Malaya Journal of Matematik, Vol. 8, No. 4, 1439-1442, 2020

https://doi.org/10.26637/MJM0804/0017

Fuzzy inventory model with allowable shortages and backorder

S. Jerline Zita[1*]

Abstract

In this paper, we are going to exhibit an inventory model in which shortages are permitted and that can be completely replaced. The model is derived to compute the economic order quantity and to minimize the total cost. To get the optimal solution closer to the reality, the fuzzy techniques are applied. We use the octagonal fuzzy numbers to fuzzify the inventory quantities like ordering cost, holding cost and backorder cost. The ranking function of octagonal fuzzy numbers is used here for defuzzification. The optimal solutions are verified with the help of numerical illustrations.

Keywords

Allowable Shortages, Backorders, Ranking Function, Octagonal Fuzzy Number.

[1] *Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Trichy-620002, Tamil Nadu, India.*

*Corresponding author: [1] jerlinezita.stephen@gmail.com

Article History: Received **19** May **2020**; Accepted **21** August **2020**

Contents

1 Introduction 1439

2 Definitions and Methodologies 1440

2.1 Ranking Function Method 1440

2.2 Arithmetic Operations under Function Principle 1440

2.3 Notations 1440

2.4 Assumpions 1440

3 Mathematical Model in Crisp Sense 1440

4 Mathematical Model in Fuzzy sense 1440

5 Numerical Example 1441

6 Conclusion 1441

References 1441

1. Introduction

Inventory means stocking the finished or in process products in a place for some extent of time. Keeping such products for a considerable period of time in business, we have to take decisions over few factors like the quantity of products, length of time etc. so as to minimize the expenses and to maximize the profit. Backorder is the quantity of products that are ordered by the customers and cannot be delivered by the company due to the non-availability. The crisp solutions we derive using mathematical formulation do not fit to the real life situations many times. Because, in reality, many factors are vague and ambiguous which cannot be represented by a single mathematical value. In such cases, the fuzzy concepts have been applied. By taking the fuzzy quantities as Octagonal fuzzy numbers, we represent each vague quantity by eight different possible values. To defuzzify, we use the specific ranking function which is used for intuitionistic octagonal fuzzy numbers.

Rezaei [8] suggestedan EOQ model in which shortages are there and are backordered. Hsu and Hsu [2] came up with an inventory model in which screening for imperfect quality items has been done for several number of times and shortages area allowed and are completely backlogged. Jagadeeswari and Chenniappan [3] contributed an inventory model for items that are deteriorating in which shortages are backloggedpartially. The inventory model developed by Khanna et al. [4] dealt with imperfect quality items with degeneration in which shortages are backlogged. Kumar and Goswami [5] introduced an EOQ model with imperfect quality products and the shortages are backordered fully. Sujatha and Parvathi [11] developed a fuzzy inventory model for degenerating items where shortages are partially backlogged. Patro et al. [7] introduced a fuzzy inventory model in which shortages are accepted and replaced partially.

This paper comes up with a proposal of a fuzzy EOQ model in which shortages are allowed and are backordered fully. The solution is discussed in both crisp and fuzzy senses. For fuzzy sense, we take quantities as octagonal fuzzy numbers. This paper has a numerical illustration for verification. The conclusion has suggestions and ideas for the further study of the paper.

2. Definitions and Methodologies

Definition 2.1. *(Fuzzy set) A fuzzy set $\tilde{A}$ in X is defined as the following set of pair $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$. Here, the membership value of $x \in X$ in a fuzzy set $\tilde{A}$ is the mapping* $\mu_{\tilde{A}} : X \to [0,1]$.

Definition 2.2. *(Octagonal Fuzzy Number) $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is called an octagonal fuzzy number and its membership function is defined as*

$$\mu_{\tilde{A}} = \begin{cases} 0, x < a_1 \\ k\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ k, a_2 \le x \le a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right), a_3 \le x \le a_4 \\ 1, a_4 \le x \le a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right), a_5 \le x \le a_6 \\ k, a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right), a_7 \le x \le a_8 \\ 0, x > a_8 \end{cases}$$

2.1 Ranking Function Method

The Ranking function is a function that maps the set of all octagonal numbers to the set of real numbers. The real number that corresponds to the octagonal fuzzy number using the ranking function is given by

$$R(\tilde{A}) = \frac{2a_1 + 3a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8}{28}$$

