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A fuzzy inventory model with green environment by controlling carbon emissions

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Abstract

The purpose of this paper is to propose a mathematical model with green environment by controlling carbon emissions. The problem is discussed in both crisp and fuzzy sense. The graded mean integration representation method is used for defuzzification. Also, the fuzzy parameters like, holding cost, setup cost, ordering cost, variable cost, social cost and carbon emission price are taken as hexagonal fuzzy numbers. The paper concludes with a numerical illustration using which the verification can be done.

Keywords

Carbon Emissions, Function Principle, Graded Mean Integration Representation Method, Hexagonal Fuzzy Numbers.

AMS Subject Classification

62A86.

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Contents

1. Introduction

Releasing the particles of carbon into the atmosphere is actually called the carbon emission. It means the emission of greenhouse gas. It also means the release of carbon di oxide and its equivalents. While discussing about the global warming and the greenhouse effect, these emissions are mentioned as carbon emission. The level of carbon di oxide has started increasing rapidly only after the industrial revolution which causes the burning of fuels in a large quantity.This results in global warming and thus our earth has become less eligible for survival.

Elhedhli and Merrick (2012) designed a green supply chain network model to minimize the cost of the carbon emission. Den elzen (2013) suggested a model in which the industrial emissions and the emissions due to transportation are determined by the life style of people. The model to be formulated should have some eco concern and the consumption of fuel and the emission of carbon molecules should have to be controlled. Sarkar Et al. (2015) proved that the emission of carbon has been more while transporting the raw materials and finished goods from one point to another. Chang (1999) introduced the calculation of economic order quantity, when the demand is not certain. Chen et al (2000) considered all the inventory parameters as fuzzy quantities. Zadeh applied fuzziness during uncertainties. Zadeh et al suggested few methods while making decisions under fuzzy environment. Jain established strategies for decision -making when the variables are fuzzy.

In this paper, we have designed an inventory model to control the carbon emissions using fuzzy concepts. The solution is discussed in both crisp and fuzzy sense. We use hexagonal fuzzy numbers to discuss the same model under fuzzy nature. A numerical example is also illustrated in both crisp and fuzzy senses to verify the above model and finally a conclusion is given for the discussed mathematical model.

2. Definitions and Methodologies

Definition 2.1. *(Fuzzy set) A fuzzy set $\tilde{A}$ in X is defined as the following set of pair $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$. Here, the membership value of $x \in X$ in a fuzzy set $\tilde{A}$ is the mapping $\mu_{\tilde{A}} : X \to [0,1]$.*

Definition 2.2. *(Hexagonal Fuzzy Number) A fuzzy number $\tilde{A}$ is a hexagonal fuzzy number denoted by $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ are real numbers satisfying $(a_2 - a_1 \le a_3 - a_2)$, $(a_5 - a_4 \le a_6 - a_5)$ and its membership function $\mu_{\tilde{A}}$ is given as*

$$\mu_{\tilde{A}} = \begin{cases} 0, x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), a_2 \le x \le a_3 \\ 1, a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), a_4 \le x \le a_5 \\ 0, x > a_6 \end{cases}$$

2.1 Graded Mean Integration Representation Method

If $\tilde{A} = (a, b, c, d, e, f)$ is a hexagonal fuzzy number, then the graded mean integration representation (GMIR) method on $\tilde{A}$ is defined as

$$P(\tilde{A}) = \frac{1}{12}[a + 2b + 3c + 3d + 2e + f]$$

2.2 Arithmetic Operations under Function Principle

The arithmetic operations amonghexagonal fuzzy numbersare given below. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ and be two hexagonal fuzzy numbers. Then

- The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$
- The subtraction of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \ominus \tilde{B} = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$
- The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5, a_6 b_6)$
- The division of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}, \frac{a_5}{b_5}, \frac{a_6}{b_6})$
- If is a scalar, $\alpha\tilde{A}$ is defined as

$$\alpha\tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6), \alpha \ge 0 \\ (\alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha \ge 0 \end{cases}$$

