Malaya Journal of Matematik, Vol. 8, No. 4, 1527-1538, 2020

https://doi.org/10.26637/MJM0804/0033

An integrated production-distribution inventory system for deteriorating products in fuzzy environment

S. Hemalatha[1*] and K. Annadurai[2]

Abstract

A key issue for the organization of responsiveness to uncertainties is intelligent manufacturing design of a complex production inventory system. To effectively handle imprecision or uncertainty, fuzzy methodologies provide a useful way to model vagueness in human recognition and judgment. Fuzzy numbers are frequently used in applications to ensure easy handling of the realistic problem. Priyan and Uthayakumar (2015) proposed an integrated production-distribution inventory system for deteriorating products that involve fuzzy deterioration rate and variable setup cost environment. They offered strategic decision-making to produce and supply products to minimize total system cost under fuzzy deterioration rate and variable setup cost environment. In this paper, their model is extended by considering the demand, production rate, deterioration rate, holding cost for both the vendor and the buyer and the ordering cost for the buyer as the triangular fuzzy number and the setup cost as a function of capital expenditure. Signed distance method is used to defuzzify the total cost and differential calculus optimization technique is employed to find optimal solutions of the model. Numerical example and sensitivity analysis are depicted to feature the contrasts among crisp and the fuzzy cases.

Keywords

Triangular fuzzy number; signed distance method; inventory costs; logarithmic function.

AMS Subject Classification

62A86.

[1] *Department of Mathematics, SSM Institute of Engineering and Technology, Dindigul-624002, Tamil Nadu, India.*
[1] *Ph.D Scholar, Mother Teresa Women's University, Kodaikanal-624101, Tamil Nadu, India.*
[2] *Department of Mathematics, M.V. Muthiah Government Arts College for Women, Dindigul-624001, Tamil Nadu, India.*
*__Corresponding author__: [1]shemalatha1974@gmail.com
Article History: Received **04** May **2020**; Accepted **17** August **2020**

Contents

1. Introduction

Inventory collaboration schemes involve mechanization of a company's replenishment processes as well as the connection of buyers and suppliers circle with real-time forecast, inventory on-hand, optimal lot sizing, quality improvements and inspections, and shipment information to reduce inventory and eliminate unnecessary expenses. Henceforth, cooperation and integration are in hot board of supply chain management. The main goal of supply chain and inventory management research is to reduce unnecessary costs without sacrificing customer service. Our main objective is to study the impact and sensitiveness of the impreciseness of cost components in the decision variables and total cost. The joint optimization concept for purchaser and merchant was initiated by Goyal (1976). Hill (1997) broadened the single-seller single-purchaser integrated production-inventory model as a generalized policy. Viswanathan and Pipalani (2001) addressed the coordinating supply chain inventory through common replenishment epochs. Yang and Wee (2003) discussed multi-item lot size inventory model for deteriorating items in just-in time (JIT) circumstances. Goyal (1987) adopted economic ordering policy for deteriorating items over an infinite time horizon. Shah (1977) investigated an order level lot size model for both exponential and Weibull distributed deterioration with backordering. Skouri and Papachristos (2003) discussed four inventory models for deteriorating items with time varying demand and partial backlogging. Wee and Widyadana (2013) expanded with diverse types of deterioration rates of the inventory models ment for deteriorating products.Abdul Jalbar et al. (2007)proposed an integrated inventory model for the single-vendor two-buyer problem. Priyan and Uthayakumar (2013) addressed pharmaceutical supply chain and inventory management strategy for a pharmaceutical company and a hospital by optimization. Sarkar (2013) developed production-inventory model with probabilistic deterioration in two-echelon supply chain management. Yan et al. (2011) developed production-distribution inventory model with the considerations of constant deterioration. Misra (1975) developed an optimal production lot size model with deterioration inventory. Sarkar (2012) proposed an EOQ model with delay in payments and time varying deterioration rate. Hall, R.W. (1983) stated that logarithmic investment function is consistent with the Japanese experience and it is more appropriate and therefore significant savings can be achieved.

In the classical economic production/order quantity (EPQ/EOQ) models, setup cost is treated as a constant. Setup cost is the investment to setup a production equipment. This includes the cost of the setup, the cost of scheduling, record keeping, moving the material and testing the first few units of output to check whether the equipment functions properly.However, in practice, setup cost can be controlled and reduced by means of various efforts. Therefore, for attaining production system efficiency, reduced lot sizes alone are not sufficient, unless accompanied by corresponding setup cost reduction and quality improvement. Thus, considerable attention is paid to the optimal lot sizing and investments in setup cost reduction and quality improvement. Affisco et al. (2002) presented a quality improvement and setup reduction in the joint economic lot size model. Annadurai and Uthayakumar (2010) proposed controlling setup cost in (Q, r, L) inventory model with defective items. To define inventory optimization tasks in such environment and to interpret optimal solution, fuzzy set theory is considered to be more convenient than probability theory. The concept of fuzzy set theory was introduced by Zadeh (1965). The membership function of a fuzzy set possesses a quantity meaning and may be viewed as a fuzzy number provided they satisfy certain conditions. Fuzzy numbers are largely applied on data analysis, artificial intelligence and decision-making. Rama and Rosario (2018) proposed a fuzzy inventory model based on different defuzzification techniques of various fuzzy numbers. Rajput et al., (2019) used signed distance method to optimize the cost of a fuzzy inventory model with shortage. Yao and Chiang (2003) proposed an inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance methods. Priyan and Uthayakumar (2014) proposed an optimal inventory management strategy for pharmaceutical company and hospital supply chain in a fuzzy-stochastic environment. Chiang et al., (2005) used the signed distance method to defuzzify the fuzzy inventory model with backorder. Priyan et al., (2015) developed two-echelon production inventory system with fuzzy production rate and promotional effort dependent demand. In particular, the various costs that impact the system, are often ill-defined and may differ from time to time. Fuzzy theory affords an alternate, flexible approach to handle such situations, because it allows the model to easily incorporate various experts' advice in developing critical parameters estimated by Zimmerman (2001). Vijayan and Kumaran (2008) developed an inventory models with a mixture of backorders and lost sales under fuzzy cost.Priyan et al., (2014) proposed a mathematical modelling for EOQ inventory system with advance payment and fuzzy parameters. Priyan and Uthayakumar (2016) developed an economic design of an inventory system involving probabilistic deterioration and variable setup cost through mathematical approach. Mahata and Goswami (2013) considered the case with imperfect quality and shortage backordering under crisp and fuzzy decision variables. Yao et al., (2003) proposed a fuzzy inventory model with two replaceable merchandises without backorder based on the signed distance method. Pu and Liu (1980) proposed a fuzzy topology1,neighborhood structure of a fuzzy point and moore-smith convergence. Priyan and Uthayakumar (2015) proposed an integrated production-distribution inventory system for deteriorating products involving fuzzy deterioration and variable setup cost. Priyan and Uthayakumar (2017) proposed an integrated production-distribution inventory system involving probabilistic defective and errors in quality inspection under variable setup cost. Vasanthi et al., (2019) addressed fuzzy EOQ model with shortages using Kuhn-Tucker conditions. This paper proposed an integrated production-distribution inventory system for deteriorating products in fuzzy environment. The fuzziness in the cost components, production rate, deterioration rate and demand are represented as the triangular fuzzy numbers and the setup cost can be reduced by ensuring extra investment. Signed distance method is used to defuzzify the total cost and differential calculus optimization technique is adopted to find the optimal solutions of the model. Numerical example and sensitivity analysis are stated to highlight the differences between crisp and the fuzzy cases. The detailed description of this article is as follows. The introduction is given in section 1. In section 2, preliminary concepts are given that have been used for model building purposes. Section 3 presents notation and assumptions. The model formulation is discussed in section 4. Solution procedure is given in section 5. In section 6, numerical examples and sensitivity analysis are given in detail to illustrate the models. Finally, conclusion of the study is presented.

