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Completely P-regular ternary semiring

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Abstract
Here, we presented the completely P-regular ternary semiring . A new form of P-regularity is defined which is complete with arbitrary ideal P, Also we defined some new concept on "completely P-regular" and discussed some theorem with suitable examples as well.

Keywords
Ternary semiring, Regular ternary semiring, Completely regular ternary semiring, P-regular ternary semiring, Completely P-regular ternary semiring.

AMS Subject Classification
15A09, 46C20, 47B20.

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1. Introduction

The concept of semiring was first introduced by Vandiver in 1934. Actually, semiring is a postulation of ring. In [5], Lister introduced ternary ring and regular ternary rings be prepared by Vasile We presently introduced the opinion of ternary semiring which is a generalization of the ternary ring presented by Lister. In fact, ternary semi ring is an algebraic system dwelling of a set T composed by a binary operation, called addition and ternary multiplication, marked by juxtaposition, which forms a commutative semi group apropos to addition, ternary semi group relative to multiplication and left, lateral, right distributive laws hold. Let take Z is a ring of integer. Now $Z^{+}$is subset of all positive integer of Z is an additive semi group which is closed under the ring product such algebraic system is said to be semi ring and Z^{-}form a ternary semiring. T.K.Dutta and S.Kar [3] introduced and studied some properties of ternary semirings which is a generalization of ternary semiring. M.K. Sen, S.K. Maity and K.P. Shum [4] have deliberated in completely regular semiring, which developed in the other paper V.R. Daddi and Y.S. Power [1] was discussed. In this paper we are introduce completely P-regular ternary semiring, where P is an arbitrary ideal .

2. Preliminaries

In this section we introduce completely p-regular ternary semiring.

Definition 2.1. *An element t_α of T is **completely P-regular** ,if there exists$_\alpha \in T$ and $p_1 \in P$ is an arbitrary ideal satisfying the following conditions,*
(i) $t_\alpha + s_\alpha + t_\alpha + p_1 = t_\alpha + p_1$
(ii)$[t_\alpha t_\alpha(t_\alpha + s_\alpha) + p_1 = t_\alpha + s_\alpha + p_1$.

Example 2.2. *If $Z_5 = 0,1,2,3,4 \in T$.It is completely regular as well as it is a completely P-regular.*

3. Completely p-regular ternary semiring

In this section we discussed some of the theorems in completely p-regular ternary semiring.

Theorem 3.1. *If T is a completely P-regular ternary semiring if and only if for any $t_\alpha \in T$ there exists $s_\alpha \in T$ such that the following conditions are,*
(i) $t_\alpha + s_\alpha + t_\alpha + p_1 = t_\alpha + p_1$
(ii)$[t_\alpha t_\alpha(t_\alpha + s_\alpha) + p_1 = t_\alpha + s_\alpha + p_1$.

Proof. Presume that T is a completely p-regular ternary semiring. If for any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$ such that $t_\alpha + s_\alpha + t_\alpha + p_1 = t_\alpha + p_1$. Therefore (i) is hold. We need

only verify that (ii) condition
For any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$
we have,

$$[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$+[t_\alpha t_\alpha t_\alpha]$$
$$=[t_\alpha t_\alpha t_\alpha]+p_1.$$

Then

$$[t_\alpha t_\alpha s_\alpha]+([t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1)$$
$$=[t_\alpha t_\alpha s_\alpha]+[t_\alpha t_\alpha t_\alpha]+p_1$$

$$([t_\alpha t_\alpha s_\alpha]+[t_\alpha t_\alpha t_\alpha])+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha(s_\alpha+t_\alpha)]+p_1$$

$$([t_\alpha t_\alpha(s_\alpha+t_\alpha)])+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha(s_\alpha+t_\alpha)]$$
$$+p_1$$

$$t_\alpha+s_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=s_\alpha+t_\alpha+p_1. \quad (3.1)$$

Since

$$[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1+[t_\alpha t_\alpha t_\alpha]=[t_\alpha t_\alpha(t_\alpha+s_\alpha+t_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha t_\alpha]+p_1,$$

we get $(t_\alpha+s_\alpha)+p_1+[t_\alpha t_\alpha t_\alpha]=t_\alpha+p_1$

$$[t_\alpha+s_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]]+p_1=t_\alpha+s_\alpha+[t_\alpha t_\alpha t_\alpha]$$
$$+[t_\alpha t_\alpha s_\alpha]+p_1$$
$$=[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha s_\alpha]+p_1$$
$$=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$

$$[t_\alpha+s_\alpha+[t_\alpha(t_\alpha+s_\alpha)]]+p_1=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1 \quad (3.2)$$

From (3.1) and (3.2) , we get

$$[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=s_\alpha+t_\alpha+p_1.$$

Similar Proof of the converse part.

