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Double Mersenne number in maximum and minimum matrices

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Abstract
We define the double Mersenne minimum matrices and double Mersenne maximum matrices separately. We calculate the determinant and inverse of double Mersenne minimum matrices and double Mersenne maximum matrices by using arithmetical functions.

Keywords
Minimum matrix, Maximum matrix, Double Mersenne Minimum matrix, Double Mersenne Maximum matrix.

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15A09, 15A15.

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Contents

1. Introduction

Let $(P,\preccurlyeq)=(P,\Lambda,\vee)$ be a lattice, let $S=\{x_1,x_2,x_3,\dots\dots,x_n\}$ be a meet- closed subset of P and let $f:P\to Z^+$ be a function. The minimum matrix $(S)_f$ and the maximum matrix $[\mathrm{S}]_f$ on S with respect to f are [1] defined by

$$(S)_f=\min(x_i,x_j)\ \text{ and }\ [S]_f=\max[x_i,x_j]$$

It is well known that $(Z_+,|)=(Z_+,\min,\max)$ is a lattice, where $|$ is the usual divisibility relation for the minimum number and the maximum number of integers. Thus minimum and maximum matrices [2] are generalizations of Minimum matrices $(\ (\mathrm{S})_f)_{ij}=\min(x_i,x_j)$ and Maximum matrices $([S]_f)_{ij}=\max[x_i,x_j]$

2. Strong Finiteness of Double Mersenne Number Minimum Matrices

Definition 2.1. *(Minimum Matrix) A partially ordered set (poset) is a pair $(P,\preceq)$, where P is a nonempty set and $\preceq$ is a reflexive, antisymmetric and transitive relation. A closed interval $[x,y]$ in P is the set $[x,y]=\{z\in P/x\leq z\leq y\},x,y\in P$. Poset $(P,\preceq)$ is said to be locally finite if the interval $[x,y]$ is finite for all $x,y\in P$. Poset $(P,\preceq)$ is a chain [3] if $x\leq y$ or $y\leq x$ for all $x,y\in P$. Let $S=\{x_1,x_2,x_3,\dots\dots\dots,x_n\}$ be a subset of P and let f be a complex valued function on P. then the $n\times n$ matrices $(S)_f=(f_{ij})$ where $f_{ij}=[\min(x_i,x_j)]]$ is called Minimum matrix on S with respect to f.*

Definition 2.2. *(Maximum Matrix) The Poset $(P,\preccurlyeq)$ is said to be locally finite if the interval $[x,y]$ is finite for all $x,y\in P$. Poset $(P,\preccurlyeq)$ is a chain if $x\leqslant y$ or $y\leqslant x$ for all $x,y\in P$. Let $S=\{x_1,x_2,x_3,\dots\dots\dots,x_n\}$ be a subset of P and let f be a complex valued function on P then the $n\times n$ matrices $[S]_f=(f_{ij})$ where $f_{ij}=f\left[\max\left(\mathrm{x_i},\mathrm{x_j}\right)\right]$ is called Maximum matrix on S with respect to f.*

Remark 2.3. *Remark: We begin by presenting the definition of Minimum matrix. Let $T=\{\mathrm{z}_1,\mathrm{z}_2,\mathrm{z}_3,\dots\dots\dots,\mathrm{z_n}\}$ be a finite multiset of real numbers, where $\mathrm{z}_1\leq\mathrm{z}_2\leq\dots\dots\dots\leq\mathrm{z}_n$ (in some cases, however, we need to assume that $\mathrm{z}_1<$*