2. Preliminaries

The pertinent definitions of fuzzy sets related to the signed distance method for the proposed model are given below.

Definition 2.1. *A fuzzy set $\tilde{B}$ on the given universal set X is a set of ordered pairs on the real line $R, \tilde{B} = \{(x, \mu_{\hat{B}}(x)) : x \in X\}$ called as membership function. The membership function is also called as degree of compatibility or a degree of truth of X in $\tilde{B}$ which is defined as $\mu_B : X \to [0,1]$*

Definition 2.2. *α -cut of a Fuzzy Set: An α -cut of a fuzzy set B is a crisp set B_α that contains all the elements of the universal set X and have a membership grade in B which is greater than or equal to the specified value α. That is $B_\alpha = \{x \in X / \mu_B(x) \geq \alpha\}$.*

Definition 2.3. *Fuzzy Point (Pu and Liu (1980)): Let $\tilde{b}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is $\mu_{\tilde{b}}(x) = \begin{cases} 1, x = b \\ 0, x \neq b \end{cases}$*

Definition 2.4. *Level α Fuzzy Interval: Let $[p, q, \alpha]$ be a fuzzy set on $R = (-\infty, \infty)$. It is called Level α fuzzy interval, $0 \leq \alpha \leq 1$, $p < q$, if its membership function is*

$$\mu_{[p,q,\alpha]}(x) = \begin{cases} \alpha, p \leq x \leq q \\ 0 \; otherwise \end{cases}$$

Definition 2.5. *Triangular Fuzzy Numbers: Let $\tilde{B} = (p, q, r)$, $p < q < r$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a triangular fuzzy number, if its membership function is*

$$\mu_B(x) = \begin{cases} \frac{x-p}{q-p}, & p \leq x \leq q \\ \frac{r-x}{r-q}, & q \leq x \leq r \\ 0, & otherwise \end{cases}$$

Let $\tilde{B}$ be a fuzzy number on R. The α -cut of the set $\tilde{B}$ is defined as $\tilde{B}(\alpha) = \{x : \mu_B(x) \geq \alpha\}$ where $\alpha \in [0,1]$. $\tilde{B}(\alpha)$ is a non-empty bounded closed interval contained in the set of real numbers and it is denoted by $\tilde{B}(\alpha) = [\tilde{B}_L(\alpha), \tilde{B}_R(\alpha)]$. $\tilde{B}_L(\alpha)$ and $\tilde{B}_R(\alpha)$ are the left and right limits of $\tilde{B}(\alpha)$ and also, called as left and right α -cut of the set $\tilde{B}$. Further, let U be the family of all these fuzzy numbers $\tilde{B}$ on R. Now, signed distance on U is taken for consideration.

Definition 2.6. *Signed distance is defined as $d_0(b,0) = b$ for $b, 0 \in R$.*

Remark 2.7. *The sense of definition is the following, if $0 < b$, then the distance between b and 0 is $d_0(b,0) = b$. If $b < 0$, then the distance between b and 0 is $-d_0(b,0) = -b$. Therefore, signed distance between b and 0 is $d_0(b,0) = b$*

For $\tilde{B} \in U$, from Definition 2.6 the signed distance of $\tilde{B}_L(\alpha)$ and $\tilde{B}_R(\alpha)$ measured from 0 is $d_0(\tilde{B}_L(\alpha), 0) = \tilde{B}_L(\alpha)$ and $d_0(\tilde{B}_R(\alpha), 0) = \tilde{B}_R(\alpha)$, respectively. Therefore, the signed distance of the interval $[\tilde{B}_L(\alpha), \tilde{B}_R(\alpha)]$, from the origin 0 is $d_0([\tilde{B}_L(\alpha), \tilde{B}_R(\alpha)], 0) = \frac{1}{2}[d_0(\tilde{B}_L(\alpha), 0) + d_0(\tilde{B}_R(\alpha), 0)] = \frac{1}{2}[\tilde{B}_L(\alpha) + \tilde{B}_R(\alpha)]$, where $\tilde{B}_L(\alpha)$ and $\tilde{B}_R(\alpha)$ exist and are integrable for $\alpha \in [0,1]$. For each $\alpha \in [0,1]$, there exists a one-to-one correspondence between the crisp interval $[\tilde{B}_L(\alpha), \tilde{B}_R(\alpha); 0]$ and α fuzzy interval $[\tilde{B}_L(\alpha), \tilde{B}_R(\alpha); \alpha]$. Henceforth, we define the signed distance from $[\tilde{B}_L(\alpha), \tilde{B}_R(\alpha); \alpha]$ to $\tilde{0}$ as $d([\tilde{B}_L(\alpha), \tilde{B}_R(\alpha); \alpha], 0) = \frac{1}{2}[\tilde{B}_L(\alpha) + \tilde{B}_R(\alpha)]$. Since $\tilde{B} \in U$, $\tilde{B}_L(\alpha)$ and $\tilde{B}_R(\alpha)$ exist and are integrable for $\alpha \in [0,1]$ the following definition is obtained.

Definition 2.8. *Let $\tilde{B} \in U$. The signed distance of $\tilde{B}$ measured from $\tilde{0}$ is defined as*

$$d(\tilde{B}, \tilde{0}) = \frac{1}{2}\int_0^1 (\tilde{B}_L(\alpha) + \tilde{B}_R(\alpha))\, d\alpha \tag{2.1}$$

3. Notations and assumptions

The following notations and assumptions are adopted for developing our model, which are almost similar to those used in Yan et al. (2011).

