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Total edge domination polynomials of Tadpole $T_{3,n}$

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Abstract

Tadpole graph $T_{3,n}$ is a special graph obtained by appending a cycle C_3 to a path P_n. In this paper we present the Total edge domination polynomial of $T_{3,n}$, $D_{te}(T_{3,n},x) = \sum_{i=\gamma_t'(T_{3,n})}^{n+3} d_{te}(T_{3,n},i)x^i$. Also we derive some properties of Total Edge domination polynomials of $T_{3,n}$.

Keywords

Tadpole $T_{3,n}$, total edge domination number, total edge domination polynomial.

AMS Subject Classification

05C70.

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1. Introduction

The graph $G(V,E)$ be a finite simple, non-trivial, undirected graph with vertex set $V(G)$ and edge set $E(G)$ [1,2]. A subset S of $E(G)$ is an edge dominating set of G if every edge not in S is adjacent to some edge in S [3]. An edge dominating set S of G is called total edge dominating set of G, if $\langle S \rangle$ has no isolated edges [6]. The total edge domination number of G is the minimum cardinality taken over all total edge dominating sets of G. The the total edge domination polynomial of G is $D_{te}(G,x) = \sum_{i=\gamma_t'(G)}^{n} d_{te}(G,i)x^i$ [1,4].

2. Definitions and results

Definition 2.1 (5). *A tadpole $T_{3,n}, n \geq 1$ be the graph obtained by joining a path P_n to a cycle C_3.*

Example 2.2. *Consider the graph $T_{3,3}$ given in Figure 1. Total Edge dominating sets of cardinality 2 is $\{\}$. Total Edge dominating sets of cardinality 3 is $\{e_2,e_3,e_4\}$,v$\{e_2,e_3,e_6\}$. Total Edge dominating sets of cardinality 4 is $\{e_1,e_2,e_3,e_4\}$, $\{e_1,e_2,e_3,e_6\}$, $\{e_1,e_2,e_4,e_5\}$, $\{e_1,e_2,e_4,e_6\}$, $\{e_1,e_2,e_5,e_6\}$, $\{e_2,e_3,e_4,e_5\}$, $\{e_2,e_3,e_4,e_6\}$, $\{e_2,e_3,e_5,e_6\}$. Total Edge dominating sets of cardinality 5 is $\{e_1,e_2,e_3,e_4,e_5\}$, $\{e_1,e_2,e_3,e_4,e_6\}$, $\{e_1,e_2,e_3,e_5,e_6\}$, $\{e_1,e_2,e_4,e_5,e_6\}$, $\{e_2,e_3,e_4,e_5,e_6\}$. Total Edge dominating sets of cardinality 6 is $\{e_1,e_2,e_3,e_4,e_5,e_6\}$. Total Edge domination polynomial for the graph $T_{3,3}$ is given by $D_{te}(T_{3,3},x) = x^6+5x^5+8x^4+2x^3$.*

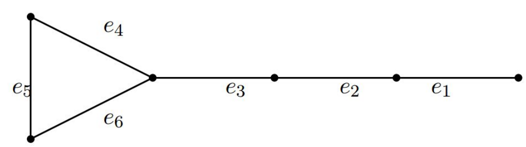

Figure 1. $T_{3,3}$

Lemma 2.3. *Let $T_{3,n}, n \geq 1$ be the tadpole with $|E(T_{3,n})| = n+3$. then $D_{te}(T_{3,n},i) = \phi$ if and only if $i > n+3$ or $i < \lfloor \frac{n+3}{2} \rfloor$.*

Lemma 2.4. *For $n \geq 1$ the total edge domination number of the tadpole, $T_{3,n}$ is given by*

$$\gamma_t'(T_{3,n}) = \begin{cases} \frac{n+3}{2}, & n \text{ is odd} \\ \left\lceil \frac{n+3}{2} \right\rceil, & n = 4k, k \geq 1 \\ \left\lfloor \frac{n+3}{2} \right\rfloor, & n = 4k-2, k \geq 1 \end{cases}$$

Lemma 2.5. *Let $T_{3,n}$, $n \geq 1$ be the tadpole with $|E(T_{3,n})| = n+3$*

(i) If $D_{te}(T_{3,n-1},i-1) = D_{te}(T_{3,n-2},i-1) = \phi$ then D_{te} $(T_{3,n},i) = \phi$ for every $n = 4k-1$.

