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Maximum eccentricity energy of globe graph, bistar graph and some graph related to bistar graph

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Abstract

The graphs considered in this article are undirected, finite and simple graphs. In this article we have proved that maximum eccentricity energy of bistar graph $B_{n,n}$ is $4\sqrt{9n+1}$. Also we have investigated maximum eccentricity energy of some graphs related to bistar graph and globe graph.

Keywords

Maximum Eccentricity Matrix, Maximum Eccentricity Eigen values, Maximum Eccentricity Energy of a Graph, Bistar Graph.

AMS Subject Classification

05C50, 05C76.

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Contents

1. Introduction

The energy of a graph introduced by Gutman[6] is an important concept of spectral graph theory which links Organic Chemistry to linear algebra of Mathematics. Generally graph's energy is summation of absolute values of eigenvalues of the adjacency matrix. Similar energies got from the eigenvalues of various graphs are considered in recent times. Several other authors also investigated energy of graph [1,2,7,10,11,13].

In a graph G, the distance between any two vertices u and v is denoted as $d(u,v)$ and it is define as the length of the minimum path between u and v in graph G, if there is no path between u and v in G then $d(u,v)$ is defined as ∞. It is useful to note that in a connected graph distance between any two vertices is always finite provided graph is finite. For a vertex v of G, the Eccentricity of a vertex v is denoted as $e(v)$ and is defined as $e(v) = \max\{d(u,v) \mid u \in V(G)\}$.
In [7] Naji and Soner introduced the concept of Maximum Eccentricity matrix $M_e(G)$ of connected graph G and it is defined as,

$$e_{ij} = \begin{cases} \max\{e(v_i), e(v_j)\}, & \text{If } v_iv_j \in E(G) \\ 0, & \text{Otherwise} \end{cases}$$

The Maximum Eccentricity eigenvalues are the eigenvalues of matrix $M_e(G)$. In this article we have considered only finite, simple and undirected graphs. To make this article self-contained, it is useful to recall some definitions from graph theory.

Definition 1.1 ([7]). *The Maximum Eccentricity energy of graph G is denoted as $EM_e(G)$ and it is defined by $EM_e(G) = \sum_{i=1}^{n}|\lambda_i|$, where $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots \lambda_n$ are the eigenvalues of $M_e(G)$, Where $M_e(G)$ is the Maximum Eccentricity matrix of graph G.*

Definition 1.2 ([3]). *A graph G is said to be bipartite if the vertex set V of G can be partitioned into two disjoint subsets V_1 and V_2 such that $V_1 \cup V_2 = V$ and for each edge has one end vertex is in V_1 and other is in V_2.*

Definition 1.3 ([3]). *A complete bipartite graph is a bipartite graph in which all the vertices of V_1 are adjacent with all the vertices of V_2. If $|V_1| = m$ and $|V_2| = n$ respectively then the corresponding complete bipartite graph is denoted as $K_{m,n}$*

Definition 1.4 ([3]). *A complete bipartite graph $K_{1,n}$ is known as star graph and the vertex of degree n is known as the apex vertex.*

Definition 1.5 ([5]). *A Bistar graph is the graph obtained by joining the centre (apex) vertices of two copies of $K_{1,n}$ by an edge and it is denoted by $B_{n,n}$. The vertex set of $B_{n,n}$ is $V(B_{n,n}) = \{v_1, v_2, \ldots\ldots, v_n, v, u, u_1, u_2, \ldots\ldots u_n\}$, where v, u are apex vertices and $v_1, v_2, \ldots\ldots, v_n, u_1, u_2, \ldots\ldots u_n$ are pendent vertices. The edge set of $B_{n,n}$ is $E(B_{n,n}) = \{vv_1, vv_2, \ldots \ldots, vv_n, vu, uu_1, uu_2, \ldots\ldots uu_n\}$.*

Definition 1.6 ([11]). *Let G be a simple connected graph. Then the square graph of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 in G.*

Definition 1.7 ([5]). *A shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' ad G" and join each vertex u in to G' the neighbours of the corresponding vertex v' in G''.*

Definition 1.8 ([9]). *A globe graph Gl(n) is a graph obtained from two isolated vertex are joined by n paths of length two.*