3.1 Notations

- N - Number of shipments per production batch (decision variable)
- Q - Order quantity (units), a decision variable
- θ - Deterioration rate
- S - Setup cost for a production batch (decision variable)
- S_0 - Original setup cost for a production batch (before any investment is made)
- P_C - Production rate (units/ time unit)
- $I(S)$ - Capital investment required to achieve setup cost $S, 0 < S \leq S_0$
- H_V - Inventory holding cost for the vendor ($/units/ time unit)
- A_C - Inventory ordering cost for the buyer ($ /order)
- D - Constant demand (units/ time unit)
- H_b - Inventory holding cost for the buyer ($ /units/ time unit)
- F_C - Fixed transportation cost per delivery ($/delivery)
- d_C - Deterioration cost per unit ($/ unit)
- V_C - Unit variable cost for both order handling and receiving ($ /unit)
- T - Duration of inventory cycle
- ATC- Average total cost of the supply chain

3.2 Assumptions

1. The production-inventory system produces single type of deteriorating item.
2. Demand is considered as a constant and deterministic.
3. The demand information and inventory position of the buyer are given to the vendor.
4. Production rate of the vendor is assumed to be constant $P > D$.
5. Backlogging and shortages are not allowed.
6. The buyer pays transportation and other handling costs.

4. Mathematical model

The expected total cost for the buyer and vendor which is similar to Yan et al. (2011) can be expressed as follows

$$ATC(\mathrm{Q,N}) = \left(\frac{D}{NQ} + \frac{\theta}{2N}\right)(A_C + S + NF_C + V_C NQ) + \frac{Q}{2}\left[(H_b + d_C\theta) + (H_V + d_C\theta) \times \left\{\frac{(2-N)D}{P_C} + N - 1\right\}\right] \quad (4.1)$$

Priyan and Uthayakumar (2015) consider setup cost S as a decision variable and seek to minimize the sum of the capital investment cost by reducing setup cost S and the inventory related costs by optimizing over Q, S and N constrained on $0 < S \leq S_0$, where S_0 is the original setup cost.They stated the logarithmic investment function which is consistent with the Japanese experience as reported in Hall (1983). Accordingly, the expected total cost is

$$ATC(\mathrm{Q,N}) = \mu L \ln\left(\frac{S_0}{S}\right) + \left(\frac{D}{NQ} + \frac{\theta}{2N}\right)(A_C + S + NF_C + V_c NQ) + \frac{Q}{2}\left[(H_b + d_C\theta) + (H_V + d_C\theta) \times \left\{\frac{(2-N)D}{P_C} + N - 1\right\}\right] \quad (4.2)$$

over $S \in (0, S_0]$, where μ is the opportunity cost of capital per year, $L\ln\left(\frac{S_0}{S}\right) = I(S)$ is the logarithmic investment function. Here, $L = \frac{1}{\xi}$, and ξ are the percentage decrease in S per dollar increase in $I(S)$.

5. Solution procedure

For fixed Q and S, the expected total cost $ATC(Q,S,N)$ is a convex function of N, which indicates that there must be an optimal $N = N^*$ to meet the following equations $ATC(Q,S,N^*) \geq ATC(Q,S,N^*+1)$ and $ATC(Q,S,N^*) \geq ATC(Q,S,N^*-1)$. Also, it is proved that the convexity of the expected total cost $ATC(Q,S,N)$ in Lemma 5.1-5.3 are based on classical differential calculus optimization technique.

Lemma 5.1. *For fixed Q and S, the expected total cost $ATC(Q,S,N)$ is convex in N.*

Proof. The first and second order partial derivatives of $ATC(Q,S,N)$ with respect to N result in

$$\frac{\partial ATC(\mathrm{Q,S,N})}{\partial N} = \frac{Q}{2}(H_V + d_C\theta)\left\{1 - \frac{D}{P_C}\right\} - \frac{(A_C + S)}{N^2}\left(\frac{D}{Q} + \frac{\theta}{2}\right)$$

and

$$\frac{\partial^2 ATC(\mathrm{Q,S,N})}{\partial N^2} = \frac{2(A_C + S)}{N^3}\left(\frac{D}{Q} + \frac{\theta}{2}\right) > 0$$

Therefore, for fixed Q and S, the expected total cost $ATC(Q,S,N)$ is convex in N. This completes the proof of Lemma 5.1. □

Lemma 5.2. *For fixed N, the expected total cost $ATC(Q,S,N)$ is convex in Q.*

Proof. Taking the first and second order partial derivatives of ATC(Q,S,N) with respect to Q, we get

$$\frac{\partial ATC(Q,S,N)}{\partial Q} = \frac{1}{2}\left[(H_b + d_C\theta) + (H_V + d_C\theta) \times \left\{\frac{(2-N)D}{P_C} + N - 1\right\} + \theta V_C\right] - \left(\frac{D}{NQ^2}\right)(A_c + S + NF_C) \quad (5.1)$$

and

$$\frac{\partial^2 ATC(Q,S,N)}{\partial Q^2} = \left(\frac{D}{NQ^3}\right)(A_C + S + NF_C) > 0$$

Therefore, for fixed N, the expected total cost $ATC(Q,S,N)$ is convex in Q. This completes the proof of Lemma 5.2. □

Lemma 5.3. *For fixed N, the expected total cost $ATC(Q,S,N)$ is convex in S.*

Proof. Taking the first and second order partial derivatives of $ATC(Q,S,N)$ with respect to S we can get

$$\frac{\partial ATC(Q,S,N)}{\partial S} = \left(\frac{2D + Q\theta}{2NQ}\right) - \frac{\mu L}{S}; \quad (5.2)$$

$$\frac{\partial^2 ATC(Q,S,N)}{\partial S^2} = \frac{2\mu L}{S^3} > 0.$$

Therefore, for fixed N, the expected total cost $ATC(Q,S,N)$ is convex in S. This completes the proof of Lemma 5.3. □

Therefore, for fixed N, optimal order quantity Q and setup cost S are obtained by setting Eqs. (5.1) and (5.2) to zero as

$$Q = \sqrt{\frac{2D(A_C + S + NF_C)}{N\left[(H_b + d_c\theta) + (H_v + d_c\theta)\left\{\frac{(2-N)D}{P_C} + N - 1\right\} + V_C\theta\right]}} \quad (5.3)$$

and

$$S = \left(\frac{2NQ}{2D+Q\theta}\right)\mu L \quad (5.4)$$

respectively. From Eqs.(5.3) and (5.4), for fixed N, optimal order quantity Q and setup cost S are obtained.