(ii) If $D_{te}(T_{3,n-1},i-1)\neq\phi$ and $D_{te}(T_{3,n-3},i-1)\neq\phi$ then $D_{te}(T_{3,n-2},i-1)\neq\phi$ for every $n\geq 4k$.

(iii) If $D_{te}(T_{3,n-1},i-1)=D_{te}(T_{3,n-3},i-1)=\phi$ then $D_{te}(T_{3,n-2},i-1)=\phi$ for $n=4k+3$.

Proof. (i) Since $D_{te}(T_{3,n-1},i-1)=D_{te}(T_{3,n-2},i-1)=\phi$, by Lemma 2.3, $i-1>n+2$ or $i-1<\lfloor\frac{n+1}{2}\rfloor$. If $i-1>n+2$, then $i>n+3$. If $i-1<\lfloor\frac{n+1}{2}\rfloor$, then $i<\lfloor\frac{n+1}{2}\rfloor+1=\lfloor\frac{n+3}{2}\rfloor$. i.e; $i<\lfloor\frac{n+3}{2}\rfloor$. In either case we have therefore $D_{te}(T_{3,n},i)=\phi$.

(ii) Suppose that $D_{te}(T_{3,n-2},i-1)=\phi$. So, by lemma 2.3 we have $i-1\geq(n+3)-2$ or $i-1<\lfloor\frac{n+1}{2}\rfloor$. If $i-1>n+1$ then $i-1>n$ and hence $D_{te}(T_{3,n-3},i-1)=\phi$, a contradiction. Hence $i-1<\lfloor\frac{n+1}{2}\rfloor$. So $i-1<\lfloor\frac{n+2}{2}\rfloor$ and hence $D_{te}(T_{3,n-1},i-1)=\phi$ which is a contradiction.

(iii) Since $D_{te}(T_{3,n-1},i-1)=D_{te}(T_{3,n-3},i-1)=\phi$, by Lemma 2.3, $i-1>n+2$ or $i-1<\lfloor\frac{n+1}{2}\rfloor$. If $i-1>n+2$, then $i-1>n+1$. If $i-1<\lfloor\frac{n+1}{2}\rfloor$, then $i-1<\lfloor\frac{n-1}{2}\rfloor$. In either case we have $D_{te}(T_{3,n-2},i-1)=\phi$. □

Lemma 2.6. *Let $T_{3,n}, n\geq 1$ be the tadpole with $|E(T_{3,n})|=n+3$. Suppose that $D_{te}(T_{3,n},i)\neq\phi$. Then we have*

(i) $D_{te}(T_{3,n-2},i-1)=\phi$, $D_{te}(T_{3,n-3},i-1)=\phi$ and $D_{te}(T_{3,n-1},i-1)\neq\phi$ if $i=n+3$.

(ii) $D_{te}(T_{3,n-1},i-1)\neq\phi$, $D_{te}(T_{3,n-2},i-1)\neq\phi$ and $D_{te}(T_{3,n-3},i-1)=\phi$ if and only if $i=n+2$.

(iii) $D_{te}(T_{3,n-1},i-1)=\phi$ and $D_{te}(T_{3,n-2},i-1)=\phi$ if and only if $n+3=4k+1$ and $i=2k$ for some positive integer k.

(iv) $D_{te}(T_{3,n-1},i-1)=\phi$, $D_{te}(T_{3,n-2},i-1)\neq\phi$ if and only if $n+3=4k$ and $i=2k$ for some integer k.