In theorem 2.1, we have shown that Maximum Eccentricity energy of Bistar graph $B_{n,n}$ is $4\sqrt{9n+1}$. for $n \in N, n \neq 1$. We also provided supportive example in Example 2.2 and in that example we have prove that $M_e(B_{5,5}) = 4(\sqrt{46})$. We also investigate the Maximum Eccentricity of square of Bistar graph in Theorem 2.3 and we have shown that its Maximum Eccentricity energy is $1+\sqrt{64n+1}$. In Theorem 2.5 we proved that Maximum Eccentricity energy of shadow graph of Bistar graph $D_2(B_{n,n})$ is $8\sqrt{9n+1}$ and in Theorem 2.7 we proved that the Maximum Eccentricity energy of Globe graph is $4\sqrt{2n}$.

2. Main Results

Theorem 2.1. *Let $n \in N, n \neq 1$. Then $EM_e(B_{n,n}) = 4\sqrt{9n+1}$, where $EM_e(B_{n,n})$ is the Maximum Eccentricity energy of graph $B_{n,n}$.*

Proof. Let $V(B_{n,n}) = \{v_1, v_2, \ldots\ldots, v_n, v, u, u_1, u_2, \ldots\ldots u_n\}$. Note that $B_{n,n}$ is graph with $2n+2$ vertices and $2n+1$ edges as shown in the folowing Figure 1. Observe that the Maximum

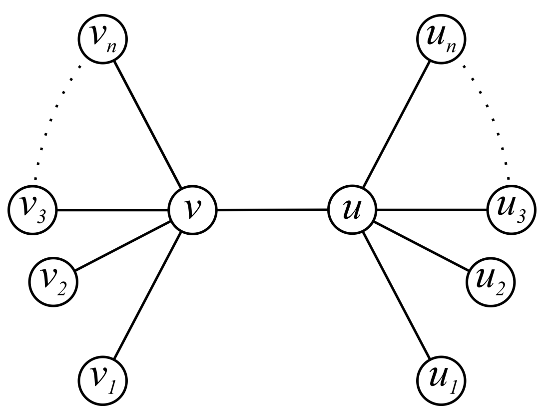

Figure 1

Eccentricity matrix $M_e(B_{n,n})$ of $B_{n,n}$ is given by

$$M_e(B_{n,n}) = \begin{array}{c} \\ v_1 \\ \vdots \\ v_n \\ v \\ u \\ u_1 \\ \vdots \\ u_n \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & \cdots & v_n & v & u & u_1 & \cdots & u_n \end{array} \\ \begin{bmatrix} 0 & \cdots & 0 & 3 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 3 & 0 & 0 & \cdots & 0 \\ 3 & \cdots & 3 & 0 & 2 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 2 & 0 & 3 & \cdots & 3 \\ 0 & \cdots & 0 & 0 & 3 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 3 & 0 & \cdots & 0 \end{bmatrix} \end{array}$$

Note that the characteristic polynomial of matrix $M_e(B_{n,n})$ is $\lambda^{2(n+1)} - 4(2n^3+5n^2+4n+1)\lambda^{2n} - 16n^2(n+1)^4\lambda^{2(n-1)}$. So, eigenvalues of $M_e(B_{n,n})$ are $0, 0, \ldots\ldots, 0(2(n-1)$ times), $1+\sqrt{9n+1}, 1-\sqrt{9n+1}, -1+\sqrt{9n+1}$ and , $-1-\sqrt{9n+1}$. Hence, Maximum Eccentricity energy of

$$\begin{aligned} B_{n,n} =& EM_e(B_{n,n}) = 0 + |1+\sqrt{9n+1}| + |1-\sqrt{9n+1}| \\ & + |-1+\sqrt{9n+1}| + |-1-\sqrt{9n+1}| \\ =& \sqrt{9n+1} + 1 + \sqrt{9n+1} - 1 + \sqrt{9n+1} - 1 \\ & + \sqrt{9n+1} + 1 \\ =& 4\sqrt{9n+1} \end{aligned}$$

□

Example 2.2. *Maximum Eccentricity energy of Bistar graph $B_{5,5} = 4\sqrt{46}$*