□

Theorem 3.2. *If T is a completely P-regular ternary semiring if and only if for any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$ such that the following condition*
(i) $t_\alpha+s_\alpha+t_\alpha+p_1=t_\alpha+p_1$
(ii) $[t_\alpha t_\alpha(t_\alpha+s_\alpha)+p_1=t_\alpha+s_\alpha+p_1$
(iii) $[t_\alpha(t_\alpha+s_\alpha)t_\alpha]+p_1=t_\alpha+s_\alpha+p_1$
(iv) $[(t_\alpha+s_\alpha)t_\alpha t_\alpha]+p_1=t_\alpha+s_\alpha+p_1$
(v) $t_\alpha+[(t_\alpha+s_\alpha)t_\alpha t_\alpha]+p_1=t_\alpha+s_\alpha+p_1$
(vi) $t_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=t_\alpha+p_1$
(vii) $t_\alpha+[t_\alpha(t_\alpha+s_\alpha)t_\alpha]+p_1=t_\alpha+p_1$
(viii) $[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=[t_\alpha(t_\alpha+s_\alpha)t_\alpha]+p_1$
$=[(t_\alpha+s_\alpha)t_\alpha t_\alpha]+p_1.$

Proof. To believe that T is a completely P- regular ternary semiring . Then for any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$ such that $t_\alpha+s_\alpha+t_\alpha+p_1=t_\alpha+p_1$. Therefore (i) is hold.

We need only verify that (ii) condition For any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$
we have,

$$[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1+[t_\alpha t_\alpha t_\alpha]$$
$$=[t_\alpha t_\alpha(t_\alpha+s_\alpha+t_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha t_\alpha]+p_1.$$

Then

$$[t_\alpha t_\alpha s_\alpha]+([t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1)$$
$$=[t_\alpha t_\alpha s_\alpha]+[t_\alpha t_\alpha t_\alpha]+p_1$$
$$[t_\alpha t_\alpha s_\alpha]+[t_\alpha t_\alpha t_\alpha])+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha(s_\alpha+t_\alpha)]+p_1$$
$$[t_\alpha t_\alpha(s_\alpha+t_\alpha)]+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1$$
$$=[t_\alpha t_\alpha(s_\alpha+t_\alpha)]+p_1.$$

$$t_\alpha+s_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=s_\alpha+t_\alpha+p_1. \quad (3.3)$$

Since $[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1+[t_\alpha t_\alpha t_\alpha]=[t_\alpha t_\alpha t_\alpha]+p_1.$
We get

$$(t_\alpha+s_\alpha)+p_1+[t_\alpha t_\alpha t_\alpha]=t_\alpha+s_\alpha+p_1+t_\alpha$$
$$=t_\alpha+s_\alpha+t_\alpha+p_1$$
$$=t_\alpha+p_1$$

$$[t_\alpha+s_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]]+p_1=t_\alpha+s_\alpha+[t_\alpha t_\alpha t_\alpha]$$
$$+[t_\alpha t_\alpha s_\alpha]+p_1$$
$$=[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha s_\alpha]+p_1$$
$$=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1.$$

$$t_\alpha+s_\alpha+[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1 \quad (3.4)$$

From (3.3) and (3.4),we get

$$[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=t_\alpha+s_\alpha+p_1.$$

Similar Proof of the converse part.

As well as For any $t_\alpha \in T$ there exists $s_\alpha \in T$ and $p_1 \in P$
we have,

$$[t_\alpha t_\alpha t_\alpha]+[t_\alpha(t_\alpha+s_\alpha)t_\alpha]+p_1=[t_\alpha(t_\alpha+s_\alpha)t_\alpha]$$
$$+p_1+[t_\alpha t_\alpha t_\alpha]$$
$$=[t_\alpha t_\alpha t_\alpha]+p_1$$

Then

$$[t_\alpha s_\alpha t_\alpha] + ([t_\alpha t_\alpha t_\alpha] + [t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1)$$
$$= [t_\alpha s_\alpha t_\alpha] + [t_\alpha t_\alpha t_\alpha] + p_1.$$