(v) $D_{te}(T_{3,n-1},i-1)\neq\phi$, $D_{te}(T_{3,n-2},i-1)\neq\phi$ and $D_{te}(T_{3,n-3},i-1)\neq\phi$ if and only if $\lfloor\frac{n+3}{2}\rfloor+1\leq i<n+3$.

Proof. (i) Since $D_{te}(T_{3,n-2},i-1)=D_{te}(T_{3,n-3},i-1)=\phi$, by Lemma 2.3, $i-1>(n+3)-2$ or $i-1<\left\lfloor\frac{(n+3)-2}{2}\right\rfloor$.If $i-1<\left\lfloor\frac{(n+3)-2}{2}\right\rfloor$, then $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$. By Lemma 2.3, $D_{te}(T_{3,n-1},i-1)=\phi$ which is a contradiction. So we have $i-1>(n+3)-2$. That is $i-1\geq(n+3)-1$. Therefore $i\geq(n+3)$. Since $D_{te}(T_{3,n-1},i-1)\neq\phi$, then $\left\lfloor\frac{(n+3)-1}{2}\right\rfloor<i-1\leq(n+3)-1$. Therefore, $i\geq(n+3)$ and $i\leq(n+3)$. Hence $i=(n+3)$.

(ii) Since $D_{te}(T_{3,n-3},i-1)=\phi$, then by Lemma 2.3, $i-1>(n+3)-3$ or $i-1<\left\lfloor\frac{(n+3)-3}{2}\right\rfloor$. Since $D_{te}(T_{3,n-2},i-1)\neq\phi$, $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor<i-1\leq(n+3)-2$. That is $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor<i-1$. Therefore, $\left\lfloor\frac{(n+3)-3}{2}\right\rfloor<\left\lfloor\frac{(n+3)-2}{2}\right\rfloor<i-1$. Hence $\left\lfloor\frac{(n+3)-3}{2}\right\rfloor<i-1$, which is a contradiction to $D_{te}(T_{3,n-3},i-1)=\phi$. So, $i-1>(n+3)-3$. That is, $i\geq(n+3)-1$. Since $D_{te}(T_{3,n-2},i-1)\neq\phi$, $i-1<(n+3)-2$. Therefore $i<(n+3)-1$. Hence $i=(n+3)-1$. That is $i=n+2$. Conversely, if $i=(n+3)-1$, then $D_{te}(T_{3,n-1},i-1)=D_{te}(T_{3,n-1},(n+3)-2)\neq\phi$, $D_{te}(T_{3,n-2},i-1)=D_{te}(T_{3,n-2},(n+3)-2)\neq\phi$. But $D_{te}(T_{3,n-3},i-1)=D_{te}(T_{3,n-3},(n+3)-2)=\phi$, since by lemma 2.3.

(iii) Since $D_{te}(T_{3,n-1},i-1)=\phi$ and $D_{te}(T_{3,n-2},i-1)=\phi$ by lemma $i-1>(n+3)-1$ or $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$ and $i-1\geq(n+3)-2$ or $i-1<\left\lfloor\frac{(n+3)-2}{2}\right\rfloor$. If $i-1<(n+3)-1$, then $i>n+3$. Therefore, $D_{te}(T_{3,n},i)=\phi$, which is a contradiction. If $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$, then $i<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor+1$. If $n+3\equiv 3\,(mod4)$, then by the definition of total edge domination number we have $\frac{(n+3)+1}{2}\leq i\leq\frac{n+3}{2}$, which is not possible. So when $n+3\equiv 3\,(mod4)$, then $\lfloor\frac{n+3}{2}\rfloor\leq i\leq i\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$. If $n\not\equiv 4k+1$, then the inequality of the form which is not possible. Therefore $n+3=4k+1$ is the only possibility. So, the above inequality becomes $\lfloor\frac{4k}{2}\rfloor\leq i\leq\lfloor\frac{4k-1}{2}\rfloor$. i.e $2k\leq i\leq\lfloor\frac{4k-1}{2}\rfloor$. Hence we obtain that $i=2k$.