$z_2 < \dots\dots\dots < z_n$). *The Minimum matrix $T_{\min}$ of the set T has $\min(z_i, z_j)$ as its ij entry. Whereas the Maximum matrix $T_{\max}$ of the set T has $\max(z_i, z_j)$ as its ij entry. Both the matrices[4] are clearly square and symmetric and they may be written explicitly as,*

$$T_{\min} = \begin{bmatrix} z_1 & z_1 & z_1 & \cdots & z_1 \\ z_1 & z_2 & z_2 & \cdots & z_2 \\ z_1 & z_2 & z_3 & \cdots & z_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_1 & z_2 & z_3 & \cdots & z_n \end{bmatrix},$$

$$T_{\max} = \begin{bmatrix} z_1 & z_2 & z_3 & \cdots & z_n \\ z_2 & z_2 & z_3 & \cdots & z_n \\ z_3 & z_3 & z_3 & \cdots & z_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_n & z_n & z_n & \cdots & z_n \end{bmatrix}$$

Definition 2.4. *(Double Mersenne Number) If the exponent of a Mersenne number is a Mersenne prime [5] M_P, then $MM_P = 2^{M_P} - 1$ is called the Double Mersenne number.*

Let $S = \{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and let S be a set of a lattice $(P, \preccurlyeq)$. Then the Double Mersenne matrix with respect to a complex valued function then the $n \times n$ matrix is denoted by $[\mathrm{A}]$ and defined as $[\mathrm{A}] = (a_{ij})$, where, $a_{ij} = 2^{2^{(x_i x_j)}-1} - 1$, and $2^{(x_i, x_j)} - 1$ is called as the Mersenne number.

Definition 2.5. *(Double Mersenne Minimum Matrix) Let $S = \{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and S be a set of a lattice $(P, \preccurlyeq)$. The Double Mersenne Minimum matrix with respect to a complex valued function f is defined by $(S)_f = (d_{ij})$, where $d_{ij} = 2^{2^{\min(x_i, x_j)}-1} - 1$, and $2^{\min}(x_i, x_j) - 1$ is known as Mersenne Minimum number.*

Definition 2.6. *(Double Mersenne Maximum Matrix) Let $S = \{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and S be a set of a lattice $(P, \preccurlyeq)$. The Double Mersenne Maximum matrix with respect to a complex valued function f is defined by $[S]_f = (g_{ij})$, where $g_{ij} = 2^{2^{\max(x_i, x_j)}-1} - 1$, and $2^{\max}(x_i, x_j) - 1$ is known as Mersenne Maximum number.*

3. Determinant Value of Double Mersenne Minimum and Maximum Matrices

Let $S = \{x_1, x_2, \dots\dots\dots x_n\}$ be a Minimum closed subset of P. Then the determinant of the Double Mersenne Minimum and Maximum matrices is denoted by $(S)_f$ and $[S]_f$ are defined as,

$$\begin{aligned} \det(S)_f =& f(x_1)\{[f(x_2) - f(x_1)][f(x_3) - f(x_2)] \\ & \dots\dots\dots [f(x_n) - f(x_{n-1})]\} \\ \det[S]_f =& [f(x_1) - f(x_2)][f(x_2) - f(x_3)] \\ & \dots\dots\dots [f(x_{n-1}) - f(x_n)] f(x_n) \end{aligned}$$

where $f(x_i) = 2^{2^{(x_i)}-1} - 1$.

4. Inverse of Double Mersenne Minimum and Maximum Matrices

(i) Suppose that the elements of the set $(S)_f$ are distinct. If $x_1 \neq 0$, then the Mersenne Minimum matrix is invertible and the inverse matrix is the $n \times n$ tridiagonal matrix $(S)_f^{-1} = R = (r_{ij})$, where

$$(S)_f^{-1} = \begin{cases} 0 & \text{if } |i-j| > 1 \\ \frac{f(x_2)}{f(x_1)[f(x_2) - f(x_1)]} & \text{if } i = j = 1 \\ \frac{1}{f(x_i) - f(x_{i-1})} + \frac{1}{f(x_{i+1}) - f(x_i)} & \text{if } 1 < i = j < n \\ \frac{1}{f(x_n) - f(x_{n-1})} & \text{if } i = j = n \\ \frac{1}{|f(x_i) - f(x_j)|} & \text{if } |i-j| = 1 \end{cases}$$