6. Fuzzy mathematical modelling of an integrated production-distribution inventory system for deteriorating products

Now, an attempt is made to modify Priyan and Uthayakumar (2015) model by fuzzifying the demand, production rate, deterioration rate, holding cost for both supplier and buyer and the ordering cost for the buyer as the triangular fuzzy number.

We represent the demand D, production rate P_C, deterioration rate θ, holding cost for both the vendor H_V and the buyer H_b and the ordering cost A_C for the buyer by triangular fuzzy numbers as given below

$\tilde{\theta} = (\theta-\Delta_1, \theta, \theta+\Delta_2), \quad 0<\Delta_1<\theta, 0<\Delta_2$
$\tilde{P}_C = (P_C-\Delta_3, P_C, P_C+\Delta_4), \quad 0<\Delta_3<P_C, 0<\Delta_4$
$\tilde{H}_b = (H_b-\Delta_5, H_b, H_b+\Delta_6), \quad 0<\Delta_5<H_b, 0<\Delta_6$

$$\tilde{H}_V = (H_V-\Delta_7, H_V, H_V+\Delta_8), \quad 0<\Delta_7<H_V, 0<\Delta_8, \quad (6.1)$$

$\tilde{D} = (D-\Delta_9, D, D+\Delta_{10}), \quad 0<\Delta_9<D, 0<\Delta_{10}$
$\tilde{A}_C = (A_C-\Delta_{11}, A_C, A_C+\Delta_{12}), \quad 0<\Delta_{11}<A_C, 0<\Delta_{12}$

The left and right limits of α -cuts of $\tilde{\delta}, \tilde{P}, \tilde{H}_B, \tilde{H}_S, \tilde{D}$ and $\tilde{A}$ are as follows.

$\tilde{\theta}_L(\alpha) = \theta-\Delta_1+\theta\Delta_1>0, \quad \tilde{\theta}_R(\alpha) = \theta+\Delta_2-\theta\Delta_2>0$
$\tilde{P}_{C_L}(\alpha) = P_C-\Delta_3+\alpha\Delta_3>0, \quad \tilde{P}_{C_k}(\alpha) = P_C+\Delta_4-\alpha\Delta_4>0$

$$\tilde{H}_{b_L}(\alpha) = \tilde{H}_b-\Delta_5+\alpha\Delta_5>0, \tilde{H}_{b_k}(\alpha) = \tilde{H}_b+\Delta_6-\alpha\Delta_6>0, \quad (6.2)$$

$\tilde{H}_{V_L}(\alpha) = \tilde{H}_V-\Delta_7+\alpha\Delta_7>0, \tilde{H}_{V_R}(\alpha) = \tilde{H}_V+\Delta_8-\alpha\Delta_8>0$
$\tilde{D}_L(\alpha) = D-\Delta_9+\alpha\Delta_9>0, \quad \tilde{D}_R(\alpha) = D+\Delta_{10}-\alpha\Delta_{10}>0$
$\tilde{A}_{C_L}(\alpha) = A_C-\Delta_{11}+\alpha\Delta_{11}>0, \tilde{A}_{C_R}(\alpha) = A_C+\Delta_{12}-\alpha\Delta_{12}>0$

Accordingly, when the parameters D, P_C, θ, H_V, H_b and A_C in Eq. (4.2) are fuzzified to be $\tilde{D}, \tilde{P}_C, \tilde{\theta}, \tilde{H}_V, \tilde{H}_b$ and $\tilde{A}_c$ as expressed in Eq. (6.1), the expected total cost function in the fuzzy sense is given by

$$A\tilde{T}C(\mathrm{Q,S,N}) = \mu L \ln\left(\frac{S_0}{S}\right) + \left(\frac{\tilde{D}}{NQ} + \frac{\tilde{\theta}}{2N}\right) \left(\tilde{A}_C + S + NF_C + V_C NQ\right) + \frac{Q}{2}\left[\left(\tilde{H}_b + d_C\tilde{\theta}\right) + \left(\tilde{H}_V + d_c\tilde{\theta}\right)\left\{\frac{(2-N)\tilde{D}}{\tilde{P}_C} + N - 1\right\}\right] \quad (6.3)$$

From Eq. (6.2), the left- hand and right-hand side of the α - cut, $0 \le \alpha \le 1$, of $A\tilde{T}C(Q,S,N)$ can be obtained in the following form

$$A\tilde{T}C(Q,S,N)_L(\alpha) = \mu L \ln\left(\frac{S_0}{S}\right) + \left(\frac{\tilde{D}_L(\alpha)}{NQ} + \frac{\tilde{\theta}_L(\alpha)}{2N}\right) \left(\tilde{A}_{C_L}(\alpha) + S + NF_C + V_C NQ\right) + \frac{Q}{2}\left[\left(\tilde{H}_{b_L}(\alpha) + d_C\tilde{\theta}_L(\alpha)\right) + \left(\tilde{H}_{V_L}(\alpha) + d_C\tilde{\theta}_L(\alpha)\right)\left\{\frac{(2-N)\tilde{D}_L(\alpha)}{\tilde{P}_{C_L}(\alpha)} + N - 1\right\}\right] \quad (6.4)$$

and

$$A\tilde{T}C(\mathrm{Q,S,N})_R(\alpha) = \mu L \ln\left(\frac{S_0}{S}\right) + \left(\frac{\tilde{D}_R(\alpha)}{NQ} + \frac{\tilde{\theta}_R(\alpha)}{2N}\right) \left(\tilde{A}_{C_R}(\alpha) + S + NF_C + V_C NQ\right) + \frac{Q}{2}\left[\left(\tilde{H}_{b_R}(\alpha) + d_C\tilde{\theta}_R(\alpha)\right) + \left(\tilde{H}_{V_R}(\alpha) + d_C\tilde{\theta}_R(\alpha)\right)\left\{\frac{(2-N)\tilde{D}_R(\alpha)}{\tilde{P}_{C_R}(\alpha)} + N - 1\right\}\right] \quad (6.5)$$

Hence, when the parameters are described with triangular number, using Eqs. (2.1),(6.4) and (6.5), the signed distance value (defuzzified value)of $A\tilde{T}C(\mathrm{Q,S,N})$ is established as

$$d(A\tilde{T}C(Q,S,N), \tilde{0}) = \mu L \ln\left(\frac{S_0}{S}\right) + \left(\frac{G_5}{NQ} + \frac{G_1}{2N}\right) \left(G_6 + S + NF_C + V_c NQ\right) + \frac{Q}{2}\left[\left(G_3 + d_C G_1\right) + \left(G_4 + d_C G_1\right)\left\{\frac{(2-N)G_5}{G_2} + N - 1\right\}\right] \quad (6.6)$$

where

$$G1 = \theta + \frac{1}{4}(\Delta_2 - \Delta_1) > 0,$$
$$G2 = P_C + \frac{1}{4}(\Delta_4 - \Delta_3) > 0,$$
$$G3 = H_b + \frac{1}{4}(\Delta_6 - \Delta_5) > 0$$
$$G4 = H_V + \frac{1}{4}(\Delta_8 - \Delta_7) > 0$$
$$G5 = D + \frac{1}{4}(\Delta_{10} - \Delta_9) > 0$$
$$G6 = A_C + \frac{1}{4}(\Delta_{12} - \Delta_{11}) > 0.$$