It exhibit,

$$([t_\alpha s_\alpha t_\alpha] + [t_\alpha t_\alpha t_\alpha]) + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1$$
$$= [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1.$$

Now

$$s_\alpha + t_\alpha + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = t_\alpha + s_\alpha + p_1. \quad (3.5)$$

Again ,

$$s_\alpha + t_\alpha + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = s_\alpha + t_\alpha + [t_\alpha s_\alpha t_\alpha] + [t_\alpha t_\alpha t_\alpha] + p_1 = [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1$$

$$s_\alpha + t_\alpha + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 \quad (3.6)$$

From (3.3) and (3.4) , we get

$$[t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = t_\alpha + s_\alpha + p_1 \quad (3.7)$$

Similar Proof of the converse part. In similar way , we get

$$[(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1 = t_\alpha + s_\alpha + p_1 \quad (3.8)$$

Using (3.5) we have , adding t_α on both sides,

$$t_\alpha + [(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1 = t_\alpha + s_\alpha + t_\alpha + p_1 = t_\alpha + p_1$$

Similar way , we get

$$t_\alpha + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = t_\alpha + p_1$$

and

$$t_\alpha + [t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1 = t_\alpha + p_1.$$

Hence

$$[t_\alpha (t_\alpha + s_\alpha)] + p_1 = [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = [(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1.$$

□

Theorem 3.3. *The following statements are equivalent for any element $t_\alpha \in T$.*
(i) t_α is completely P-regular.
(ii)There exist a unique elementw $\in V^+(t)$ such that $[t_\alpha t_\alpha (t_\alpha + w)] + p_1 = t_\alpha + w + p_1$, $t_\alpha + [(t_\alpha + w) t_\alpha t_\alpha] + p_1 = t_\alpha + p_1$,
$[t_\alpha t_\alpha (t_\alpha + w)] + p_1 = t_\alpha + p_1$, $[t_\alpha (t_\alpha + w) t_\alpha] + t_\alpha + p_1 = t_\alpha + p_1$ *and* $[t_\alpha t_\alpha (t_\alpha + w)] + p_1 = [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = [(t_\alpha + w) t_\alpha t_\alpha] + p_1 = t_\alpha + p_1$.
(iii)There exists an unique element $w \in V^+(t_\alpha)$ such that $[t_\alpha t_\alpha (t_\alpha + w)] + p_1 = t_\alpha + w + p_1$.
(iv)$H^+_{(t_\alpha)}$ is a ternary subring of T, where $H^+_{(t_\alpha)}$is the H-class on $(T, +)$ containing $t_\alpha \in T$.

Proof. Let $t_\alpha \in T$ be completely p-regular .There exists an element $s_\alpha \in T$ satisfying the following conditions:
$t_\alpha + s_\alpha + t_\alpha + p_1 = t_\alpha + p_1$
$[t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1 = t_\alpha + s_\alpha + p_1$
$[t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = t_\alpha + s_\alpha + p_1$
$[(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1 = t_\alpha + s_\alpha + p_1$
$t_\alpha + [(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1 = t_\alpha + p_1$
$t_\alpha + [t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1 = t_\alpha + p_1$
$t_\alpha + [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = t_\alpha + p_1$
$[t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1 = [t_\alpha (t_\alpha + s_\alpha) t_\alpha] + p_1 = [(t_\alpha + s_\alpha) t_\alpha t_\alpha] + p_1$
Let $w = s_\alpha + t_\alpha + s_\alpha + p_1$
Hence

$$t_\alpha + w + t_\alpha = t_\alpha + ((s_\alpha + t_\alpha + s_\alpha) + p_1) + t_\alpha = (t_\alpha + s_\alpha + t_\alpha) + p_1 = t_\alpha + p_1.$$

And

$$w + t_\alpha + w = ((s_\alpha + t_\alpha + s_\alpha) + p_1 + t_\alpha + ((s_\alpha + t_\alpha + s_\alpha) + p_1) = (s_\alpha + t_\alpha + s_\alpha) + 2p_1 = s_\alpha + t_\alpha + s_\alpha + p_1 = w.$$

