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Odd triangular graceful labeling on simple graphs

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Abstract
In 2001, Devaraj et. al. [1] defined Triangular graceful graphs. We shall define an odd triangular graceful graphs as follows:Let $V(G)$ and $E(G)$ denote the vertex set and edge set of the graph G respectively. Consider an injective function $f: V(G) \to \{0,1,2,\ldots,T_{2q-1}\}$, where q is the number of edges of G and T_i is the i^{th} triangular number. That is $T_1 = 1, T_2 = 3, T_3 = 6$ etc., and $T_n = \frac{n(n+1)}{2}$. If the function f induces the function f^* on $E(G)$ such that $f^*(uv) = \mid f(u) - f(v) \mid$ for all edges $(uv) \in E(G)$ with $\{f^*(E(G))\} = \{T_1, T_3, \ldots, T_{2q-1}\}$, we say that f is an odd triangular graceful graph and a graph which admits such a labeling is called an odd triangular graceful labeling.

Keywords
Star graph, double star graph, path graphs, odd triangular graceful graphs, triangular graceful graphs.

AMS Subject Classification
05C38, 05C78.

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1. Introduction

Unless specified otherwise, a graph in this paper shall mean a simple finite graph without isolated vertices. In 1967, Rosa [7] introduced the β-valuation of a graph G. Golomb [4] called such labeling graceful and this terminology is now commonly used. In 2001, Devaraj et al. [1] defined triangular graceful graph as follows: Let G be a (p,q) graph. Let $V(G)$, $E(G)$ respectively the vertex set and edge set of G. Consider an injective function $f: V(G) \to X$ where $X = \{0,1,2,\ldots,T_q\}$ where T_q is the q^{th} triangular number and q is the number of edges of G. That is $T_1 = 1, T_2 = 3, T_3 = 6$ etc., and $T_n = \frac{n(n+1)}{2}$. Define the function $f^*: E(G) \to \{1,2,3,\ldots,T_q\}$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges (uv). If $f^*(E(G))$ is a sequence of distinct consecutive triangular numbers say $\{T_1, T_2, \ldots, T_q\}$ then the function is said to be triangular graceful graph and a graph which admits such a labeling is called a triangular graceful graph. In this paper, we define an odd triangular graceful graphs. Let $V(G)$ and $E(G)$ denote the vertex set and edge set of the graph G respectively. Consider an injective function $f: V(G) \to \{0,1,2,\ldots,T_{2q-1}\}$, where q is the number of edges of G and T_i is the i^{th} triangular number. That is $T_1 = 1, T_2 = 3, T_3 = 6$ etc., and $T_n = \frac{n(n+1)}{2}$. If the function f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $(uv) \in E(G)$ with $\{f^*(E(G))\} = \{T_1, T_3, \ldots, T_{2q-1}\}$, we say that f is an odd triangular graceful and a graph which admits such a labeling is called an odd triangular graceful graph.

2. Main Result

Definition 2.1. *Let $V(G)$ and $E(G)$ denote the vertex set and edge set of the graph G respectively. Consider an injective function $f: V(G) \to \{0,1,2,\ldots,T_{2q-1}\}$, where q is the number of edges of G and T_i is the i^{th} triangular number. That is $T_1 = 1, T_2 = 3, T_3 = 6$ etc., and $T_n = \frac{n(n+1)}{2}$. If the function f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all edges $(uv) \in E(G)$ with $\{f^*(E(G))\} = \{T_1, T_3, \ldots, T_{2q-1}\}$, we say that f is an odd triangular graceful and a graph which admits such a labeling is called an odd triangular graceful graph.*

Remark 2.2. *Even though triangular graceful labeling is an analog of graceful labeling, every graceful graphs are not be triangular graceful. Similarly every triangular graceful graphs need not be graceful.*

Remark 2.3. *There are some graphs which are triangular* *graceful graphs but not odd triangular graphs.*

Theorem 2.4. *All paths P_n are odd triangular graceful graphs.*

Proof. Let G be a path with n vertices. Let $v_1, v_2, \dots, v_n$ be the vertices of G. Clearly G has $q = n-1$ edges.Let T_q is the q^{th} triangular number.
Define $f : V(G) \to \{0, 1, 2, \dots, T_{2q-1}\}$ as follows:

$$\begin{aligned} f(v_1) &= T_{2q-1}, f(v_2) = 0, f(v_3) = 1 \\ f(v_{i+3}) &= f(v_{i+2}) + (-1)^{i+1} T_{2q-(2i+1)},\ 1 \le i \le q-2 \\ f(v_n) &= f(v_{n-1}) + (-1)^n T_3. \end{aligned}$$

Therefore, f is injective and f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all

$$(uv) \in E(G) = \{T_1, T_3, \dots, T_{2q-1}\}.$$

Thus $\{f^*(E(G))\} = \{T_1, T_3, \dots, T_{2q-1}\}$ Therefore f is an odd triangular graceful labeling. Hence G is an odd triangular graceful graph. □

Example 2.5.