2.2 Arithmetic Operations under Function Principle

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ and be two octagonal fuzzy numbers. Then

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$
- $\tilde{A} \ominus \tilde{B} = (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1)$
- $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8)$
- $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}, \frac{a_5}{b_5}, \frac{a_6}{b_6}, \frac{a_7}{b_7}, \frac{a_8}{b_8})$
- If is a scalar, $\alpha\tilde{A}$ is defined as

$$\alpha\tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6, \alpha a_7, \alpha a_8), \alpha \ge 0 \\ (\alpha a_8, \alpha a_7, \alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha \ge 0 \end{cases}$$

2.3 Notations

$D-$	Demand Rate
$A-$	Ordering cost
$H-$	Holding cost
$B-$	Backorder cost
$s-$	Shortage cost
$Q-$	Economic Order Quantity
T_C-	Total cost
$\tilde{A}-$	Fuzzy Ordering cost
$\tilde{H}-$	Fuzzy Holding cost
$\tilde{B}-$	Fuzzy Backorder cost
$\tilde{Q}^*-$	Fuzzy Economic Order Quantity
$\tilde{T}_C-$	Fuzzy Total Cost

2.4 Assumpions

- The demand rate is a known constant.
- There is no limit for time.
- Shortages are permitted and that can be fully backlogged.

3. Mathematical Model in Crisp Sense

For the given mathematical notations and taking the assumptions into account, the total cost is calculated as

$$T_C = \frac{H(Q-s)^2D}{2Q} + \frac{Bs^2D}{2Q} + \frac{AD}{Q} \tag{3.1}$$

By differentiating partially the equation (3.1) with respect to Q and equating it to zero, we have

$$Q^* = \sqrt{\frac{s^2(H+BD)+2AD}{H}} \tag{3.2}$$

4. Mathematical Model in Fuzzy sense

The fuzzy mathematical model is arrived by considering the ordering cost, holding cost and the backorder cost as octagonal fuzzy numbers. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$, $\tilde{b} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$, $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8)$ be octagonal fuzzy numbers. Now, the fuzzy total cost can be derived as

$$\tilde{T}_C = \frac{\tilde{H}(Q-s)^2D}{2Q} + \frac{\tilde{B}s^2D}{2Q} + \frac{\tilde{A}D}{Q} \tag{4.1}$$

Substituting the octagonal fuzzy numbers, we get

$$\begin{aligned} \tilde{T}_C = & \frac{(h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8)\otimes((Q-s)^2D)}{2Q} \\ & \oplus \frac{(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8)\otimes(s^2D)}{2Q} \\ & \oplus \frac{(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)\otimes D}{Q} \end{aligned} \tag{4.2}$$

$$\tilde{T_C} = \begin{bmatrix} \frac{h_1(Q-s)^2D+b_1s^2D+2a_1D}{2Q}, \frac{h_2(Q-s)^2D+b_2s^2D+2a_2D}{2Q} \\ \frac{h_3(Q-s)^2D+b_3s^2D+2a_3D}{2Q}, \frac{h_4(Q-s)^2D+b_4s^2D+2a_4D}{2Q} \\ \frac{h_5(Q-s)^2D+b_5s^2D+2a_5D}{2Q}, \frac{h_6(Q-s)^2D+b_6s^2D+2a_6D}{2Q} \\ \frac{h_7(Q-s)^2D+b_7s^2D+2a_7D}{2Q}, \frac{h_8(Q-s)^2D+b_8s^2D+2a_8D}{2Q} \end{bmatrix} \tag{4.3}$$

By defuzzifying equation (4.3) using ranking function, we get

$$\begin{aligned} \tilde{T_C}^* = & \frac{(Q-s)^2D}{56Q}(2h_1+3h_2+4h_3+5h_4+5h_5+4h_6 \\ & +3h_7+2h_8)+\frac{s^2D}{56Q}(2b_1+3b_2+4b_3+5b_4+5b_5 \\ & +4b_6+3b_7+2b_8)+\frac{D}{28Q}(2a_1+3a_2+4a_3+5a_4 \\ & +5a_5+4a_6+3a_7+2a_8) \end{aligned} \tag{4.4}$$

