq F$ and F is μ-closed$\}$. By definition, $x \notin \cap\{F : A \subseteq F$ and F is μ-pre-closed$\}$ Then by theorem 3.5, $x \notin \cap\{F : A \subseteq F$ and F is $\mu-$ pre*-closed$\}$ Therefore $x \notin \cap F$, Where F is in A. Hence, it is proved.
(v) In theorem 4.2, replace A by $pre^*\ c_\mu(A)$, then clearly get the result. □

Remark 4.4. *In GTS(X, μ), $pre^*\ c_\mu(\phi) \neq \phi$ if μ does not having X.*

Consider the GTS (X, μ), where X= $\{1, 2, 3, 4, 5\}$ and $\mu = \{\phi, \{2\}, \{3, 4\}, \{3, 5\}, \{4,5\}, \{2,3,4\}, \{2,3,5\}, \{2,4,5\}, \{3,4,5\}, \{2,3,4,5\}\}$. In this example, $pre^*\ c_\mu(\phi)$= $\{1\}$. In general, $pre^*\ c_\mu(\phi) \neq \phi$ if X is not a member of μ.

Theorem 4.5. *If A and B are subsets of (X, μ) then $pre^* c_\mu(A \cup B) = pre^* c_\mu(A) \cup pre^* c_\mu(B)$.*

Proof. Let $x \in pre^* c_\mu(A \cup B)$ then $x \in \cap F$ such that F is $\mu-$ pre*-closed containing $A \cup B$. Thus $x \in \cap F$ such that F is a $\mu-pre^*$-closed set containing A or B. Therefore $x \in pre^*\ c_\mu(A) \cup pre^*\ c_\mu(B)$.
Conversely, we know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. By theorem 4.3, $pre^*\ c_\mu(A) \subseteq pre^* c_\mu(A \cup B)$ and $pre^* c_\mu(B) \subseteq pre^* c_\mu(A \cup B)$. Therefore, $pre^*\ c_\mu(A) \cup pre^* c_\mu(B) \subseteq pre^* c_\mu(A \cup B)$. □

Remark 4.6. *In general, $pre^*\ c_\mu(\underset{i\in N}{U} A_i) = \underset{i\in N}{U}\ pre^*\ c_\mu(A_i)$.*

Theorem 4.7. *$pre^*\ c_\mu(A \cap B) \subseteq pre^*\ c_\mu(A) \cap pre^*\ c_\mu(B)$ where A and B are in (X, μ).*

Proof. As We know that, $A \cap B \subseteq A$ and $A \cap B \subseteq B$. By theorem 4.3, $pre^*\ c_\mu(A \cap B) \subseteq pre^*\ c_\mu(A)$ and $pre^*\ c_\mu(A \cap B) \subseteq pre^*\ c_\mu(B)$. Therefore, $pre^*\ c_\mu(A \cap B) \subseteq pre^*\ c_\mu(A) \cap pre^*\ c_\mu(B)$. □

Remark 4.8. *In (ii), (iii) and (iv) of theorem 4.3, the inclusions may be strict and equality may also hold. This can be seen from the ensuing illustration.*

Example 4.9. *Let X= { a, b, c, d} be endowed with* $\mu = \{ \phi , \{ b\} ,\{ a, b\} , \{ a , d\} , \{ a, b, d\} \}$.

For (ii), Let A= { a, c} then $pre^{*}\ c_{\mu}(A) = \{ a, c\}$ *Therefore, A =* $pre^{*}\ c_{\mu}\ (A)$.

For (iii), Let A= { b} and B={ b, c } . Here $A \subset B$ *then* $pre^{*}c_{\mu}(A)= \{ b, c\}$ *and* $pre^{*}\ c_{\mu}(B)= \{ b, c\}$ *Therefore* $pre^{*}c_{\mu}(A) = pre^{*}\ c_{\mu}(B)$.

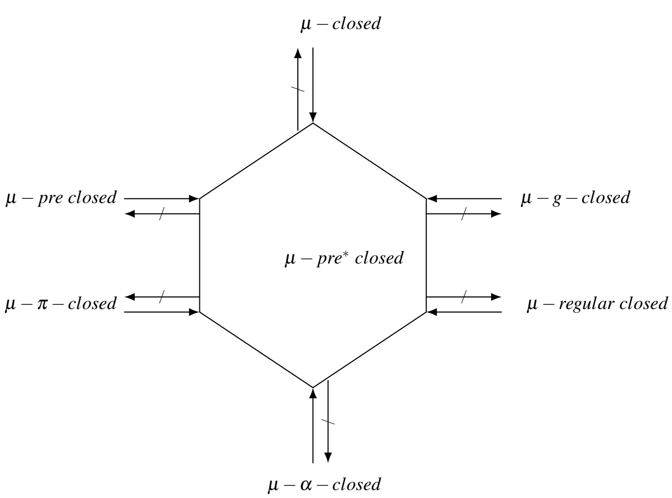
Example 4.10. *Let X={ a,b,c,d} be endowed with* $\mu = \{ \phi ,\{ a,b\} ,\{ a,c\} ,\{ b,c\} , \{ a,b,c\} \}$.