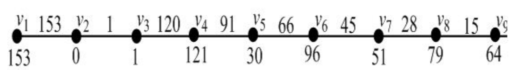

Figure 1. P_{10} is an odd triangular graceful graph

Theorem 2.6. *All stars $K_{1,n}$ are odd triangular graceful graphs.*

Proof. Let $v_0, v_1, v_2, \dots, v_n$ be the vertices of star $K_{1,n}$. The edge set of $K_{1,n}$ is, $E(K_{1,n}) = \{v_0 v_i / 1 \le i \le n\}$. Then $K_{1,n}$ has $n+1$ vertices and $q = n$ edges.Let T_q is the q^{th} triangular number.
Define $f : V(K_{1,n}) \to \{0, 1, 2, \dots, T_{2q-1}\}$ as follows:

$$\begin{aligned} f(v_0) &= 0, f(v_i) = T_{2i-1}, 1 \le i \le n-1, \\ f(v_n) &= T_{2n-1} = T_{2q-1}. \end{aligned}$$

Then f is injective, f induces the function f^* on $E(K_{1,n})$ such that $f^*(uv) = |f(u) - f(v)|$ for all $(uv) \in E(K_{1,n}) = \{T_1, T_3, \dots, T_{2q-1}\}$. Thus

$$\{f^*(E(K_{1,n}))\} = \{T_1, T_3, \dots, T_{2q-1}\}$$

and f is an odd triangular graceful labeling Hence $K_{1,n}$ is an odd triangular graceful graph. □

Example 2.7.

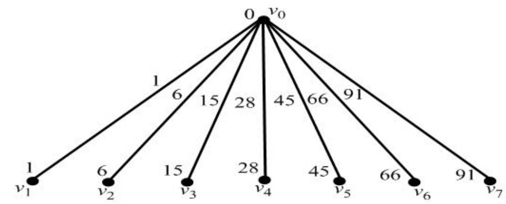

Figure 2. S_7 is an odd triangular graceful graph

Theorem 2.8. *The double star $S(n,m)$ is an odd triangular graceful graph.*

Proof. Let the double star $S(m,n)$ has vertex set

$$\{u_0, u_1, u_2, \dots, u_n\} \cup \{v_0, v_1, v_2, \dots, v_m\}$$

and the edge set

$$E(\mathrm{S(m,n)}) = \{u_0 v_0\} \cup \{u_0 u_i / 1 \le i \le n\} \cup \{v_0 v_i / 1 \le i \le m\}.$$

Then $S(m,n)$ has $n+m+2$ vertices and $q = n+m+1$ edges. Let T_q is the q^{th} triangular number.
Define $f : V(\mathrm{S(m,n)}) \to \{0, 1, 2, \dots, T_{2q-1}\}$ as follows:

$$\begin{aligned} f(u_0) &= T_1, f(u_i) = T_{2i+1} + 1, 1 \le i \le n \\ f(v_0) &= 0 f(v_i) = T_{2n+2i+1}, 1 \le i \le m \\ f(u_n) &= T_{2n+1} + 1. \end{aligned}$$

Therefore, f is injective, f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all

$$(uv) \in E(S(\mathrm{m,n})) = \{T_1, T_3, \dots, T_{2q-1}\}.$$

Thus $\{f^*(E(\mathrm{S(m,n)}))\} = \{T_1, T_3, \dots, T_{2q-1}\}$.
Therefore f is an odd triangular graceful labeling. Hence $S(m,n)$ is an odd triangular graceful graph. □

Example 2.9.