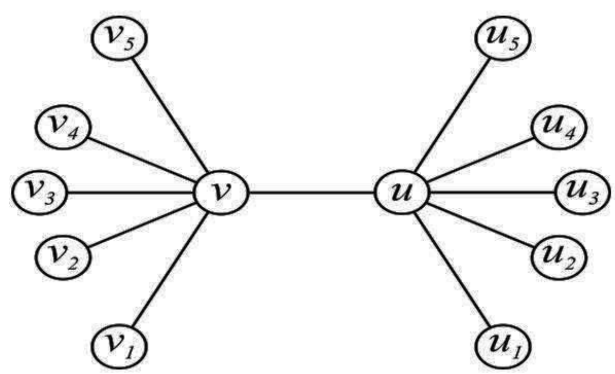

Figure 2

Proof. From the following matrix we have the characteristic polynomial of matrix $M_e(B_{5,5})$ is $\lambda^{12} - 1584\lambda^{10} - 518400\lambda^8$. So, Maximum Eccentricity eigenvalues of $B_{5,5}$ are $0, 0, \ldots, 0(8$ times), $(\sqrt{46}+1), (\sqrt{46}-1), (-\sqrt{46}+1)$ and $(-\sqrt{46}-1)$. Hence, Maximum Eccentricity energy of $B_{5,5} = 0 + |\sqrt{46}+1| + |\sqrt{46}-1| + |-\sqrt{46}+1| + |-\sqrt{46}-1| = 4\sqrt{46}$.
The Maximum Eccentricity matrix of $B_{5,5}$ is given by

$M_e(B_{5,5}) =$

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v \\ u \\ u_1 \\ u_3 \\ u_4 \\ u_5 \\ \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 3 & 3 & 3 & 3 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 & 3 & 3 & 3 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

□

Theorem 2.3. *Let $n \in N, n \neq 1$. Then $EM_e\left(B^2_{n,n}\right) = 1+\sqrt{64n+1}$, where $EM_e\left(B^2_{n,n}\right)$ is the Maximum Eccentricity energy of graph $B^2_{n,n}$.*

Proof. Let $V\left(B^2_{n,n}\right) = \{v_1, v_2, \ldots\ldots, v_n, v, u, u_1, u_2, \ldots\ldots u_n\}$. Note that $B^2_{n,n}$ is graph with $2n+2$ vertices and $4n+1$ edges as shown in the following Figure 3. Observe that the Maximum

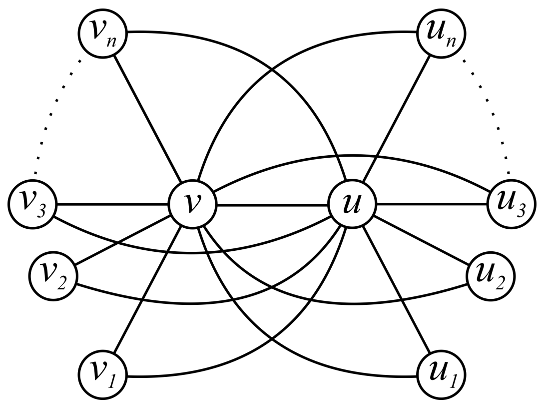

Figure 3

Eccentricity matrix $M_e\left(B^2_{n,n}\right)$ of $B^2_{n,n}$ is given by

$$
B^2_{n,n} = \begin{array}{cc}
 & \begin{array}{cccccccc} v_1 & \cdots & v_n & v & u & u_1 & \cdots & u_n \end{array} \\
\begin{array}{c} v_1 \\ \vdots \\ v_n \\ v \\ u \\ u_1 \\ \vdots \\ u_n \end{array} &
\begin{bmatrix}
0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\
2 & \cdots & 2 & 0 & 1 & 2 & \cdots & 2 \\
2 & \cdots & 2 & 1 & 0 & 2 & \cdots & 2 \\
0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 2 & 2 & 0 & \cdots & 0
\end{bmatrix}
\end{array}
$$