The defuzzified value $d(A\tilde{T}C(Q,S,N), \tilde{0})$ is taken as the estimate of fuzzy cost function in Eq. (4.2) denoted by $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q,S,N}))$. The estimate in Eq. (6.6) is a convex function of Q and S similar to Eq. (4.2). Then optimal delivery lot size Q^* and setup cost S^* of $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q,S,N}))$ are obtained by equating the first order partial derivatives of $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q,S,N}))$ with respect to Q and S to zero. That is

$$\frac{\partial \mathrm{S}_d(A\tilde{T}C(\mathrm{Q,S,N}))}{\partial Q} = 0 \quad (6.7)$$

Solving Eq. (6.7), the optimal order quantity

$$Q^* = \sqrt{\frac{2G_5\,(G_6+S+NF_C)}{N\,(G_3+d_cG_1)+(G_4+d_cG_1)\left\{\frac{(2-N)G_5}{G_2}+N-1\right\}+V_cG_1}} \tag{6.8}$$

and

$$\frac{\partial \mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))}{\partial S} = 0 \tag{6.9}$$

Solving Eq. (6.9), we obtain

$$S^* = \left(\frac{2NQ}{2G_5+QG_1}\right)\mu L \tag{6.10}$$

Hence, for fixed N, from Eqs. (6.8) and (6.10), the fuzzy optimal delivery lot size Q^* and fuzzy setup cost S^* are obtained.Thus, the fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ is minimized.

7. Numerical analysis

Numerical analysis illustrates the above solution procedure for both crisp and fuzzy models.The same numerical data from Sarkar (2013) are used to verify the results obtained in this paper. The solutions are obtained by using MatLab software: $D = 4800$ units/year, $P_C = 10000$ units/year, $\theta = 0.2, H_V = \$6/$ units/year, $H_b = \$7/$ units/yearand $A_c = \$25/$ order ,$F_c = \$50/$ delivery ,$V_c = \$1/$ unit, $d_c = \$50/$ unit and $\mu = 0.1/$ dollar/year. In Sarkar (2013) model, he considered the setup cost $S = \$800$ per production run. In addition, the same numerical example as in Priyan and Uthayakumar (2015) model is used: initial setup cost $S_0 = 800$, and different parameters of investment function $L = 7250, 5800$ and 4350. Based on these values,optimal lot size Q, setup cost S, total number of deliveries N, and the minimum expected total cost $ATC(Q,S,N)$ for the crisp model developed in section 5 are summarized in Table 1.

In Table 2 , some triangular fuzzy numbers are set to the input parameters $(D, P_C, \theta, H_V,\ H_b$ and A_C) to represent the components of fuzzy models developed in section 2 . For each of these parameters, the variations in the values are arranged arbitrarily and their defuzzified values are determined by applying the signed distance method. Based on these values optimal lot size Q^*, setup cost S^*, total number of deliveries N^*, and minimum fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ developed in section 6 are summarized in Table 3. The comparison of minimum fuzzy expected total cost $S_d(A\tilde{T}C(Q,S,N))$ and minimum expected total cost $ATC(Q,S,N)$ against n are summarized in Table 4. The corresponding curves of minimum fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ and minimum expected total cost$ATC(Q,S,N)$ against n are plotted in Figure 1.

7.1 Sensitivity analysis

To further validate the model, the effects of parameters D, P_C, θ, H_V, H_b and A_C on the optimal lot size Q^*, setup cost S^*, total number of deliveries N^*, and the minimum expected total cost for both crisp and fuzzy models are analyzed. Some triangular fuzzy numbers of the input parameters $(D, P_C, \theta, H_V, H_b$ and $A_c)$ for the set of values of D, P_C, θ, H_V, H_b and A_C are assumed to be $\theta = 0.4, 0.5, 0.7, 0.9, P_C = 15000, 12500, 7500, 5000, H_b = 10.5\ 8.75, 5.25, 3.5, H_V = 9, 7.5, 4.5, 3, D = 7200, 6000, 3600,$ 2400 and $A_c = 37.5, 31.25\ 18.75$, 12.5. The corresponding defuzzified values are determined by applying the signed distance method. Sensitivity analysis is performed by taking one parameter at a time and keeping the remaining parameters unchanged. Meanwhile the other parameter values follow those data mentioned above in the numerical analysis. The results of sensitivity analysis are given in Tables $5-10$. Further, managerial implications of the proposed model based on the numerical results are presented.

1. Table 5 shows that when the deterioration rate θ increases, optimal lot size Q, setup cost S, total number of deliveries N, for both crisp and fuzzy models decrease. The results indicate that the expected total cost for both crisp and fuzzy model increases as the deterioration rate θ escalates. This occurs due to the impact of deterioration on the inventory cycle. Moreover, our results indicate that the optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model.The corresponding curves of the minimum fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ and the minimum expected total cost$ATC(Q,S,N)$ against θ are plotted in Figure 2.

2. In Table 6, optimal lot size Q, setup cost S, total cost for both crisp and fuzzy models decrease when theproduction rate P_C decreases. These results indicate that the optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model.The corresponding curves of the minimum fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ and the minimum expected total cost $ATC(Q,S,N)$ against P_C are plotted in Figure 3.

3. It is observed from Table 7 that optimal lot size Q, setup cost S, total cost for both crisp and fuzzy model decrease when the holding cost for the buyer H_b decreases. Thus, optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model.The corresponding curves of the minimum fuzzy expected total cost $\mathrm{S}_d(A\tilde{T}C(\mathrm{Q},\mathrm{S},\mathrm{N}))$ and the minimum expected total cost $ATC(Q,S,N)$ against H_b are plotted in Figure 4.

4. From Table 8, it is observed that optimal lot size Q, setup cost S, total cost for both crisp and fuzzy model decrease when the holding cost for the supplier H_V decreases. Also, our results indicate that optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model.The corresponding curves of the minimum fuzzy expected total

cost $S_d(A\tilde{T}C(Q,S,N))$ and the minimum expected total cost $ATC(Q,S,N)$ against H_V are plotted in Figure 5.