where $f(x_i) = 2^{2^{(x_i)}-1} - 1, \quad i = 1, 2, \dots\dots n.$

(ii) Suppose that the elements of the set $[S]_f$ are distinct. If $x_n \neq 0$, then the S- prime Maximum matrix is invertible and the inverse matrix is the $n \times n$ tridiagonal matrix $[S]_f^{-1} = T = (t_{ij})$, where

$$[S]_f^{-1} = \begin{cases} 0 & \text{if } |i-j| > 1 \\ \frac{1}{[f(x_1) - f(x_2)]} & \text{if } i = j = 1 \\ \frac{1}{f(x_{i-1}) - f(x_i)} + \frac{1}{f(x_i) - f(x_{i+1})} & \text{if } 1 < i = j < n \\ \frac{1}{f(x_{n-1}) - f(x_n)} + \frac{1}{f(x_n)} & \text{if } i = j = n \\ \frac{1}{|f(x_i) - f(x_j)|} & \text{if } |i-j| = 1 \end{cases}$$

where $f(x_i) = 2^{2^{(x_i)}-1} - 1, i = 1, 2, \dots\dots n.$

Example 4.1. *If $S = \{2, 3\}$ is a lower closed set. Construct 2×2 Double Mersenne minimum matrix on S and also find its inverse.*

Solution: *By the definition of Double Mersenne minimum matrix, $(S)_f = (d_{ij})$, where $d_{ij} = 2^{2^{\min(x_i x_j)}-1} - 1$.*

$$\begin{aligned} \therefore (S)_f = (d_{ij}) &= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \\ &= \begin{bmatrix} 2^{2^{\min(2,2)}-1} - 1 & 2^{2^{\min(2,3)}-1} - 1 \\ 2^{2^{\min(3,2)}-1} - 1 & 2^{2^{\min(3,3)}-1} - 1 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 7 \\ 7 & 127 \end{bmatrix}. \end{aligned}$$

The determinant value of the double Mersenne minimum matrix (by theorem) is,

$$f(x_1) = 7, f(x_2) = 127.$$

By theorem,

$$\det(S)_f = f(x_1)[f(x_2) - f(x_1)] = 840.$$

The inverse of the Double Mersenne minimum matrix is,

$$(S)_f^{-1} = R = (r_{ij}) = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

Here, $r_{11}=\frac{127}{840}, r_{12}=\frac{1}{120}, r_{21}=\frac{1}{120}, r_{22}=\frac{1}{120}$

$$\therefore (S)_f^{-1}=\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}=\begin{bmatrix} \frac{127}{840} & \frac{1}{120} \\ \frac{1}{120} & \frac{1}{120} \end{bmatrix}.$$

By the definition of Double Mersenne maximum matrix,

$$[S]_{f_f}=(g_{ij}), \quad \text{where} \quad g_{ij}=2^{2^{\max(x_i,x_j)}-1}-1$$

$$\therefore [S]_f=(g_{ij})=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$
$$=\begin{bmatrix} 2^{2^{\max(2,2)}-1}-1 & 2^{2^{\max(2,3)}-1}-1 \\ 2^{2^{\max(3,2)}-1}-1 & 2^{2^{\max(3,3)}-1}-1 \end{bmatrix}$$
$$=\begin{bmatrix} 7 & 127 \\ 127 & 127 \end{bmatrix}.$$

The determinant value of the double Mersenne maximum matrix (by theorem) is,

$$f(x_1)=7, f(x_2)=127$$
$$\therefore \det[S]_f=[f(x_1)-f(x_2)]\,f(x_2)=-15240.$$

The inverse of the Double Mersenne maximum matrix is,

$$[S]_f^{-1}=T=(t_{ij})=\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}.$$

Here, $t_{11}=-\frac{1}{120}, t_{12}=\frac{1}{120}, t_{21}=\frac{1}{120}, t_{22}=\frac{-7}{15240}$.