2.3 Notations

$D-$	Annual demand
$P-$	Production rate
$A-$	Ordering cost
$S-$	Setup cost
$H-$	Holding cost per unit time
A_C-	Carbon emission quantity from order per cycle
S_C-	Carbon emission from setting up inventory
H_C-	Carbon emission quantity from holding
$b-$	Variable cost
$g-$	Social cost
$C-$	Carbon emission cost
$n-$	Number of orders made
$d-$	Distance travelled
$Q-$	Economic order quantity
T_C-	Total cost
$\tilde{A}-$	Fuzzy Ordering cost
$\tilde{S}-$	Fuzzy Setup cost
$\tilde{H}-$	Fuzzy Holding cost
$\tilde{b}-$	Fuzzy Variable cost
$\tilde{g}-$	Fuzzy Social cost
$\tilde{C}-$	Fuzzy Carbon emission cost
$\tilde{Q}-$	Fuzzy Economic Order Quantity
$\tilde{T_C}-$	Fuzzy Total Cost

2.4 Assumptions

- The carbon emission quantities obey EOQ assumptions.
- Total demand is contemplated as constant.
- A single product is reviewed over the given period of time.
- Time of plan is constant.

3. Mathematical Model in Crisp Sense

The total cost for the given mathematical model is arrived by adding the ordering cost, setup cost, social cost, holding cost and the variable cost.

$$T_C = \frac{D}{Q}\left[(A + CA_c) + \left(\frac{S + CS_c}{n}\right) + 2g\right] + \frac{Q}{2}\left[(H + CH_c) + \frac{nCD}{P} + \frac{bd}{D}\right] \tag{3.1}$$

By differentiating partially the equation (3.1) with respect to Q and equating it to zero, we obtain the optimal order quantity

$$Q^* = \sqrt{\frac{2D\left[(A + CA_c) + \left(\frac{S + CS_c}{n}\right) + 2g\right]}{\left[(H + CH_c) + \frac{nCD}{P} + \frac{bd}{D}\right]}} \tag{3.2}$$

Hence, equation (3.2) gives the economic order quantity and equation (3.1) gives the minimum total cost.

4. Mathematical Model in Fuzzy sense

Now, the above mathematical model is considered under fuzzy environment. We take the parameters like ordering cost, setup cost and holding cost etc. as hexagonal fuzzy numbers. Let

$$\begin{aligned}\tilde{A} &=(a_1,a_2,a_3,a_4,a_5,a_6)\\ \tilde{S} &=(s_1,s_2,s_3,s_4,s_5,s_6)\\ \tilde{H} &=(h_1,h_2,h_3,h_4,h_5,h_6)\\ \tilde{C} &=(c_1,c_2,c_3,c_4,c_5,c_6)\\ \tilde{g} &=(g_1,g_2,g_3,g_4,g_5,g_6)\\ \tilde{b} &=(b_1,b_2,b_3,b_4,b_5,b_6)\end{aligned}$$

be hexagonal fuzzy numbers. From equation (3.1), the fuzzy total cost is given by

$$\begin{aligned}\tilde{T_C} = \Bigg[\frac{D}{Q} \otimes \Bigg[(\tilde{A} \oplus \tilde{C}A_c) \oplus \left(\frac{\tilde{S} \oplus \tilde{C}S_c}{n}\right) \oplus 2\tilde{g}\Bigg]\\ \oplus \frac{Q}{2} \otimes \Bigg[(\tilde{H} \oplus \tilde{C}H_c) \oplus \frac{n\tilde{C}D}{P} \oplus \frac{\tilde{b}d}{D}\Bigg]\Bigg] \qquad (4.1)\end{aligned}$$

By substituting the hexagonal fuzzy numbers in the above equation, we get

$$\tilde{T_C} = \begin{bmatrix} \frac{D}{Q}\left(a_1 + c_1A_c + \frac{s_1+c_1S_c}{n} + 2g_1\right) + \frac{Q}{2}\left(h_1 + c_1H_c + \frac{nDc_1}{P} + \frac{db_1}{D}\right)\\ \frac{D}{Q}\left(a_2 + c_2A_c + \frac{s_2+c_2S_c}{n} + 2g_2\right) + \frac{Q}{2}\left(h_2 + c_2H_c + \frac{nDc_2}{P} + \frac{db_2}{D}\right)\\ \frac{D}{Q}\left(a_3 + c_3A_c + \frac{s_3+c_3S_c}{n} + 2g_3\right) + \frac{Q}{2}\left(h_3 + c_3H_c + \frac{nDc_3}{P} + \frac{db_3}{D}\right)\\ \frac{D}{Q}\left(a_4 + c_4A_c + \frac{s_4+c_4S_c}{n} + 2g_4\right) + \frac{Q}{2}\left(h_4 + c_4H_c + \frac{nDc_4}{P} + \frac{db_4}{D}\right)\\ \frac{D}{Q}\left(a_5 + c_5A_c + \frac{s_5+c_5S_c}{n} + 2g_5\right) + \frac{Q}{2}\left(h_5 + c_5H_c + \frac{nDc_5}{P} + \frac{db_5}{D}\right)\\ \frac{D}{Q}\left(a_6 + c_6A_c + \frac{s_6+c_6S_c}{n} + 2g_6\right) + \frac{Q}{2}\left(h_6 + c_6H_c + \frac{nDc_6}{P} + \frac{db_6}{D}\right) \end{bmatrix}$$