For (iv), Let A= { a, b} then $pre * c_{\mu}(A) = \{ a, b,d \}$ *and* $pre\ c_{\mu}(A) = c_{\mu}(A)= X$. *Therefore, A=* $pre^{*}\ c_{\mu}(A) \subset pre\ \ c_{\mu}(A) = c_{\mu}(A)$.

Let B= { b, c} then $pre^{*}\ c_{\mu}(B) = \{ b, c, d\}$ *and* $pre\ c_{\mu}(B) = c_{\mu}(B)= X$. *Therefore,* $B \subset pre^{*}\ c_{\mu}(B) \subset pre\ c_{\mu}(B) = c_{\mu}(B)$.

Let C= { c} then $pre^{*}\ c_{\mu}(C) = pre\ c_{\mu}(C)= c_{\mu}(C)= \{ c, d\}$. *Therefore,*
$C \subset pre^{*}\ c_{\mu}(C) = pre\ c_{\mu}(C) = c_{\mu}(C)$.

The above results can be represented in the succeeding diagram.



Where $A \rightarrow B$ represents A implies B and $A \nrightarrow B$ represents A does not implies B.

Remark 4.11. *$\mu-\ \beta$- closed set, $\mu-\ \delta$- closed set, $\mu-\ \theta$- closed set and $\mu-\ \alpha^{*}$- closed set are independent with $\mu - pre^{*}$-closed.*

5. $\mu - pre^{*}$-open set

In this sector, we bring up $\mu - pre^{*}$-open sets and analysis their basic properties.

Definition 5.1. *Let* $(X,\ \mu)$ *be a GTS. A subset A of X is called a* $\mu - pre^{*}$*-open set if* $X\backslash A$ *is a* $\mu - pre^{*}$*-closed set.*

$pre^{*}O_{\mu}(X)$ *or* $P^{*}O_{\mu}(X)$ *is the collection of all* $\mu - pre^{*}$*-open sets in X.*

Theorem 5.2. *Arbitrary union of* $\mu - pre^{*}$*-open set is* $\mu - pre^{*}$*-open.*

Proof. Assume $\{ U_{\alpha} \}$ is a collection of $\mu - pre^{*}$-open sets then $\{ X \setminus \{ U_{\alpha} \} \}$ is a collection of $\mu - pre^{*}$-closed sets. By theorem 3.2, $\cap\{ X \setminus\{ U_{\alpha}\} \}$ *is* $\mu - pre^{*}$-closed. It follows that $\cup\{ U_{\alpha} \}$ is $\mu - pre^{*}$-open. □

Theorem 5.3. *Intersection of two* $\mu - pre^{*}$*-open sets is not* $\mu - pre^{*}$*-open which is shown from the following example.*

Example 5.4. *Let X={ a, b, c, d} and* $\mu = \{ \phi , \{ b\} , \{ a , c\} , \{ a , d\} , \{ a, b, c\} , \{ a, b, d\} ,\{ a , c, d\} , \{ b , c, d\}$ *Here* $\{ a, b\} , \{ a, d\}$ *are* $\mu - pre^{*}$*-open but* $\{ a, b\} \cap \{ a, d\}$ *is not* $\mu - pre^{*}$*-open.*

Theorem 5.5. *Suppose* $(X,\ \mu)$ *is a GTS, then the following theorems are hold*

(i) All μ*- open is* $\mu-\ pre^{*}$*-open.*

(ii) All $\mu - g$*- open is* $\mu-\ pre^{*}$*-open.*

(iii) All $\mu - pre - open$ *is* $\mu-\ pre^{*}$*-open.*

(iv) All $\mu - \alpha$*- open is* $\mu-\ pre^{*}$*-open.*

(v) All μ*-regular open is* $\mu-\ pre^{*}$*-open.*

(vi) All $\mu - \pi$*- open is* $\mu-\ pre^{*}$*-open.*

Proof. Clearly, the result is true by the compliment concept of theorem 3..8 , 3.6, 3.5, 3.10, 3.12 and 3.15. But the converse of all the above statements are not true which are shown form the following examples. □

Example 5.6. *Consider the GTS* $(X,\ \mu)$*, X={ a, b, c, d} and* $\mu = \{ \phi , \{ b\} ,\{ a, c\} , \{ a, d\} , \{ a, b, c\} \} \{ a, b, d\} , \{ a, c, d\} , \{ b, c, d\} , X\}$. *Here { a, b} and { c, d} are* $\mu-\ pre^{*}$*-open but not* μ*- open and* $\mu - g$*- open.*