Note that the characteristic polynomial of $M_e\left(B^2_{n,n}\right)$ is $\lambda^{2(n+1)} - (4n+1)(2n+1)^2\lambda^{2n} - 4n(2n+1)^3\lambda^{(2n-1)}$ Hence, Maximum Eccentricity eigenvalues are, $0, 0, \ldots\ldots, 0((2n-1)$ times $), -1$, $\frac{1}{2}(1+\sqrt{64n+1})$ and $\frac{1}{2}(1-\sqrt{64n+1})$ Therefore, Maximum Eccentricity energy of

$$
\begin{aligned}
B^2_{n,n} =& EM_e\left(B^2_{n,n}\right) = 0 + |-1| + \frac{1}{2}|1+\sqrt{64n+1}| \\
& + \frac{1}{2}|1-\sqrt{64n+1}| \\
=& 1 + \frac{1}{2}(1+\sqrt{64n+1} - 1 + \sqrt{64n+1}) \\
=& 1 + \sqrt{64n+1}.
\end{aligned}
$$

□

Example 2.4. *Maximum Eccentricity energy of square of Bistar graph $B^2_{5,5} = 1+\sqrt{321}$.*

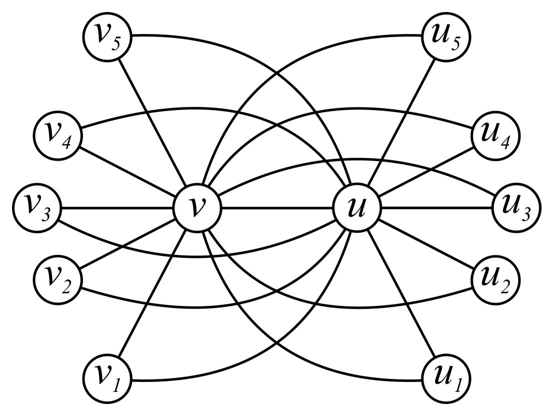

Figure 4

Proof. The Maximum Eccentricity matrix of $B^2_{5,5}$ is given by $M_e(B^2_{5,5}) =$

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v & u & u_1 & u_2 & u_3 & u_4 & u_5 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v \\ u \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
2 & 2 & 2 & 2 & 2 & 0 & 1 & 2 & 2 & 2 & 2 & 2 \\
2 & 2 & 2 & 2 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Note that the characteristic polynomial of $M_e\left(B^2_{5,5}\right)$ is $\lambda^{12} - 2541\lambda^{10} - 26620\lambda^9$. Therefore, the Maximum Eccentricity eigenvalues of $B^2_{5,5}$ are $0, 0, \ldots\ldots, 0(9$ times $), -1, \frac{1}{2}(1+\sqrt{321}$ and $\frac{1}{2}(1-\sqrt{321})$ Hence, Maximum Eccentricity energy of $B^2_{n,n} = EM_e\left(B^2_{n,n}\right) = 0 + |-1| + \left|\frac{1}{2}(1+\sqrt{321})\right| + |\frac{1}{2}(1-\sqrt{321})| = 1+\sqrt{321}$. □

Theorem 2.5. *Let $n \in N, n \neq 1$. Then $EM_e\left(D_2\left(B_{n,n}\right)\right) = 8(\sqrt{9n+1})$, where $EM_e\left(D_2\left(B_{n,n}\right)\right)$ is the Maximum Eccentricity energy of graph $D_2\left(B_{n,n}\right)$.*

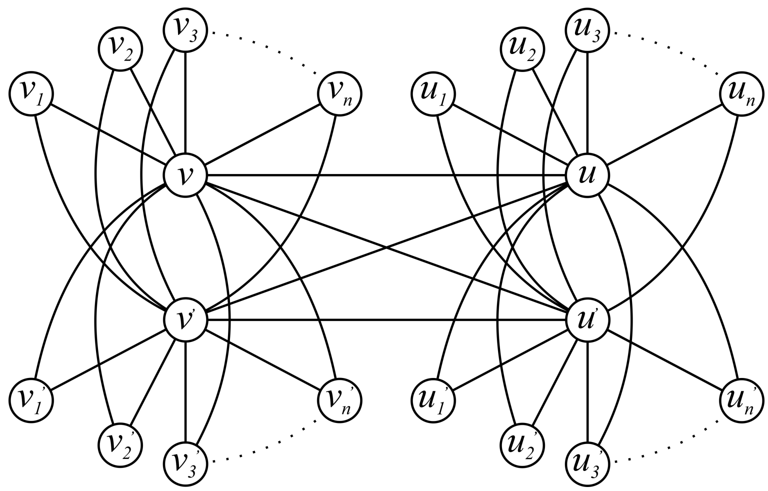

Figure 5

Proof. Let $V(D_2(B_{n,n})) = \{v_1, v_2, \ldots\ldots, v_n, v, u, u_1, u_2, \ldots\ldots u_n\}$. Note that $D_2(B_{n,n})$ is graph with $4(n+1)$ vertices and $4(2n+1)$ edges as shown in the above Figure 5. □

