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The Mersenne S-prime matrices on A–sets

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Abstract
In this paper we study about some properties of Mersenne S-prime Meet matrices on A -sets. $(S)_f$ are defined to verify a recursive structure theorem and det (S) and $(S_f)^{-1}$ are calculated using the recursive formula on A-sets.

Keywords
a-Set, A-Set, Meet Matrices, Mersenne S-Prime Meet Matrices.

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15A09, 03G10.

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Contents

1. Introduction

The concept of Meet and Join matrices plays an important role in the number theory concept and the theory of generalized inverses. The GCD and LCM matrices are called as the Meet and Join matrices. S is a finite subset of P which is a lattice. If $f: P \rightarrow \mathrm{C}$ is a complex valued function then the Meet matrix is $\left((S)_f\right)_{ij} = f(x_i \wedge x_j)$. Haukkanen [3] derived formula for $\det(S_f)$ and $\left(S_f\right)^{-1}$. To study the Meet matrices Korkee and Haukkanen [5] have used incidence functions in which the new upper and lower bounds for $\det(S_f)$, were obtained. The generalizations of GCD matrices and LCM matrices are the Meet and Join matrices [1]. The Mersenne S-prime matrices are defined and certain results are studied. The results from structure theorem will be used to find $\det(S_f)$ and $\left(S_f\right)^{-1}$.

2. Preliminaries

Definition 2.1. *(a-set) We define the binary operation $\sqcap$ on non empty subsets S_1 and S_2 as $S_1 \sqcap S_2 = \{x \wedge y / x \in S_1, y \in S_2, x \neq y\}$ If $a \in P$, then $S \subset P$ is said to be an a-set if $S \sqcap S = \{a\}$.*

Definition 2.2. *The Meet matrix of size nxn is defined as $\left((S)_f\right)_{ij} = f(x_i \wedge x_j)$ Mersenne S-Prime Meet matrix [2] is considered as $\left((S)_f\right)_{ij} = 2^{4\left(x_i \wedge x_j\right)} - 1$.*

Example 2.3. *Consider $(P, \leq) = (\mathbf{Z}+, |)$ and $S = \{1,2,3\}$ Then*

$$(S)_f = \begin{bmatrix} 2^{4\times1}-1 & 2^{4\times1}-1 & 2^{4\times1}-1 \\ 2^{4\times1}-1 & 2^{4\times2}-1 & 2^{4\times1}-1 \\ 2^{4\times1}-1 & 2^{4\times1}-1 & 2^{4\times3}-1 \end{bmatrix}.$$

S is an A -set, with $A = \{1,1\}$.

3. Results on Mersenne S-prime Meet matrices on A-sets

Result I: I The Mersenne S-prime Meet matrices satisfies

$$(S)_f \cdot \mathrm{Adj}\left((S)_f\right) = \mathrm{Adj}\left((S)_f\right) \cdot (S)_f = \left(\det(S)_f\right).$$

Proof. If the Mersenne S-prime Meet matrix is denoted as M then by the property of the determinant, the $(\mathrm{i}, j)^{\text{th}}$ element of $M \cdot Adj M = f_{i1}M_{1j} + f_{i2}M_{2j} + f_{i3}M_{3j} + \ldots + f_{in}M_{nj}$. Also

$$M \cdot Adj M = \begin{cases} 0 & \text{as } i \neq j \\ \det M & \text{as } i = j, \end{cases}$$

$\therefore M(\text{ Adj } M) = \mathrm{diag}[\det(M), \det(M), \ldots, \det(M)]$. Similarly the other result can also be proved. Hence $(S)_f$. Adj $((S)_j) =$ Adj $((S)_j) \cdot (S)_f = \left(\det(S)_f\right)$. I is proved. □

Example 3.1. *For the given S, We get Det* $(S)_f = 1,46,88,000$. *Also*

$$\mathrm{Adj}(S)_f = \begin{bmatrix} 1044000 & -61200 & -3600 \\ -61200 & 61200 & 0 \\ -3600 & 0 & 3600 \end{bmatrix}$$

It satisfies the given property.

Result II: Every Mersenne S-Prime Meet matrix $(S)_f$ is invertible then $\left(\left(S_f\right)^{-1}\right)^{-1} = (S)_f$.

Proof. By the definition of inverse of a matrix, $\left(S_f\right)^{-1}$ is invertible and the inverse of $\left(S_f\right)^{-1}$ is S_f. □

Result III: If the Mersenne S-Prime Meet matrix $(S)_f$ is non singular then so $\left(S_f\right)^T$ and also $\left(\left(S_f\right)^T\right)^{-1} = \left(\left(S_f\right)^{-1}\right)^T$.

Proof. As $(S)_f$ is non singular matrix $\det(S)_f \neq 0$ and $\left(S_f\right)^{-1}$ exists. Also $\det(S)_f \neq 0 \Rightarrow \det\left(S_f\right)^T \neq 0$ and $\left(S_f\right)^T$ is non singular and $\left(\left(S_f\right)^T\right)^{-1}$ exists. Hence the proof. □

Theorem 3.2. *Mersenne S-prime Meet matrices on A-sets satisfies structure theorem [4]. If S is an A-set ,then* $(S) = M^T DM$ *where the functions are* $f_1 = f$ *and*

$$f_{k+1}(x) = f_k(x)\frac{f_k(a_k)^2}{f_k(x_k)},$$

for $\mathrm{k} = 1,2,\ldots\ldots\ldots,\mathrm{n}-1$.