(iv) Since $D_{te}(T_{3,n-1},i-1)=\phi$ by lemma $i-1>(n+3)-1$ or $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$. If $i-1>(n+3)-1$, then $i-1\geq(n+3)-2$. Therefore $D_{te}(T_{3,n-2},i-1)=\phi$ and $D_{te}(T_{3,n-3},i-1)=\phi$ which is a contradiction. If $n+3\equiv 3\,(mod4)$ then $n+2\equiv 3\,(mod4)$, $n+1\equiv 3\,(mod4)$ and $n\equiv 3\,(mod4)$. Therefore $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$, $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor\leq i-1<n+1$, $\lfloor\frac{n}{2}\rfloor\leq i-1\leq n$. i.e $i<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$, $1+\left\lfloor\frac{(n+3)-2}{2}\right\rfloor\leq i$, $1+\lfloor\frac{n}{2}\rfloor\leq 1$. Since $D_{te}(T_{3,n},i)\neq\phi$, then $i>1+\frac{(n+3)}{2}$. Hence $1+\frac{(n+3)}{2}\leq i\leq\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$ which is not possible. Hence $n+3\equiv 3\,(mod4)$. So here we have three possible cases. If $n+3\equiv 1\,(mod4)$, then $n+2\equiv 0\,(mod4)$, $n+1\equiv 3\,(mod4)$ and $n\equiv 2\,(mod4)$. Since $D_{te}(T_{3,n-1},i-1)=\phi$, then $i-1<\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$. Therefore, $i-1\leq\left\lfloor\frac{(n+3)-1}{2}\right\rfloor$. Since $D_{te}(T_{3,n-2},i-1)=\phi$, $D_{te}(T_{3,n-3},i-1)=\phi$, $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor\leq i-1\leq(n+3)-2$ and $1+\frac{n}{2}\leq i-1\leq n$. Therefore $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor+1\leq i$ and $2+\frac{n}{2}\leq i$. So $\lfloor\frac{n+1}{2}\rfloor+1\leq i\leq\lfloor\frac{n+2}{2}\rfloor$ and $2+\frac{n}{2}\leq i\leq\lfloor\frac{n+2}{2}\rfloor$ which are not possible.

(iv) Since $D_{te}(T_{3,n-1},i-1)\neq\phi$, $D_{te}(T_{3,n-2},i-1)\neq\phi$ and $D_{te}(T_{3,n-3},i-1)=\phi$, then we have $\left\lfloor\frac{(n+3)-1}{2}\right\rfloor\leq i-1\leq(n+3)-1$, $\left\lfloor\frac{(n+3)-2}{2}\right\rfloor\leq i-1\leq(n+3)-2$, and $\lfloor\frac{n}{2}\rfloor\leq i-1\leq n$. Therefore, $\left\lfloor\frac{(n+3)-1}{2}\right\rfloor\leq i-1\leq n$.

Table 1. $d_{te}(T_{3,n},i)$:The number of total edge dominating sets of $T_{3,n}$ with cardinality i