Observe that the Maximum Eccentricity matrix $M_e(D_2(B_{n,n}))$ of $D_2(B_{n,n})$ is given by

Hence, Maximum Eccentricity energy of

$$\begin{aligned} D_2(B_{n,n}) =& EM_e(D_2(B_{n,n})) = 0 + |2(1+\sqrt{9n+1})| \\ &+ |2(1-\sqrt{9n+1})| + |2(-1+\sqrt{9n+1})| \\ &+ |2(-1-\sqrt{9n+1})| \\ =& 2(1+\sqrt{9n+1}-1+\sqrt{9n+1}-1+\sqrt{9n+1} \\ &+1+\sqrt{9n+1}) \\ =& 8(\sqrt{9n+1}). \end{aligned}$$

$$M_e(D_2(B_{n,n})) = \begin{array}{cc} & \begin{array}{cccccccccccccccc} v_1 & \ldots & v_n & v_1' & \ldots & v_n' & v & v' & u & u' & u_1 & \ldots & u_n & u_1' & \ldots & u_n' \end{array} \\ \begin{array}{c} v_1 \\ \vdots \\ v_n \\ v_1' \\ \vdots \\ v_n' \\ v \\ v' \\ u \\ u' \\ u_1 \\ \vdots \\ u_n \\ u_1' \\ \vdots \\ u_n' \end{array} & \begin{bmatrix} 0 & \ldots & 0 & 0 & \ldots & 0 & 3 & 3 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 3 & 3 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 3 & 3 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 3 & 3 & 0 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 3 & \ldots & 3 & 3 & \ldots & 3 & 0 & 0 & 2 & 2 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 3 & \ldots & 3 & 3 & \ldots & 3 & 0 & 0 & 2 & 2 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 2 & 2 & 0 & 0 & 3 & \ldots & 3 & 3 & \ldots & 3 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 2 & 2 & 0 & 0 & 3 & \ldots & 3 & 3 & \ldots & 3 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 3 & 3 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 3 & 3 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 3 & 3 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & 0 & 0 & 3 & 3 & 0 & \ldots & 0 & 0 & \ldots & 0 \end{bmatrix} \end{array}$$

Note that the characteristic polynomial of $M_e(D_2(B_{n,n}))$ is $\lambda^{4(n+1)} - 16\left(2n^3+5n^2+4n+1\right)\lambda^{4n+2} - 64n^2(n+1)^4\lambda^{4n}$ So, Maximum Eccentricity eigenvalues of $M_e(D_2(B_{n,n}))$ are $0, 0, \ldots\ldots, 0[4n \text{ times}], 2(1+\sqrt{9n+1})\ 2(1-\sqrt{9n+1}), 2(-1+\sqrt{9n+1})$ and $2(-1-\sqrt{9n+1})$

Example 2.6. *Maximum Eccentricity energy of shadow graph of Bistar graph* $D_2(B_{5,5}) = 8(\sqrt{46})$.