Proof. Using the given definition of f_i we obtain $f_2(x) = f_1(x) - \frac{f_1(a_1)^2}{f(x_1)} = f(x)$, $f_3(x) = f_2(x) - \frac{f_2(a_2)^2}{f_2(x_2)} = f_2(x)$ and finally $f_n(x) = f_{n-l}(x) - \frac{f_{n-1}(a_{n-1})^2}{f_{n-1}(x_{n-1})} = f_{n-1}(x)$. By combining all these results $D = \mathrm{diag}\left(f_1(x_1), f_2(x_2), \ldots\ldots\ldots f_n(x_n)\right)$ becomes $D = \mathrm{diag}\{f(x), f(x), \ldots\ldots., f(x)(\text{n times }), (M)_{ij} = \frac{f_i(a_i)}{f_i(x_i)}$ for all $i < j$

$$\begin{array}{lllll} M_{11}=1 & M_{12}=0 & M_{13}=0 & \cdots & M_{1n}=0 \\ M_{21}=0 & M_{22}=1 & M_{23}=0 & \cdots & M_{2n}=0 \\ M_{31}=0 & M_{32}=0 & M_{33}=1 & \cdots & M_{3n}=0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ M_{n1}=0 & M_{n2}=0 & M_{n3}=0 & \cdots & M_{nn}=1 \end{array}$$

Then

$$\begin{aligned} \left(M^{\mathrm{T}}DM\right)_{ij} =& f_i(a_i) + \left(f_I(a_i) - f_2(a_i)\right) \\ &+ \left(f_2(a_i) - f_3(a_i)\right) + \left(f_3(a_i) - {}_4(a_i)\right) \\ &+ \cdots + \left(f_{i-1}(a_i) - f_i(a_i)\right) \\ =& f_1(a_i) \\ =& f(x_i \wedge x_j) \end{aligned}$$

Therefore $\quad (S)_f = M^T DM.$ □

Theorem 3.3. *S is an A-set and* f_i*'s are functions as defined in the previous theorem then* $\det(S)_f = \prod_{i=1}^n f_i(x_i)$

Proof. If we consider $n = 3$ Then

$$(\mathrm{S})_f = \begin{bmatrix} f(x_1 \wedge x_1) & f(x_1 \wedge x_2) & f(x_1 \wedge x_3) \\ f(x_2 \wedge x_1) & f(x_2 \wedge x_2) & f(x_2 \wedge x_3) \\ f(x_3 \wedge x_1) & f(x_3 \wedge x_2) & f(x_3 \wedge x_3) \end{bmatrix}$$

If $x_i \leq x_j$ for $i \leq j$, then $(\mathrm{S})_f = \begin{bmatrix} x_1 & x_1 & x_1 \\ x_1 & x_2 & x_2 \\ x_1 & x_2 & x_3 \end{bmatrix}$

$$\begin{aligned} \left|(S)_f\right| =& x_1\left(x_2x_3 - x_2^2\right) - x_1\left(x_1x_3 - x_1x_2\right) \\ &+ x_1\left(x_1x_2 - x_1x_2\right) \\ =& x_1x_2x_3 - x_1x_2^2 - x_1^2x_3 + x_1^2x_2 \end{aligned}$$

Hence the proof. □

Example 3.4. *S is considered as earlier then* $f(x) = 2^{4x} - 1$ *gives* $f_1(x) = 2^{4x} - 1$ *and* $f_2(x) = 2^{4x} - 16, f_3(x) = 2^{4x} - 16$ *Also* $(S)_f = \mathrm{M^TDM}$, *where* $\mathrm{D} = \mathrm{diag}(15, 240, 4080)$ *and*

$$\mathrm{M} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

gives $\det(S)_f = f_1(1)f_2(2)f_3(3) = 14,688,000$ *by using the result from recursion formula for determinant evaluation in [4].*

Theorem 3.5. *The inverse of Mersenne S-prime Meet matrix on A-sets is found from* $\left(S_f\right)^{-1} = N\Delta N^{\mathrm{T}}$ *[4].*

Proof. As $(S)_f = \mathrm{M^TDM}$ we get $\left(S_f\right)^{-1} = N\Delta N^{\mathrm{T}}$ where $\mathrm{N} = \mathrm{M}^{-1}$ and $\Delta = \mathrm{diag}\left(\frac{1}{f_1(x_1)}, \frac{1}{f_2(x_2)}, \ldots, \frac{1}{f_n(x_n)}\right)$. □

Example 3.6. *If S is considered the same as earlier then* $\left(S_f\right)^{-1} = N\Delta N^{\mathrm{T}}$ *gives* $\Delta = \mathrm{diag}(1/15, 1/240, 1/4080)$,

$$\mathrm{N} = \mathrm{M}^{-1}, \quad \mathrm{N} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\left(S_f\right)^{-1} = \begin{bmatrix} \frac{29}{408} & \frac{-1}{240} & \frac{-1}{4080} \\ \frac{-1}{240} & \frac{1}{240} & 0 \\ \frac{-1}{4080} & 0 & \frac{1}{4080} \end{bmatrix}$$

4. Conclusion

In this paper, we have discussed certain properties of Mersenne S-prime Meet matrices on A-sets.Thus the Mersenne S-prime Meet matrices on A sets $\left(S_f\right)$ satisfying structure theorem is proved also $\det\left(S_f\right)$ along with $\left(S_f\right)^{-1}$ are found from the outcome of the relations on A-sets. Mersenne S-prime Join matrices on A sets could be considered for future study.

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