Example 5.7. *Consider the GTS* $(X,\ \mu)$*, X={ a, b, c, d} and* $\mu = \{ \phi, \{ a, b\} , \{ a, c\} , \{ b, c\} , \{ a, b, c\} \}$. *Here { a} , { b} and { c} are* $\mu-\ pre^{*}$*-open but not* $\mu - pre - open$*,* $\mu - \alpha$*- open,* μ*-regular open and* $\mu - \pi$*- open.*

6. $\mu - pre^{*}$-Interior

Definition 6.1. *Let A be a subset of* $(X,\ \mu)$*, then the interior of* $\mu-\ pre^{*}$*-open set is defined by* $\cup\{ U / U \subseteq A \text{ and } U \in pre^{*}O_{\mu}(X)\}$ *and denoted as* $pre^{*}i_{\mu}(A)$ *or* $p^{*}i_{\mu}(A)$. $pre^{*}i_{\mu}(A)$ *is the largest* $\mu - pre^{*}$ *open set contained in A.*

Theorem 6.2. *If $(X,\ \mu)$ is a GTS, then the following results hold*

i) $pre^*\ i_\mu(\phi)\ =\ \phi$

ii) $pre^*\ i_\mu(A)\ \subseteq\ A$

iii) If $A\ \subseteq\ B$ *then* $pre^*\ i_\mu(A)\ \subseteq\ pre^*\ i_\mu(B)$

iv) $i_\mu(A) \subseteq pre\ i_\mu(A) \subseteq pre^* i_\mu(A) \subseteq A$

v) $pre^*\ i_\mu(\ pre^*\ i_\mu(A))\ =\ pre^*\ i_\mu(A)$

Proof. Obviously, the result is true by compliment concept of theorem 4.3. □

Remark 6.3. *In GTS*$(X,\ \mu)$, $pre^*\ i_\mu(X) \neq\ X\ if\ \mu$ *does not having X.*

Consider the example, Let $(X,\ \mu)$ be GTS, where X= { 1, 2, 3, 4, 5} and $\mu =$ { ϕ, { 2} , { 3, 4} , { 3, 5} , { 4,5} , { 2,3,4} ,{ 2,3,5} , { 2,4,5} , { 3,4,5} ,{ 2,3,4,5} } In this example, $pre^*\ i_\mu(X)$= { 2, 3, 4, 5} In general, $pre^*\ i_\mu(X) \neq\ X$, X is not a member of μ.

Theorem 6.4. *Let A be a subset of X then A is $\mu-\ pre^*$-open if and only if $pre^*\ i_\mu(A)\ =\ A$.*

Proof. If A is $\mu-\ pre^*$-open, by definition $pre^*\ i_\mu(A)$ = A. Conversely, suppose $pre^*\ i_\mu(A)\ =\ $A. Since $pre^*\ i_\mu(A)\ =\ \cup\{\ U\ /\ U \subseteq A,\ U$ is $\mu-\ pre^*$-open$\}$ By theorem 5.2, $pre^*\ i_\mu(A)$ is $\mu-\ pre^*$-open. Hence A is $\mu-\ pre^*$-open. □

Theorem 6.5. *If A and B are $\mu-\ pre^*$-open sets in $(X,\ \mu)$, then $pre^*\ i_\mu(A\ \cup\ B)\ \supseteq pre^* i_\mu(A) \cup\ pre^*\ i_\mu(\ B)$.*

Proof. Since $A \subseteq\ A\ \cup\ B$ and $B\ \subseteq\ A\ \cup\ B$. By theorem 6.2, it follows that $pre^*\ i_\mu(A) \subseteq\ pre^*\ i_\mu(A\ \cup\ B)$ and $pre^*\ i_\mu(B)\ \subseteq\ pre^*\ i_\mu(A\ \cup\ B)$. Hence $pre^*\ i_\mu(A\ \cup\ B)\ \supseteq\ pre^*\ i_\mu(A\ )\ \cup\ pre * i_\mu(\ B)$. □

Remark 6.6. *The inclusion of theorem 6.5 may be strict and equality may also hold. This can be seen from the ensuing illustration.*

Example 6.7. *Consider the GTS$(X,\ \mu)$, where X={ a,b,c,d} and μ = { ϕ,{ b} , { a,b} , { a,d} , { a,b,d} } .*

Let A = { a} and B= { b,d} then A ∪B = { a,b,d} . Also $pre^* i_\mu(A)$ = { a} , $pre^* i_\mu(B)$ = { b,d} and $pre^* i_\mu(A \cup B)$ = { a,b,d} . Here $pre^* i_\mu(A) \cup pre^* i_\mu(B)$ = { a,b,d} . Hence $pre^* i_\mu(A) \cup pre^* i_\mu(B) = pre^* i_\mu(A \cup B)$