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$T_{3,1}$	0	5	4	1																
$T_{3,2}$	0	2	5	4	1															
$T_{3,3}$	0	0	2	8	5	1														
$T_{3,4}$	0	0	0	10	13	6	1													
$T_{3,5}$	0	0	0	7	19	18	7	1												
$T_{3,6}$	0	0	0	2	14	31	24	8	1											
$T_{3,7}$	0	0	0	0	4	32	49	31	9	1										
$T_{3,8}$	0	0	0	0	0	21	64	73	39	10	1									
$T_{3,9}$	0	0	0	0	0	9	54	113	104	48	11	1								
$T_{3,10}$	0	0	0	0	0	2	27	117	186	143	58	12	1							
$T_{3,11}$	0	0	0	0	0	0	4	80	230	290	191	69	13	1						
$T_{3,12}$	0	0	0	0	0	0	0	34	198	416	433	249	81	14	1					
$T_{3,13}$	0	0	0	0	0	0	0	11	115	428	706	624	318	94	15	1				
$T_{3,14}$	0	0	0	0	0	0	0	2	42	312	844	1139	873	399	108	16	1			
$T_{3,15}$	0	0	0	0	0	0	0	0	4	156	740	1550	1763	1191	493	123	17	1		
$T_{3,16}$	0	0	0	0	0	0	0	0	0	49	469	1584	2689	2636	1590	601	139	18	1	
$T_{3,17}$	0	0	0	0	0	0	0	0	0	13	206	1209	3134	4452	3827	2083	724	156	19	1

Hence $\left\lfloor \frac{(n+3)-1}{2} \right\rfloor \leq i-1 \leq (n+3)-2$. Converse part is obvious.

□

Theorem 2.7. *If $D_{te}(T_{3,n},i)$ is the family of the edge dominating set of $T_{3,n}$ with cardinality $i \geq \left\lfloor \frac{n+8}{2} \right\rfloor$, then $d_{te}(T_{3,n},i) = d_{te}(T_{3,n-1},i-1) + d_{te}(T_{3,n-2},i-1)$.*

Proof. Here, we consider all the three cases as given below, where $i > \left\lceil \frac{n+3}{2} \right\rceil + 1$.
Case 1. If $D_{te}(T_{3,n-1},i-1) = D_{te}(T_{3,n-2},i-1) = \phi$, then $D_{te}(T_{3,n},i) = \phi$. Therefore $|D_{te}(T_{3,n-1},i-1)| + |D_{te}(T_{3,n-2}, i-1)| = 0$ and $|D_{te}(T_{3,n-1},i)| = 0$. Hence $d_{te}(T_{3,n},i) = d_{te}(T_{3,n-1},i-1) + d_{te}(T_{3,n-2},i-1)$.
Case 2. If $D_{te}(T_{3,n-1},i-1) = \phi$ and $D_{te}(T_{3,n-2},i-1) = \phi$, then $D_{te}(T_{3,n},i) = \{\{n+3\} \cup X / X \in D_{te}(T_{3,n-1},i-1)\}$. Therefore $|D_{te}(T_{3,n-1},i-1)| = |D_{te}(T_{3,n},i)|$ and $|D_{te}(T_{3,n-2},i-1)| = 0$. Hence $d_{te}(T_{3,n},i) = d_{te}(T_{3,n-1},i-1) + d_{te}(T_{3,n-2},i-1)$.
Case 3. If $D_{te}(T_{3,n-1},i-1) \neq \phi$ and $D_{te}(T_{3,n-2},i-1) \neq \phi$