Therefore w is inverse of t_α.Hence $w \in V^+(t)$.
Therefore $t_\alpha + w + t_\alpha = t_\alpha$
and $[t_\alpha t_\alpha (t_\alpha + s_\alpha)] + p_1 = [t_\alpha t_\alpha (t_\alpha + w + p_1)] = t_\alpha + w + p_1$.
And

$$[t_\alpha t_\alpha (t_\alpha + w + p_1)] + t_\alpha = (t_\alpha + w + p_1) + t_\alpha = t_\alpha + w + t_\alpha + p_1 = t_\alpha + p_1.$$

Further

$$[(t_\alpha + w + p_1) t_\alpha t_\alpha] = [(t_\alpha + (s_\alpha + t_\alpha + s_\alpha) + p_1) t_\alpha t_\alpha] = (t_\alpha + s_\alpha + t_\alpha) + t_\alpha + p_1) t_\alpha t_\alpha = t_\alpha + s_\alpha + p_1 t_\alpha t_\alpha$$

$$t_\alpha + s_\alpha + p_1 = (t_\alpha + s_\alpha + t_\alpha) + s_\alpha + p_1 = t_\alpha + (s_\alpha + t_\alpha + s_\alpha) + p_1 = t_\alpha + w$$

$$[t_\alpha (t_\alpha + w + p_1) t_\alpha] = [t_\alpha (t_\alpha + s_\alpha + t_\alpha + s_\alpha + p_1 + p_1) t_\alpha] = [t_\alpha (t_\alpha + w + p_1) t_\alpha] = t_\alpha + w + p_1$$

$$[(t_\alpha+w+p_1)t_\alpha t_\alpha]+t_\alpha=t_\alpha+p_1$$

and $[t_\alpha(t_\alpha+w+p_1)t_\alpha]+t_\alpha=t_\alpha+p_1$.

Hence $[t_\alpha t_\alpha(t_\alpha+w+p_1)]=[t_\alpha(t_\alpha+w+t_\alpha)t_\alpha]=[(t_\alpha+w+p_1)t_\alpha t_\alpha]$ and

$$\begin{aligned}t_\alpha+[t_\alpha t_\alpha(t_\alpha+w+p_1)]&=t_\alpha+[t_\alpha t_\alpha(t_\alpha+w+p_1)\\ ]&=t_\alpha+w+t_\alpha+p_1\\ &=2t_\alpha+w+p_1.\end{aligned}$$

Uniqueness:
Let $x\in V^+(t)$ be another element satisfying the conditions, Hence

$$\begin{aligned}w&=w+t_\alpha+w\\ &=2w+t_\alpha\\ &=2w+t_\alpha+x+t_\alpha\\ &=2w+2t_\alpha+x\\ &=2w+3t_\alpha+2x\\ &=2t_\alpha+w+t_\alpha+w+2x\\ &=2t_\alpha+w+2x\\ &=t_\alpha+2x\\ &=x.\end{aligned}$$

Therefore $w=x$.
Thus $(i)\Rightarrow(ii)$ and $(ii)\Rightarrow(iii)$ is obviously true. Let us prove that $(iii)\Rightarrow(iv)$, Assume that there exist an unique element $w\in V^+(t_\alpha)$ such that $[t_\alpha(t_\alpha+w+t_\alpha)]=t_\alpha+w+t_\alpha$.
To prove $H^+_{(t_\alpha)}$ is a ternary subring of T, where $H^+_{(t_\alpha)}$ is the H-class on (T,+) containing $t_\alpha\in T$.
We have $[t_\alpha t_\alpha(t_\alpha+w+p_1)]=t_\alpha+w+p_1$.
Adding t_α on bothsides , Weget
$[t_\alpha t_\alpha(t_\alpha+w+p_1)]+t_\alpha=t_\alpha+w+p_1+t_\alpha$.

$$\begin{aligned}t_\alpha+p_1&=t_\alpha+w+p_1+t_\alpha\\ &=t_\alpha+(w+t_\alpha+p_1)\\ &=(t_\alpha+w+p_1)+t_\alpha.\end{aligned}$$

Therefore $t_\alpha H^+(t_\alpha+w+p_1)$.Hence $H^+_{(t_\alpha)}$ contains an additive idempotent element $t_\alpha+w+p_1(=w+t_\alpha+p_1)$.Therefore $H^+_{(t_\alpha)}$ is a group. Now

$$\begin{aligned}t_\alpha+p_1&=(t_\alpha+w+p_1)+t_\alpha\\ &=[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(w+p_1)]+t_\alpha\\ &=[t_\alpha t_\alpha(t_\alpha+w+p_1)]+t_\alpha\\ &=[t_\alpha t_\alpha t_\alpha]+[t_\alpha t_\alpha(w+p_1)+t_\alpha]\end{aligned}$$