5. From Table 9, it is evident that optimal lot size Q, setup cost S, total cost for both crisp and fuzzy model decrease when demand D decreases. Optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model.The corresponding curves of the minimum fuzzy expected total cost $S_d(A\tilde{T}C(Q,S,N))$ and the minimum expected total cost $ATC(Q,S,N)$ against D are plotted in Figure 6.

6. From Table 10, it is inferred that optimal lot size Q , setup cost S , total cost for both crisp and fuzzy model decrease when the ordering cost for the buyer A_C decreases. In addition, these results indicate that the optimal solutions and the expected total cost of the fuzzy model slightly fluctuate from the solutions of the crisp model. The corresponding curves of the minimum fuzzy expected total cost $S_d(A\tilde{T}C(Q,S,N))$ and the minimum expected total cost $ATC(Q,S,N)$ against A_C are plotted in Figure 7.

Table 1. The optimal solution for a given example

L	(Q,S,N)	$ATC(Q,S,N)$
7250	(202, 30, 1)	12199
	(158, 48, 2)	12099
	(135, 61, 3)	12273
	(120, 72, 4)	12514
	(109, 82, 5)	12772
	(100, 90, 6)	13035
	(93, 98, 7)	13295
5800	(195, 23, 1)	11706
	(153, 37, 2)	11672
	(131, 47, 3)	11882
	(116, 56, 4)	12147
	(106, 64, 5)	12424
	(98, 71, 6)	12701
	(91, 77, 7)	12973
4350	(188, 17, 1)	11172
	(148, 27, 2)	11203
	(127, 34, 3)	11450
	(113, 41, 4)	11740
	(103, 47, 5)	12036
	(95, 52, 6)	12328
	(89, 56, 7)	12612

Table 2. Input parameters θ, P_C, H_b, H_V, D and A_c as fuzzy triangular values

Input parameters as fuzzy triangular values	Defuzzified values
$\tilde{\theta} - (0.1, 0.2, 0.3)$	$d(\tilde{\theta},\tilde{0}) \cdot 0.21$
$\tilde{P}_C - (9000, 10000, 11000)$	$d\left(\tilde{P}_C,\tilde{0}\right) - 9562.5$
$\tilde{H}_b - (6,7,8)$	$d\left(\tilde{H}_b,\tilde{0}\right) - 6.75$
$\tilde{H}_V - (5,6,7)$	$d\left(\tilde{H}_V,\tilde{0}\right) - 5.81$
$\tilde{D} - (3800, 4800, 5800)$	$d(\tilde{D},\tilde{0}) - 4687.5$
$\tilde{A}_C - (20, 25, 30)$	$d\left(\tilde{A}_c,\tilde{0}\right) \cdot 24.375$

Table 3. Optimal solution for fuzzy parameters

L	(Q^*,S^*,N^*)	$S_d(A\tilde{T}C(Q,S,N))$
7250	(196, 30, 1)	12070
	(155, 48, 2)	11960
	(132, 61, 3)	12126
	(117, 73, 4)	12358
	(106, 82, 5)	12610
	(98, 91, 6)	12867
	(91, 99, 7)	13121
5800	(190, 23, 1)	11577
	(150, 37, 2)	11533
	(128, 48, 3)	11735
	(114, 56, 4)	11992
	(103, 64, 5)	12263
	(96, 71, 6)	12534
	(90, 77, 7)	12801
4350	(183, 17, 1)	11042
	(145, 27, 2)	11065
	(125, 35, 3)	11304
	(111, 41, 4)	11586
	(101, 47, 5)	11875
	(94, 52, 6)	12162
	(88, 57, 7)	12441

Table 4. Summary of the optimal solution for Crisp and Fuzzy parameters

L	(Q,S,N)	ATC (Q,S,N)	(Q^*,S^*,N^*)	$S_d(A\tilde{T}C$ $(Q,S,N))$
7250	(202, 30, 1)	12199	(196, 30, 1)	12070
	(158, 48, 2)	12099	(155, 48, 2)	11960
	(135, 61, 3)	12273	(132, 61, 3)	12126
	(120, 72, 4)	12514	(117, 73, 4)	12358
	(109, 82, 5)	12772	(106, 82, 5)	12610
	(100, 90, 6)	13035	(98, 91, 6)	12867
	(93, 98, 7)	13295	(91, 99, 7)	13121
5800	(195, 23, 1)	11706	(190, 23, 1)	11577
	(153, 37, 2)	11672	(150, 37, 2)	11533
	(131, 47, 3)	11882	(128, 48, 3)	11735
	(116, 56, 4)	12147	(114, 56, 4)	11992
	(106, 64, 5)	12424	(103, 64, 5)	12263
	(98, 71, 6)	12701	(96, 71, 6)	12534
	(91, 77, 7)	12973	(90, 77, 7)	12801
4350	(188, 17, 1)	11172	(183, 17, 1)	11042
	(148, 27, 2)	11203	(145, 27, 2)	11065
	(127, 34, 3)	11450	(125, 35, 3)	11304
	(113, 41, 4)	11740	(111, 41, 4)	11586
	(103, 47, 5)	12036	(101, 47, 5)	11875
	(95, 52, 6)	12328	(94, 52, 6)	12162
	(89, 56, 7)	12612	(88, 57, 7)	12441

Table 5. Effects of deterioration rate θ on optimal solutions

Different Values of θ	Fuzzy triangular values	Defuzzified values $d(\tilde{\theta},\tilde{0})$	L	(Q,S,N)	$ATC(Q,S,N)$	(Q,S^*,N^*)	$S_d(A\tilde{T}q(\text{QSN}))$	Savings %
0.4	(0.3, 0.4, 0.5)	0.394	7250	(120, 36, 1)	13488	(121, 36, 1)	13452	0.3
			5800	(117, 28, 1)	13022	(118, 28, 1)	12986	0.3
			4350	(114, 21, 1)	12515	(115, 21, 1)	12480	0.3
0.5	(0.4, 0.5, 0.6)	0.488	7250	(109, 33, 1)	14071	(110, 33, 1)	14004	0.5
			5800	(107, 26, 1)	13590	(108, 26, 1)	13525	0.5
			4350	(104, 19, 1)	13070	(105, 19, 1)	13006	0.5
0.7	(0.6, 0.7, 0.8)	0.675	7250	(94, 28, 1)	15097	(95, 29, 1)	14977	0.8
			5800	(92, 22, 1)	14595	(93, 22, 1)	14477	0.8
			4350	(90, 16, 1)	14053	(91, 16, 1)	13937	0.8
0.9	(0.8, 0.9, 1)	0.863	7250	(83, 25, 1)	15996	(85, 25, 1)	15837	1.0
			5800	(81, 20, 1)	15476	(83, 20, 1)	15320	1.0
			4350	(80, 14, 1)	14917	(82, 15, 1)	14764	1.0