Proof. The Maximum Eccentricity matrix of $D_2(B_{5,5})$ is given by

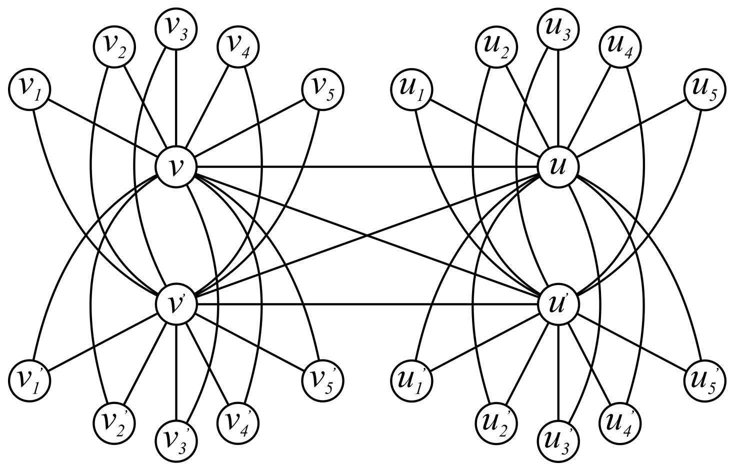

Figure 6

$M_e(D_2\,(B_{5,5})) =$

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccccccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_1' & v_2' & v_3' & v_4' & v_5' & v & v' & u & u' & u_1 & u_2 & u_3 & u_4 & u_5 & u_1' & u_2' & u_3' & u_4' & u_5' \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_1' \\ v_2' \\ v_3' \\ v_4' \\ v_5' \\ v \\ v' \\ u \\ u' \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_1' \\ u_2' \\ u_3' \\ u_4' \\ u_5' \end{array} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

The characteristic polynomial of $M_e(D_2(B_{5,5}))$ is $\lambda^{24} - 6336\lambda^{22} + 8294400\lambda^{20}$. Hence, the Maximum Eccentricity eigenvalue of $D_2\,(B_{5,5})$ are $0, 0, \ldots\ldots, 0$(20 times) $, 2(1+\sqrt{46}), 2(1-\sqrt{46}), 2(-1+\sqrt{46})$ and $2(-1-\sqrt{46})$.

Therefore, Maximum Eccentricity energy

$$
\begin{aligned}
EM_e\,(D_2\,(B_{5,5})) &= 0 + |2(1+\sqrt{46})| + |2(1-\sqrt{46})| \\
&\quad + |2(-1+\sqrt{46})| + |2(-1-\sqrt{46})| \\
&= 2(1+\sqrt{46} - 1 + \sqrt{46} - 1 + \sqrt{46} + 1 + \sqrt{46}) \\
&= 8(\sqrt{46}).
\end{aligned}
$$

□

Theorem 2.7. *Let $n \in N, n \neq 1$. Then $EM_e(Gl(n)) = 4\sqrt{2n}$, where $EM_e(Gl(n))$ is the Maximum Eccentricity energy of graph $Gl(n)$.*

Proof. Let $V\,(Gl(n)) = \{v, v_1, v_2, \ldots\ldots, v_n, u\}$. Note that $Gl(n)$ is graph with $n+2$ vertices and $2n$ edges as shown in the following Figure 7. Observe that the Maximum Eccentricity matrix $M_e(Gl(n))$ of $Gl(n)$ is given by

$$
M_e(Gl(n)) = \begin{array}{cc}
 & \begin{array}{ccccc} v & v_1 & \ldots & v_n & u \end{array} \\
\begin{array}{c} v \\ v_1 \\ \vdots \\ v_n \\ u \end{array} &
\begin{bmatrix}
0 & 2 & \cdots & 2 & 0 \\
2 & 0 & \cdots & 0 & 2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
2 & 0 & \cdots & 0 & 2 \\
0 & 2 & \cdots & 2 & 0
\end{bmatrix}
\end{array}
$$

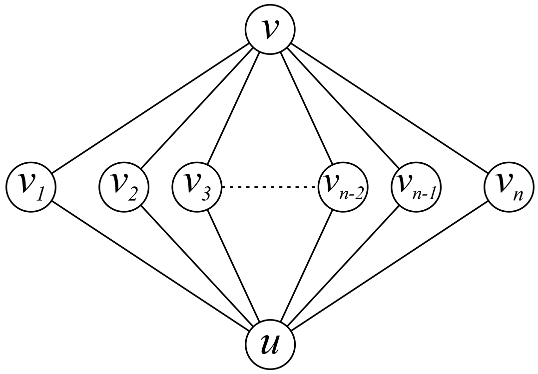

Figure 7

Note that the characteristic polynomial of matrix $M_e(Gl(n))$ is $\lambda^7 - 8n\lambda^5$. So, eigenvalues of $M_e(Gl(n))$ are $0, 0, \ldots\ldots, 0$(n times), $2\sqrt{2n}$ and $-2\sqrt{2n}$.
Hence, Maximum Eccentricity energy of

$$\begin{aligned} Gl(n) =& EM_e(Gl(n)) = 0 + |2\sqrt{2n}| + |-2\sqrt{2n}| \\ =& 2\sqrt{2n} + 2\sqrt{2n} \\ =& 4\sqrt{2n}. \end{aligned}$$

□

Example 2.8. *Maximum Eccentricity energy of Globe graph* $Gl(5) = 4(\sqrt{10})$.