Also

$$\begin{aligned}[t_\alpha t_\alpha t_\alpha]&=[t_\alpha(t_\alpha+w+t_\alpha)]\\ &=[t_\alpha(t_\alpha+w)]+[t_\alpha t_\alpha t_\alpha]\\ &=[t_\alpha+(w+[t_\alpha t_\alpha t_\alpha])].\end{aligned}$$

This implies that $t_\alpha R^+t^3_\alpha$. Similarly $t_\alpha L^+t^3_\alpha$.Therefore $t_\alpha H^+t^3_\alpha$. Let $m,n,o\in H^+_{(t_\alpha)}$. Therefore $m,n,o\in L^+_{(t_\alpha)}$ and $m,n,o\in R^+_{(t_\alpha)}$. Hence , there exists $a,b,c,d,g,f\in T$ such that $t_\alpha=a+m,m=d+t_\alpha,t_\alpha=b+n,n=g+t_\alpha,t_\alpha=c+o$, $o=f+t_\alpha$.
Now

$$\begin{aligned}[mno]&=[(d+t_\alpha)(g+t_\alpha)(f+t_\alpha)]\\ &=[(d+t_\alpha)(g+t_\alpha)f]+[(d+t_\alpha)(g+t_\alpha)t_\alpha]\\ &=[dgf]+[dt_\alpha f]+[t_\alpha gf]+[t_\alpha t_\alpha f]+[dgt_\alpha]\\ &\quad+[dt_\alpha t_\alpha]+[t_\alpha gt_\alpha]+[t_\alpha t_\alpha t_\alpha]\end{aligned}$$

Also

$$\begin{aligned}[t_\alpha t_\alpha t_\alpha]&=[(a+m)(b+n)(c+o)]\\ &=[(a+m)(b+n)c]+[(a+m)(b+n)o]\\ &=[abc]+[anc]+[mbc]+[mnc]+[abo]\\ &\quad+[ano]+[mbo]+[mno].\end{aligned}$$

Therefore,$[mno]L^+\in[t_\alpha t_\alpha t_\alpha]\Rightarrow[mno]\in L^+_{[t_\alpha t_\alpha t_\alpha]}=L^+_{(t^3_\alpha)}=L^+_{(t_\alpha)}$. similarly $[mno]\in R^+_{[t_\alpha t_\alpha t_\alpha]}=R^+_{(t^3_\alpha)}=R^+_{(t_\alpha)}$.
Hence $[mno]\in H^+_{([t_\alpha t_\alpha t_\alpha])}=H_(t^3_\alpha)^+=H_(t_\alpha)^+$.
Therefore $(H^+_{(t_\alpha)},+,\bullet)$ is a ternary semigroup.
Hence $(H^+_{(t_\alpha)},+,\bullet)$ is a ternary ring.
Let us prove that $(iv)\Rightarrow(i)$. Let $(H^+_{(t_\alpha)},+,\bullet)$ is a ternary subring of T. Every ternary ring has a ternary subring of T. Every element of a ternary ring is being completely P-regular. Hence $t_\alpha\in T$ is a completely P-regular . □

Corollary 3.4. *If T is a completely P-regular ternary semiring, and* $[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=t_\alpha+s_\alpha+p_1$ *where* $p_1\in P$ *is an arbitrary multiplication identity then* $t_\alpha=s_\alpha=p_1(=e)$.

Proof. Let $[t_\alpha t_\alpha(t_\alpha+s_\alpha)]+p_1=t_\alpha+s_\alpha+p_1$.
Take $t_\alpha=s_\alpha=p_1$
Now $[t_\alpha t_\alpha(t_\alpha+t_\alpha)]+t_\alpha=t_\alpha+t_\alpha+t_\alpha$
$\Rightarrow[t_\alpha t_\alpha t_\alpha]+t_\alpha=t_\alpha\Rightarrow t_\alpha\subseteq t_\alpha$.
Hence $t_\alpha\in T$ is an completely P-regular. □

4. Conclusion

Here,we defined completely P-regular ternary semiring and discussed some of the theorems.Throughout the paper,we only discussed about the completely P-regular with arbitrary ideal P. In further research we will develop some ideals other than arbitrary ideal P.

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