Table 6. Effects of production rate P_C on optimal solutions

% of change	Fuzzy triangular values	Defuzzified values $d\left(\tilde{P}_C,\tilde{0}\right)$	L	(Q,S,N)	$ATC(Q,S,N)$	(Q^*,S^*,N^*)	$\text{S}_d(A\tilde{T}Q(\text{Q,S,N}))$	Savings %
+50%	(12000, 15000, 18000)	14625	7250	(101, 76, 5)	13175	(102, 77, 5)	13155	0.2
			5800	(98, 60, 5)	12817	(99, 60, 5)	12797	0.2
			4350	(96, 44, 5)	12418	(96, 44, 5)	12399	0.2
+25%	(10000, 12500, 15000)	12187.50	7250	(115, 70, 4)	12694	(116, 70, 4)	12676	0.1
			5800	(113, 54, 4)	12323	(113, 54, 4)	12305	0.1
			4350	(110, 40, 4)	11911	(110, 40, 4)	11894	0.1
-25%	(7000, 7500, 8000)	7125	7250	(140, 63, 3)	12098	(141, 64, 3)	12060	0.3
			5800	(135, 49, 3)	11712	(137, 49, 3)	11675	0.3
			4350	(131, 36, 3)	11285	(132, 36, 3)	11249	0.3
-50%	(4500, 5000, 5500)	4781.25	7250	(152, 69, 3)	11725	(154, 69, 3)	11671	0.5
			5800	(147, 53, 3)	11351	(149, 54, 3)	11299	0.5
			4350	(142, 39, 3)	10935	(144, 39, 3)	10885	0.5

Table 7. Effects of buyer's holding cost H_b on optimal solutions

% of change	Fuzzy triangular values	Defuzzified values $d\left(\tilde{H}_b,\tilde{0}\right)$	L	(Q,S,N)	ATC (Q,S,N)	(Q^*,S^*,N^*)	$\text{S}_d(A\tilde{T}Q$ $(\text{Q,S,N}))$	Savings %
+50%	(9.5,10.5,11.5)	10.031	7250	(187, 28,1)	12538	(189, 28, 1)	12494	0.4
			5800	(181, 22,1)	12035	(183, 22, 1)	11992	0.4
			4350	(175, 16,1)	11490	(177, 16, 1)	11449	0.4
+25%	(7.75,8.75,9.75)	8.391	7250	(194, 29,1)	12372	(195, 29, 1)	12337	0.3
			5800	(188, 23,1)	11874	(189, 23, 1)	11840	0.3
			4350	(182, 16,1)	11334	(183, 17, 1)	11301	0.3
-25%	(4.25,5.25,6.25)	5.109	7250	(210, 32,1)	12019	(211, 32, 1)	12004	0.1
			5800	(203, 24,1)	11532	(204, 25, 1)	11518	0.1
			4350	(196,18,1)	11004	(197, 18, 1)	10990	0.1
-50%	(3,3.5,4)	3.375	7250	(220, 33,1)	11830	(221, 33, 1)	11817	0.1
			5800	(212, 26,1)	11350	(213, 26, 1)	11337	0.1
			4350	(205, 18,1)	10829	(206, 19, 1)	10816	0.1

Table 8. Effects ofvendor's holding cost H_V on optimal solutions

% of change	Fuzzy triangular values	Defuzzified values $d\left(\tilde{H}_V,\tilde{0}\right)$	L	(Q,S,N)	ATC (Q,S,N)	(Q^*,S^*,N^*)	$S_d(A\tilde{T}Q$ (Q,S,N))	Savings %
+50%	(8, 9, 10)	8.625	7250	(195, 29,1)	12342	(196, 29, 1)	12324	0.15
			5800	(189, 23,1)	11844	(190, 23, 1)	11827	0.14
			4350	(183, 16,1)	11306	(183, 17, 1)	11289	0.15
+25%	(7, 7.5, 8)	7.125	7250	(198, 30,1)	12271	(199, 30, 1)	12253	0.15
			5800	(192,23,1)	11776	(193, 23, 1)	11759	0.14
			4350	(185, 17,1)	11240	(186, 17, 1)	11223	0.15
-25%	(4, 4.5, 5)	4.313	7250	(205, 31,1)	12126	(206, 31, 1)	12116	0.08
			5800	(198, 24,1)	11636	(199, 24, 1)	11627	0.08
			4350	(191, 17,1)	11104	(192, 17, 1)	11095	0.08
-50%	(2.5, 3, 3.5)	2.91	7250	(209, 31,1)	12051	(208, 31, 1)	12047	0.03
			5800	(202, 24,1)	11564	(202, 24, 1)	11559	0.04
			4350	(195, 18,1)	11034	(195, 18, 1)	11030	0.04

Table 9. Effects of demand D on optimal solutions

% of change	Fuzzy triangular values	Defuzzified values $d\left(\tilde{D},\tilde{0}\right)$	L	(Q,S,N)	ATC (Q,S,N)	(Q^*,S^*,N^*)	$S_d(A\tilde{T}Q$ (Q,S,N))	Savings %
+50%	(76200,7200, 8200)	14625	7250	(170, 51,3)	15597	(166, 52, 3)	15251	2.2
			5800	(165,40,3)	15181	(162, 40, 3)	14837	2.3
			4350	(161, 29,3)	14724	(158, 30, 3)	14383	2.3
+25%	(5000,6000, 7000)	12187.50	7250	(173, 42,2)	13913	(171, 43, 2)	13634	2.0
			5800	(169, 33,2)	13467	(166, 33, 2)	13191	2.0
			4350	(164, 24,2)	12981	(162, 24, 2)	12708	2.1
-25%	(2600,3600, 4600)	7125	7250	(188, 38,1)	10142	(187, 38, 1)	10077	0.6
			5800	(181, 29,1)	9681	(180, 29, 1)	9616	0.7
			4350	(173, 21,1)	9177	(173, 21, 1)	9113	0.7
-50%	(2000,2400, 2800)	4781.25	7250	(169, 51,1)	7976	(168, 52, 1)	7834	1.8
			5800	(161, 39,1)	7557	(160, 40, 1)	7419	1.8
			4350	(153, 28,1)	7094	(151, 28, 1)	6959	1.9