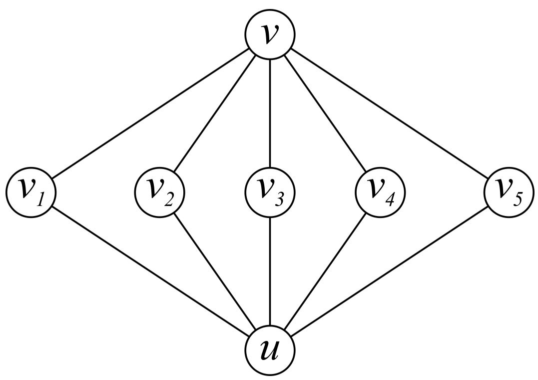

Figure 8

Proof. The Maximum Eccentricity matrix of $Gl(5)$ is given by

$$M_e(B_{n,n}) = \begin{array}{c} \\ v \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ u \end{array} \begin{array}{c} \begin{array}{ccccccc} v & v_1 & v_2 & v_3 & v_4 & v_5 & u \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

Note that the characteristic polynomial of matrix $M_e(Gl(5))$ is $\lambda^7 - 40\lambda^5$. So, Maximum Eccentricity eigenvalues of $Gl(5)$ are $0, 0, \ldots, 0$(5 times), $2\sqrt{10}$, and $-2\sqrt{10}$.
Hence, Maximum Eccentricity energy of

$$\begin{aligned} Gl(5) =& EM_e(Gl(5)) = 0 + |2\sqrt{10}| + |-2\sqrt{10}| \\ &= 4\sqrt{10}. \end{aligned}$$

□

References

[1] C. Adiaga and M. Smitha, On maximum degree energy of a graph, *Int. J. Contemp. Math. Sciences,* 4(8), (2009), 385–396.

[2] J. Akiyama, K. Ando and D. Avis, Eccentric graph, *Discr. Math,* 56(1985), 1–6.

[3] R.Balakrishnan and K. Ranganathan, *A Textbook of Graph Theory*, Universitext Springer (2000).

[4] J.A. Bondy and U.R. Murty, *Graph Theory*, Springer, Berlin (2008).

[5] G.V. Ghodasara and M.J. Patel, Some Bistar related square sum graphs, *IJMTT*, (2017), 172–177.

[6] I. Gutman, The energy of a graph, Ber. Math-Statist, *Sekt.Forschungz.Graz,* 103(1978), 1–22.

[7] Ahmed M. Naji and N.D. Soner, Maximum eccentricity energy of a graph, *IJSER*, 7(5), (2016).

[8] N. Prabhavathy, A new concept of energy from eccentricity matrix of graphs, *Malaya Journal of Matematik*, (2019), 400–402.

[9] A.H.Rokad, Fibonacci cordial labelling of some special graphs, *Orient.J.Comp.Sci. and Technol*, 10(4).

[10] S. K. Vaidya and K. M. Popat, Some new results on energy of graphs, MATCH *Commun. Math. Comput. Chem,* 77(2017), 589–594.

[11] S. K. Vaidya and K. M. Popat, Energy of m- Splitting and m-Shadow Graphs, *Far East Journal of Mathematical Sciences,* 102(2017), 1571–1578.

[12] S. K. Vaidya and N.H. Shah, some Star and Bistar Related Divisor Cordial Graphs, *Annals of Pure and Applied Mathematics*, (2013), 67–77.

[13] Jianfeng Wang, Mei Lu, FranscoBelardo and Milan Randic, The anti-adjacency matrix of a graph: Eccentricity matrix, *Discrete Applied Mathematics*, Elsevier (2018).

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