$$D_{te}(T_{3,n},i) = \begin{cases} \{X \cup \{n+3\}, \text{ if } n-1 \notin X, n \text{ or } n-2 \in X\} \cup \\ \{X \cup \{n+3\}, \text{ if } n-3 \notin X, 1, n-2 \text{ or } n, \\ n-4 \in X\} \cup X - \{n-1\} \cup \{n-2, n+3\}, \\ \text{ if } n-3, n-1 \in X, n, n-2 \notin X \cup \\ X - \{n+2\} \cup \{n+1, n+3\}, \text{ if } n+1 \notin X, \\ n-2, n+2 \text{ or } n, n+2 \in X \cup \{Y \cup \{n+3\}, \\ \text{ if } 1 \in Y\} \cup \{Y \cup \{n+2\} \text{ if } n+1 \in Y\} \cup \\ \{Y \cup \{n+3\}, \text{ if } 1, n+1 \notin Y\} \cup \\ \{Y \cup \{n+1\}, \text{ if } n, n-1 \in Y, n+1 \notin Y\} \cup \\ \{Y - \{1\}\} \cup \{n+2, n+3\}, \text{ if } 1,2,3,n \in Y, \\ n+1 \notin Y \cup \{Y - \{n-1\}\} \cup \{n-2, n+3\}, \\ \text{ if } n-1, n-3 \in Y, n-2 \notin Y \cup \{Y - \{n\}\} \cup \\ \{n+2, n+3\}, \text{ if } n-4 \notin Y, n-1 \in Y \cup \\ \{Z \cup \{n+3\}\}, \text{ if } n, n-1 \notin Z \text{ and } 1, \\ n+1 \in Z \cup \{Z \cup \{n+2\}\}, \text{ if } n \text{ or } n-1 \notin Z \\ \text{ and } 1,2,n+1 \in Z \cup \{Z \cup \{n+2\}\}, \text{ if} \\ n \notin Z \text{ and } n-2, n-1, n+1 \in Z \cup \{Z - \{1\}\} \cup \\ \{n+2, n+3\}, \text{ if } n \notin Z, n-4, n-2, n-1 \in Z \\ \cup \{Z - \{n-1\}\} \cup \{n, n+1\}, \text{ if } n+1 \notin Z \end{cases}$$

where $X \in D_{te}(T_{3,n-1},i-1) - D_{te}(T_{3,n-2},i-1)$, $Y \in D_{te}(T_{3,n-1},i-1) \cap D_{te}(T_{3,n-2},i-1)$, $Z \in \{D_{te}(T_{3,n-2},i-1) - D_{te}(T_{3,n-1},i-1) \cap D_{te}(T_{3,n-2},i-1)\}$. In this construction, $|D_{te}(T_{3,n},i)|$ is the sum of $|D_{te}(T_{3,n-1},i-1) - D_{te}(T_{3,n-2},i-1)|$, $|D_{te}(T_{3,n-1},i-1) \cap D_{te}(T_{3,n-2},i-1)|$ and $|D_{te}(T_{3,n-2},i-1) - [D_{te}(T_{3,n-1},i-1) \cap D_{te}(T_{3,n-2},i-1)]|$. Hence we have $d_{te}(T_{3,n},i) = d_{te}(T_{3,n-1},i-1) + d_{te}(T_{3,n-2},i-1)$.
From the above construction in each case, we obtain that $d_{te}(T_{3,n},i) = d_{te}(T_{3,n-1},i-1) + d_{te}(T_{3,n-2},i-1)$. □

Using all the above theorems and lemmas, we obtain the coefficients of $D_{te}(T_{3,n},i)$ for $1 \leq n \leq 17$ in Table 1.

Theorem 2.8. *Let $T_{3,n}$, $n \geq 1$ be the tadpole with $|E(T_{3,n})| = n+3$. Then*

(i) $D_{te}(T_{3,4k+2},x) = x\left[D_{te}(T_{3,4k+1},x) + D_{te}(T_{3,4k},x)\right] - x^{2k+1} - 3x^{2k+3} + 2x^{2k+2}$.

(ii) For any $k > 1$, $D_{te}(T_{3,4k+3},x) = x[D_{te}(T_{3,4k+2},x) + D_{te}(T_{3,4k+1},x)] - x^{2k+4} - (2k+3)x^{2k+3}$.

(iii) $D_{te}(T_{3,4k},x) = x\left[D_{te}(T_{3,4k-1},x) + D_{te}(T_{3,4k-2},x)\right] - 2x^{2k+1} - 3x^{2k+2} + x^{2k+3}$.

(iv) For any $k > 1$ $D_{te}(T_{3,4k+1},x) = x[D_{te}(T_{3,4k},x) + D_{te}(T_{3,4k-1},x)] + x^{2k+3} + (2k+1)x^{2k+2}$.

3. Conclusion

In this paper we obtain the total edge domination polynomial of tadpole. Similarly we can find the total edge domination polynomial of any specified graph.

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