Differentiating the above equation w.r.t Q and equating to zero, we get By simplifying, we have

$$\tilde{Q}^* = \sqrt{\frac{s^2\begin{pmatrix}(2h_1+3h_2+4h_3+5h_4+5h_5+4h_6 \\ +3h_7+2h_8)+(2b_1+3b_2+4b_3 \\ +5b_4+5b_5+4b_6+3b_7+2b_8)D\end{pmatrix} +2(2a_1+3a_2+4a_3+5a_4+5a_5+4a_6 +3a_7+2a_8)D}{(2h_1+3h_2+4h_3+5h_4+5h_5+4h_6+3h_7+2h_8)}}$$

This gives the fuzzy EOQ and equation (4.4) gives the fuzzy total cost.

5. Numerical Example

Crisp sense:
$D = 250$ units per year
$A = Rs.10$ per unit
$s = Rs.50$ per year
$H = Rs.60$ per unit
$B = Rs.2$ per unit, then
$Q^* = 153.02$
$T_C^* = Rs.6181.50$

Fuzzy Sense:
$D = 250$ units per year
$A = (4, 6, 8, 10, 12, 14, 16, 18)$
$s = Rs.50$ per year
$H =(40, 45, 50, 55, 60, 65, 70, 75)$
$B = (0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4)$ then
$Q^* = 163.47$
$T_C^* = Rs.6581.80.$

6. Conclusion

This paper came up with a fuzzy inventory model in which both shortages and backorder are allowed. The model derived is established in both crisp and fuzzy senses.The ranking function for octagonal intuitionistic numbersis used for defuzzification. Also the inventory costs like cost ordering cost, holding cost and backorder cost are taken as octagonal fuzzy numbers. The arrived solutions are verified using numerical illustrations.This paper can be developed in future for further research work.

References

[1] S. K. Goyal, L. E. Cardenas-Barron, Note on: Economic production quantity model for items with imperfect quality- a practical approach, *International Journal of Production Economics,* 77(2002), 85–87.

[2] J. T. Hsu, L. F. Hsu, A note on: Optimal inventory model for items with imperfect quality and shortage backordering, *International Journal of Industrial Engineering Computations* 3(5), (2012), 939–948.

[3] J. Jagadeeswari, P. K. Chenniappan, An order level inventory model for deteriorating items with time-quadratic demand and partial backlogging, *Journal of Business and Management Sciences,* 2(3), (2014), 79–82.

[4] A. Khana, P. Gautam, C. K. Jaggi, Inventory Modelling for deteriorating imperfect quality items with selling price dependent demand and shortage backordering under credit financing, *International Journal of Mathematical, Engineering and Management Sciences* 2(2), (2017), 110–124.

[5] R. S. Kumar, A. Goswami, Fuzzy stochastic EOQ inventory model for items with imperfect quality and shortages are backlogged, *AMO - Advanced Modeling and Optimization,* 15(2), (2013), 261–279.

[6] G. Menaka, Ranking of Octagonal Intuitionistic Fuzzy Numbers, *IOSR Journal of Mathematics*, 13(3), (2017), 63–71.

[7] R. Patro, M. Acharya, M. M. Nayak, S. Patnaik, A fuzzy inventory model with time dependent Weibull deterioration, quadratic demand and partial backlogging, *International Journal Management and Decision Making,* 16(3), (2017), 243–279.

[8] J. Rezaei, Economic order quantity model with backorder for imperfect quality items, *Proceeding of IEEE International Engineering Management Conference,* (2005), 466–470.

[9] R. Saranya, R. Varadarajan, A fuzzy inventory model with acceptable shortage using graded mean integration value method, *Journal of Physics*, 1000(2018), 012009.

[10] M. K. Salameh, M. Y. Jaber, Economic production quantity model for items with imperfect quality, *International Journal of Production Economics,* 64(2000), 59–64.

[11] J. Sujatha, P. Parvathi, Fuzzy EOQ model for deteriorating items with weibull demand and partial backlogging under trade credit, *International Journal of Informative and Futuristic Research,* 3(2)(2015), 526–537.

[12] M. I. M. Wahab, M. Y. Jaber, Economic order quantity

model for items with imperfect quality, different holding costs, and learning effects: A note, *Computers and Industrial Engineering,* 58(1)(2010), 186–190.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