Table 10. Effects of buyer's ordering cost A_C on optimal solutions

% of change	Fuzzy triangular values	Defuzzified values $d\left(\tilde{A}_C,\tilde{0}\right)$	L	(Q,S,N)	ATC (Q,S,N)	(Q^*,S^*,N^*)	$S_d(A\tilde{T}Q$ (Q,S,N))	Savings %
+50%	(35,37.5,40)	35.63	7250	(215, 32,1)	12488	(213, 32, 1)	12446	0.3
			5800	(208, 25,1)	12005	(206, 25, 1)	11962	0.4
			4350	(202, 18,1)	11481	(200, 18, 1)	11436	0.4
+25%	(30,31.25,32.25)	29.53	7250	(208, 31,1)	12346	(207, 31, 1)	12306	0.3
			5800	(202, 24,1)	11858	(200, 24, 1)	11817	0.3
			4350	(195, 18,1)	11329	(193, 17, 1)	11287	0.4
-25%	(18,18.75,19.5)	17.72	7250	(194, 29,1)	12047	(193, 29, 1)	12021	0.2
			5800	(188, 23,1)	11549	(187, 22, 1)	11522	0.2
			4350	(181, 16,1)	11009	(180, 16, 1)	10982	0.2
-50%	(12,12.5,13)	11.81	7250	(187, 28,1)	11889	(186, 28, 1)	11871	0.2
			5800	(180, 22,1)	11385	(179, 22, 1)	11367	0.2
			4350	(174, 16,1)	10840	(173, 16, 1)	10820	0.2

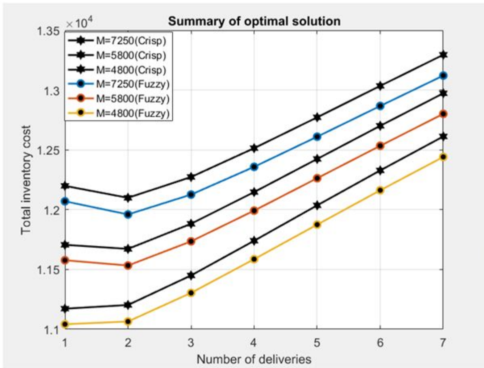

Figure 1. Summary of optimal solution

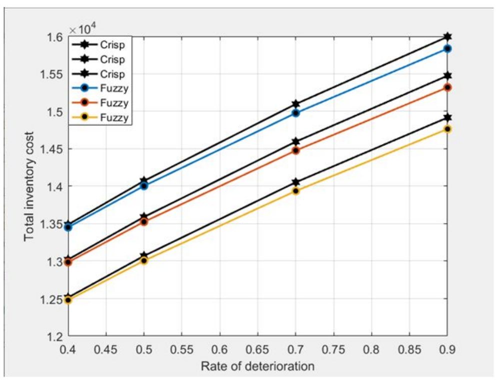

Figure 2. Effects of deterioration rate θ on optimal solutions

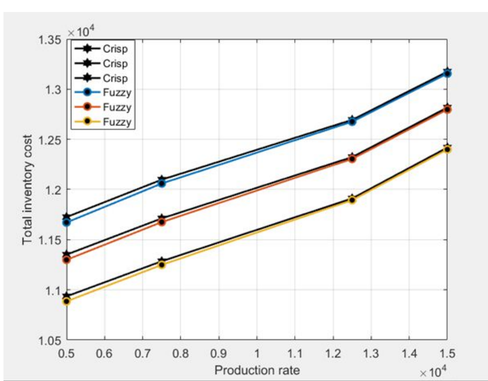

Figure 3. Effects of Effects of production rate P_C on optimal solution

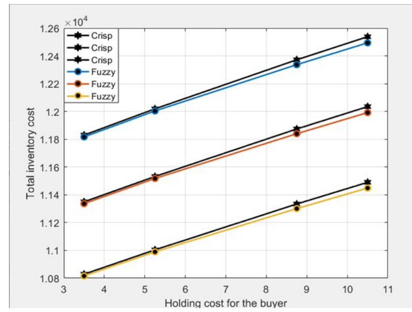

Figure 4. Effects ofbuyer's holding cost H_b on optimal solution

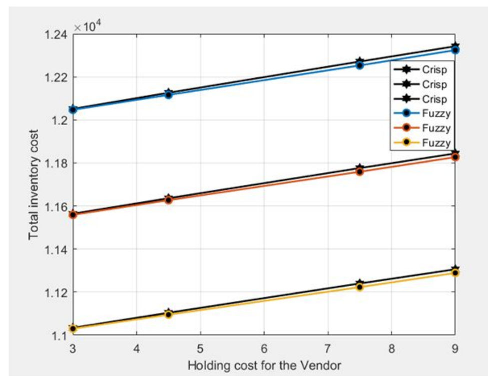

Figure 5. Effects of vendor's holding cost H_V on optimal solution

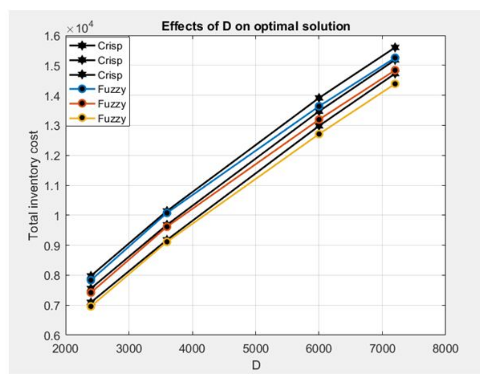

Figure 6. Effects of demand D on optimal solutions

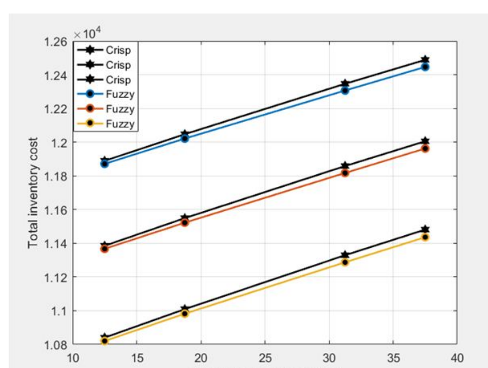

Figure 7. Effects of buyer's ordering cost A_C on optimal solutions

8. Conclusion

In real world applications, the input cost and other parameters in the EOQ inventory model may not be precisely known or it may be uncertain due to some unmanageable factors. To effectively handle imprecision or uncertainty, fuzzy methodologies provide a useful way to model vagueness in human recognition and judgment.Fuzzy numbers are frequently used in applications to ensure easy handling of the realistic problem.

This paper proposes an integrated production-distribution inventory system for deteriorating products in fuzzy environment. The fuzziness in the cost components, production rate, deterioration rate and demand are represented as the triangular fuzzy numbers and the setup cost can be reduced by means of extra investment. Signed distance method is applied to defuzzify the total cost and differential calculus optimization technique is adopted to find the optimal solutions of the model. Numerical example and sensitivity analysis are provided to highlight the differences between crisp and the fuzzy cases.

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ISSN(P):2319 − 3786
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ISSN(O):2321 − 5666
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