The graded mean integration representation method is applied here for defuzzification.

$$\tilde{T}_C^* = \begin{bmatrix} \frac{D}{12Q}[(a_1+2a_2+3a_3+3a_4+2a_5+a_6)+A_c(c_1+2c_2\\ +3c_3+3c_4+2c_5+c_6)+\frac{1}{n}[(s_1+2s_2+3s_3+3s_4+2s_5\\ +s_6)+S_c(c_1+2c_2+3c_3+3c_4+2c_5+c_6)]\\ +2(g_1+2g_2+3g_3+3g_4+2g_5+g_6)\\ \frac{Q}{24}[(h_1+2h_2+3h_3+3h_4+2h_5+h_6)+H_c(c_1+2c_2\\ +3c_3+3c_4+2c_5+c_6)+\frac{nD}{P}(c_1+2c_2+3c_3+3c_4+2c_5\\ +c_6)+\frac{d}{D}(b_1+2b_2+3b_3+3b_4+2b_5+b_6)] \end{bmatrix} \qquad (4.2)$$

Now, differentiating the above equation partially with respect to Q and equating to zero, we get

$$Q^* = \sqrt{\frac{2D\begin{bmatrix}(a_1+2a_2+3a_3+3a_4+2a_5+a_6)+A_c(c_1\\ +2c_2+3c_3+3c_4+2c_5+c_6)+\frac{1}{n}[(s_1+2s_2\\ +3s_3+3s_4+2s_5+s_6)+S_c(c_1+2c_2\\ +3c_3+3c_4+2c_5+c_6)]\\ +2(g_1+2g_2+3g_3+3g_4+2g_5+g_6)\end{bmatrix}}{\begin{bmatrix}(h_1+2h_2+3h_3+3h_4+2h_5+h_6)+H_c(c_1\\ +2c_2+3c_3+3c_4+2c_5+c_6)+\frac{nD}{P}(c_1\\ +2c_2+3c_3+3c_4+2c_5+c_6)\\ \frac{d}{D}(b_1+2b_2+3b_3+3b_4+2b_5+b_6)\end{bmatrix}}}$$

This gives the fuzzy optimum order quantity and equation (4.2) gives the fuzzy total cost.

5. Numerical Example

5.1 In Crisp Sense

Let $P = 8000$ units per year, $D = 6000$ units per year, $A = 20$ per cycle, $C = Rs.0.2$, $A_c = 40$ Order per cycle, $S = Rs.80$, $S_c = 40$, $H = 0.8$ per unit time, $H_c = 2$, $n = 1$, $g = Rs.0.5$, $b = 5$ per unit transported, $d = 250Km$.
The optimal order quantity's given by $Q^* = 948.68$.
And the total cost is given by $T_C^* = 1479.95$.

5.2 In Fuzzy Sense

Let $D = 6000$ units per year, $A_c = 40$ order per cycle, $S_c = Rs.40$, $H_c = Rs.2$, $d = 250Km$, $n = 1$, $\tilde{A} = (17,18,19,21,22,23)$, $\tilde{S} = (50,60,70,90,100,110)$, $\tilde{H} = (0.4,0.6,0.7,0.9,1.0,1.2)$, $\tilde{b} = (1,3,4,6,7,9)$, $\tilde{g} = (0.1,0.3,0.4,0.6,0.7,0.9)$, $\tilde{C} = (0.05,0.1,0.15,0.25,0.3,0.35)$
Then the fuzzy optimal order quantity $Q^* = 948.68$
And, the fuzzy total cost is given by $\tilde{T_C}^* = 1479.94$

6. Conclusion

In this paper, we have suggested a fuzzy inventory model with green environment by minimizing carbon emissions. The above model is discussed in both crisp and fuzzy senses. The graded mean integration representation method is used for defuzzification. Also, the fuzzy parameters like holding cost, setup cost, ordering cost, variable cost, social cost and carbon emission price are considered as hexagonal fuzzy numbers. The derived solutions are verified using numerical examples. This paper can be developed for future research work.

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