$$\therefore [S]_f^{-1}=\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}=\begin{bmatrix} \frac{-1}{120} & \frac{1}{120} \\ \frac{1}{120} & \frac{-7}{15240} \end{bmatrix}.$$

Example 4.2. *If* $S=\{1,2,3\}$ *is a lower closed set. Construct* 3×3 *Double Mersenne minimum matrix on* S *and also find its inverse.*

Solution:

By the definition of Double Mersenne minimum matrix,

$$(S)_f=(d_{ij}), \text{ where } d_{ij}=2^{2^{\min(x_i x_j)}-1}-1$$

$$\therefore (S)_f=(d_{ij})=\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 7 & 7 \\ 1 & 7 & 127 \end{bmatrix}.$$

The determinant value of the double Mersenne minimum matrix (by theorem) is,

$$f(x_1)=1, f(x_2)=7, f(x_3)=127.$$

$$\therefore \det(S)_f=f(x_1)[f(x_2)-f(x_1)][f(x_3)-f(x_2)]=720.$$

The inverse of the Double Mersenne minimum matrix is,

$$(S)_f^{-1}=R=(r_{ij})=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}.$$

Here, $r_{11}=\frac{7}{6}, r_{12}=\frac{1}{6}, r_{13}=0, r_{21}=\frac{1}{6}, r_{22}=\frac{7}{40}, r_{23}=\frac{1}{120}, r_{31}=0, r_{32}=\frac{1}{120}, r_{33}=\frac{1}{120}$.

$$\therefore (S)_f^{-1}=\begin{bmatrix} \frac{7}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{7}{40} & \frac{1}{120} \\ 0 & \frac{1}{120} & \frac{1}{120} \end{bmatrix}.$$

By the definition of Double Mersenne maximum matrix,

$$[S]_f=(g_{ij}), \quad \text{where } g_{ij}=2^{2^{\max(x_i,x_j)-1}-1}$$

$$\therefore [S]_f=\begin{bmatrix} 1 & 7 & 127 \\ 7 & 7 & 127 \\ 127 & 127 & 127 \end{bmatrix}$$

$$\det[S]_f=[f(x_1)-f(x_2)][f(x_2)-f(x_3)]\,f(x_3)=91440.$$

The inverse of the Double Mersenne maximum matrix is,

$$[S]_f^{-1}=T=(t_{ij})=\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}.$$

Here, $t_{11}=\frac{1}{6}, t_{12}=\frac{1}{6}, t_{13}=0, t_{21}=\frac{1}{6}, t_{22}=\frac{-7}{40}, t_{23}=\frac{1}{120}, t_{31}=0, t_{32}=\frac{1}{120}, t_{33}=\frac{-7}{15240}$.

$$\therefore [S]_f^{-1}=\begin{bmatrix} \frac{1}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{-7}{40} & \frac{1}{120} \\ 0 & \frac{1}{120} & \frac{-7}{15240} \end{bmatrix}.$$

5. Characteristic of Double Mersenne Minimum Matrices

1. If a Double Mersenne Minimum matrix contains prime number then the double Mersenne Minimum matrix is called Double Mersenne Prime Minimum matrix.
2. If Mersenne Prime matrices are infinite then the Double Mersenne minimum matrix is also Infinite matrix.
3. Exponents of all Mersenne Prime matrices are called basic sequence of number of double Mersenne Prime matrices.
4. The original continuous prime number sequence of Double Mersenne Prime matrices is $p=2,3,5,7$.
5. Double Mersenne Minimum matrices are strongly finite, since the sum of the original prime number sequence of double Mersenne prime is $2+3+5+7=17$ is a Fermat prime.

6. Conclusion

In this paper, the different properties of Double Mersenne Minimum and Double Mersenne Maximum matrices of the set S with $\min(x_i,x_j)$ and $\max(x_i,x_j)$ as their (i,j) entries like determinant value and inverse of Double Mersenne Minimum and Double Mersenne Maximum matrices have been studied. The study is carried out by applying known results of meet and joins matrices to Mersenne minimum and Mersenne maximum matrices.

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