Let C = { d} and D= { b,c} then $C\ \cup\ D$ = { b,c,d} . Also $pre^* i_\mu(C) = \phi, pre^* i_\mu(D)$ = { b} and $pre^* i_\mu(C \cup D)$ = { b,d} . Here $pre^* i_\mu(C) \cup pre^* i_\mu(D)$ ={ b} . Hence $pre^* i_\mu(C) \cup pre^* i_\mu(D) \subset pre^* i_\mu(C \cup D)$

Theorem 6.8. *Let $x\ \in\ X$, then $x\ \in\ pre^*\ c_\mu(A)$ if and only if $V\ \cap\ A\ \neq\ \phi$ for every $\mu-\ pre^*$-open set containing x.*

Proof. Let $x\ \in\ pre^*\ c_\mu(A)$. Suppose $V\ \cap\ A\ =\ \phi$ for every $\mu-\ pre^*$-open set V containing x. clearly $A\ \subseteq\ X \backslash V$ and $X \backslash V$ is $\mu-\ pre^*$-closed. Since $pre^* c_\mu(A)$ is the smallest $\mu - pre^*$ closed set containing A. Therefore $pre^* c_\mu(A) \subseteq X \backslash V$. Hence $x \in X \backslash V$, which is contradiction to $x \in V$.

Suppose $x\ \notin\ pre^*\ c_\mu(A)$, then there exists $\mu-\ pre^*$-closed set F such that A ⊆ F and x ∉ F. Obviously, x ∉ A, $x \in X \backslash F$ and $X \backslash F$ is $\mu - pre^*$ open. It follows that $A \cap X \backslash F = \phi$, which is contradiction to our assumption. Therefore, $x \in pre^* c\mu(A)$ □

Theorem 6.9. *If A is a subset of (X,μ). Prore that*
(i) $pre^* i_\mu(X \backslash A) = X \backslash pre^* c_\mu(A)$
(ii) $pre^* c_\mu(X \backslash A) = X \backslash pre^* i_\mu(A)$

Proof. (i) Let $x \in pre^* i_\mu(X \backslash A)$, then there exists $\mu - pre^*$ open set V such that $x \in V \subseteq X \backslash A$. Therefore, $x \in V$ and $V \cap A = \phi$. By theorem 6.6, $x \notin pre^* c_\mu(A)$. Hence $x \in X \backslash pre^* c_\mu(A)$. Conversely, Suppose $x \in X \backslash pre^* c_\mu(A)$, that implies $x \notin pre^* c_\mu(A)$. By theorem 6.6, there exists $\mu - pre^*$ open set V such that $V \cap A = \phi$. This shows that $V \subseteq X \backslash A$. Hence $x \in pre^* i_\mu(X \backslash A)$.
(ii) It follows from (i) □

7. Conclusion

The goal of the paper is to find basic properties of $\mu - pre^*$-closed sets in generalized topological spaces. Further, I raise my research towards some special areas in generalized topological spaces.

Acknowledgements

The authors are beholden to the reviewers for useful suggestions which improved the contents of the paper.

References

[1] Abd El-Monsef M. E,El-Deep S.N and Mahmoud. R.A, β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77–90.

[2] Andrijevic D, Semi-pre open sets, *Mat. Vesnik*, 38 (1986), 24-32.

[3] Bishwambhar Roy, Ritu Sen, On a class of sets between – closed sets and – closed sets, *Journal of Taibah University for Science*, 2018, 1–10.

[4] Csaszar A, Generalized open sets in Generalized Topologies, *Acta Math, Hungar,*, 96(2002), 351–357.

[5] Akos Csaszar, Further remarks on the formula for γ-interior, *Acta Mathematica Hungarica*, (2006), 10–25.

[6] Csaszar A, Generalized open sets in Generalized Topologies, *Acta Math, Hungar,*, 106(2005), 53–66.

[7] Devi R, Maki H, Balachandran K, Semi generalized closed maps and generalized closed maps, *Mem. Fac. Kochi univer. A. Math.*, 14(1993), 41–54.

[8] Hari siva Annam. G, On some new class of nearly closed and open sets, *IJMA*, 5(11)(2014), 47–52.

[9] Maki H; Devi R, Balachandran K, Associated topologies of generalized α- closed maps and α- generalized closed maps, *Mrm. Fac. Kochi univer. A. Math.*, 15(1994), 57–63.

[10] Palaniappan N and Rao K. C, Regular Generalized closed sets, *Kyngpook. Math. J.*, 1733(2)(1993), 211–219.

[11] Roy.B, on a type of generalized open sets, *Appl, Gen. Topol.,* 12(2)(2011), 163–173.

[12] Sivagami.P, Sivaraj. D, Note on Generalized open sets, *Acta Mathematica Hungarica*, 117(4)(2002), 335–340.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