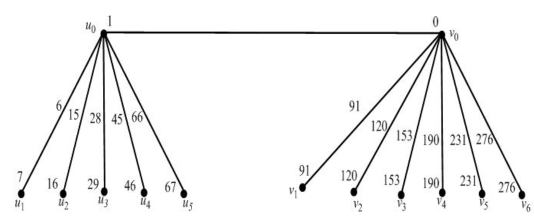

Figure 3. $S(5,6)$ is an odd triangular graceful graph

Theorem 2.10. *$S(n,1,m)$ is an odd triangular graceful graphs.*

Proof. Let $G = S(n,1,m)$ be the double star with one vertex joining the end vertices of the stars. Let

$$\begin{aligned} V(G) = &\{u_0, u_1, u_2, \dots, u_n\} \\ &\cup \{w\} \cup \{v_0, v_1, v_2, \dots, v_m\}, \end{aligned}$$

and

$$E(G) = \{u_0 w, w v_0\} \cup \{u_0 u_i / 1 \le i \le n\} \cup \{v_0 v_i / 1 \le i \le m\}.$$

Number of edges $q = m+n+2$. Also $|V(G)| = m+n+3$. Let T_q is the q^{th} triangular number.
Define $f : V(G) \to \{0, 1, 2, \ldots, T_{2q-1}\}$ as follows:

$$\begin{aligned} f(w) &= 0, f(u_0) = 1 = T_1 \\ f(u_i) &= f(u_0) + T_{2i+1}, 1 \leq i \leq \\ f(v_0) &= T_{2q-1} \\ f(v_i) &= f(v_0) - T_{2n+2i+1}, 1 \leq i \leq m. \end{aligned}$$

The edge values of $u_0 w = T_1$. The edge values of $wv_0 = T_{2q-1}$. The edge values of $u_0 u_i = \{T_3, T_5, \ldots, T_{2n+1}\}, 1 \leq i \leq n$. The edge values of

$$v_0 v_i = \{T_{2n+3}, T_{2n+5}, \ldots, T_{2n+2m+1}\}, 1 \leq i \leq m.$$

Therefore the f is injective, and f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all

$$uv \in E(G) = \{T_1, T_3, \ldots, T_{2q-1}\}.$$

Thus $\{f^*(E(G))\} = \{T_1, T_3, \ldots, T_{2q-1}\}$. Therefore f is an odd triangular graceful labeling. Hence $S(n, 1, m)$ is an odd triangular graceful graph. □

Example 2.11.

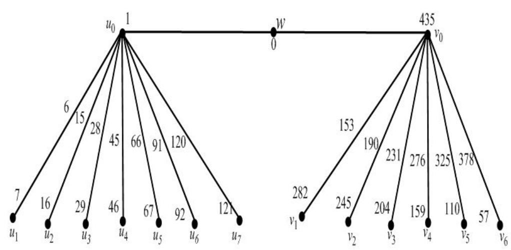

Figure 4. $S(7, 1, 6)$ is an odd triangular graceful graph

Theorem 2.12. *The Fork graph G is an odd triangular graceful graph.*

Proof. Let $V(G) = \{v_i / 1 \leq i \leq 5\}$ be the vertices of G. Let the edge set be $E(G) = \{v_1 v_i / i = 2, 3\} \cup \{v_3 v_i / i = 4, 5\}$. Let T_q is the q^{th} triangular number.
Define

$$f : V(G) \to \{0, 1, 2, \ldots, T_{2q-1}\} = \{0, 1, 2, \ldots, 28\},$$

as follows: $f(v_1) = 1, f(v_2) = 7, f(v_3) = 0,\ f(v_4) = 15,$ $f(v_5) = 28$. Clearly f is injective and f induces the function f^* on $E(G)$ such that $f^*(uv) = |f(u) - f(v)|$ for all $uv \in E(G) = \{T_1, T_3, \ldots, T_{2q-1}\}$. Thus

$$\{f^*(E(G))\} = \{T_1, T_3, \ldots, T_{2q-1}\}.$$

Hence G is an odd triangular graceful graph. □

Example 2.13.

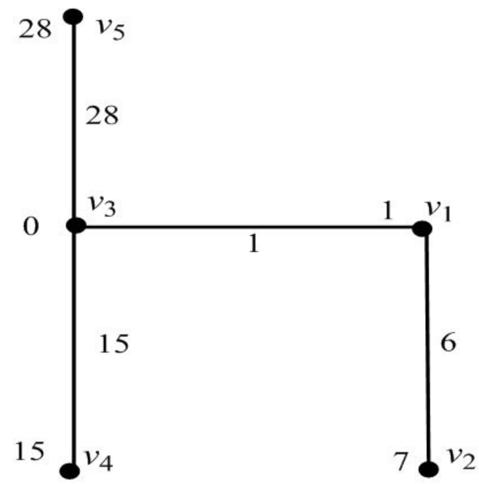

Figure 5. Forkgraph is an odd triangular graceful graph

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